[redacted]

*GSI, Darmstadt, Germany,*

*Moscow Institute for Physics and Engineering, Russia*

**Challenge to describe NS and their evolution:**

one needs construction of bridge from micro to macro world

- 20 orders in distance (from $\sim 10^{-13} cm$ to $\sim 10^3\ km$)
- 15 orders in density (from mean solar density $\sim g/cm^3$ to $10\rho_0$,
  $\rho_0 \simeq 2.7 \cdot 10^{14} g/cm^3 \simeq 0.17 fm^{-3}$ density of atomic nucleus
- $\sim 40$ orders in time (from $10^{-23}\ s$ to $\sim 10^{10} yr$)
- $\sim 60$ orders in mass/energy (from $m_\pi \sim 10^{-25} g$ to $(10 - 50) M_\odot \sim (2 \cdot 10^{34} - 10^{35}) g$

Strong, el. mag., weak, grav. interactions are involved

**Triumph of Universality of Physical Laws**

Lect. I

- Historical remarks
  (important results related to further discussion of physics of NS interiors)
- Relevant info.
  (related to further discussion of physics of NS interiors)
- NS interiors:
  Nucleon Fermi liquid:
  NN interaction
  Pion softening
  Pion condensation
  Other possible phase transitions

Lect. II

- NS cooling
  "Standard" scenario
  "Standard+exotics" scenario
  "Nuclear medium cooling" scenario
  General radiation problem
  Comparison with data

# Historical Remarks

Preliminaries to discovery of NS

- **1932** Discovery of "$n$" by Chadwick.
  NBI seminar: L. Landau suggests possibility of NS (not published, from recollection of L. Rosenfeld 1974).

- **1934** W. Baade, F. Zwicky, Phys. Rev., suggest that SN end evolution as NS.
  NS were considered as an exotics, more seriously was discussed a possibility of $n$ cores in usual stars, as Sun, see L. Landau, Sov. DAN, 1937.

- **1939** J. Oppenheimer, G. Wolkoff, Phys. Rev., considered ideal $n$ gas in grav. field.

**Why NS is compact?**

(Stability: balance of repulsion and attraction)

$$E_{Fn} \sim \frac{p_{Fn}^2}{2m_n} A, \quad E_G \sim -\frac{GM^2}{R}$$

$$p_{Fn} = \hbar(3\pi^2 \rho_n)^{1/3}, \quad \rho_n = \frac{3A}{4\pi R^3}, \quad M \simeq A m_n$$

$$\to p_{Fn} \sim \hbar A^{1/3}/R \to$$

$$R \sim \hbar^2/(GM^{1/3} m_n m_n^{5/3}) \sim 10 km \quad for \quad M \sim M_\odot,$$

Quantum object!

**Why NS is of neutrons?**

$$E_{Fn} \sim \frac{p_{Fn}^2}{2m_n} A \propto A \quad at\ fixed \quad \rho$$

$$E_{sym} \sim (N-Z)^2/A \sim A \quad for \quad N \sim A, \qquad but$$

$$E_{Coul} \sim Z^2 e^2/R \sim Z^2 A^{-1/3} > A \quad if \quad Z \sim A.$$

$\to$ Profitable to suppress $Z$. How much?

# Historical Remarks

- **1933** $n \to p + e + ?$. Energy-momentum dis-balance: N.Bohr suggests energy non-conservation in micro world $\to$ problem with relativity.
  W.Pauli suggests $\nu$: $n \to p + e + \bar{\nu}$.

- **1941** G.Gamov calls "Urca" - process ("Casino da Urca" in Rio, "thief" in Odessa slang). Modern name: "Direct Urca" (DU)
  Energy-momentum balance $n \leftrightarrow p + e$ (for $T = 0$ $\omega_\nu = 0$) $\to p_{Fn} \gg p_{Fp}, E_{Fn} = \frac{p_{Fn}^2}{2m_n} \simeq p_{Fe}$ ($e$ is relativistic))
  Charge neutrality $\to$ $p_{Fp} = p_{Fe} \to$ several % of protons and DU is forbidden up to high densities, as follows from momentum conservation: $p_{Fn} \geq 2p_{Fp}$. $\rho_{c,DU} \sim 5\rho_0$ for realistic EoS like Urbana-Argone EoS.

**Period of courageous theorists**

- **1942** W. Baade et al, suggested NS in Crab Nebula. (Was observed in 1968.)

- **1959** A.B. Migdal suggests $1S_0$ $nn$ pairing in NS. (Now pulsar glitches are associated with superfluidity, cooling)

- **1960** B. Ambarzumian, G. Saakian suggest hyperonization in NS interiors. $n \leftrightarrow \Lambda$, for $\mu_n \simeq E_{Fn} > m_\Lambda - m_n$. (Now hyperonization at $\rho \gtrsim (2 \div 4)\rho_0$ is discussed including interactions.)

- **1965** S. Tsuruta, A. Cameron, and J. Bahcall, R. Wolf: First scenario for NS cooling.

- **1967** F. Pacini, considered emission of rapidly rotating NS (explains nature of future pulsars)

# Historical Remarks

## Experimental Era

- **08.1967** S.J. Bell discovered radio pulsar "LGM1" or "CP". "Little Green Men" LGM1 or Cambridge Pulsar "CP". Precise cosmic clock! Was unpublished during 6 months: May be signal of out of Earth civilization?

- **02. 1968** A. Hewish, S.J. Bell, et al, Nature.
Did not know work of Pacini and thought about white dwarf. Electron gas pressure + gravity:

$$R \sim 10^3 km,$$

$P \sim 1s$ can be explained:

$$\frac{mv^2}{R} < \frac{GMm}{R^2}, \quad v = \frac{2\pi R}{P} \rightarrow R < (GM)^{1/3}(\frac{P}{2\pi})^{2/3}.$$

- **06. 1968** T. Gold Nature 218, F. Pacini Nature 219, suggested that pulsars are rapidly rotating NS.

- **end 1968** Discovery of pulsars in Vela ($P = 88ms$) and Crab ($P = 33\,ms$). Clearly NS, connection to SN

# Historical Remarks

Observation of Crab born: Japan, China 1054

Yang Wei: Exp.: I am humbly prostrating myself. In constellation of Tven Huan (Taurus) I observed the appearance of the guest-star. It was slightly rainbowed. Theory: According to the Emperor order I respectably made a prediction. The quest star will not spoil Aldebaran ($\alpha$-Tauri). This indicates that the country will acquire the great power (finding of I. Shklovskii).

Now $>$ 3000 supernovas are observed. NS are born in SNII type explosions of stars with $M \gtrsim 10 M_\odot$ with a frequency several times per century in the Galaxy.

- **23.02.1987** First registration of $\nu$ from SN by Kamiokande (Japan), IBM(USA), Baksan (USSR): 19 $\nu$ during $\sim 10s$ of $\omega \sim 10 \div 40 MeV$.

  Now Superkamiokande may allow to observe $\sim 10^3 \div 10^4 \nu$ for SN explosion within our Galaxy or in naboring galaxies.

# Historic

## Yet exotics

- **1971** A.B.Migdal, $\pi$ condensate for $\rho \sim \rho_0$, possibility of abnormal superdense nuclei glued by $\pi$ condensate (depends on unknown $\rho$ dependence of Landau-Migdal parameter $g'(\rho)$); **1974** A.B.Migdal, **1976** , A.B.Migdal,G.Sorokin, O.Markin, I.Mishustin: $\pi$ condensate superdense nuclei $N \sim Z$, $A \lesssim 10^2$, neutron nuclei $A \gtrsim 10^3$, **1977** D.V., G.Sorokin, A.Chernoutsan: superdence nuclei-stars of arbitrary size.
- **1971** A.R. Bodmer: $u, d, s$ quark very small size systems.
- **1984** E.Witten: $u, d, s$ strange nuclei-stars (depends on uncertainty in $B$, $m_s$).
- **1974** T.D.Lee, G.Wick: $\sigma$ superdense nuclei.
- **1978** A.B.Migdal, A.Chernoutsan, I.Mishustin, Dynamics of I order phase transition to superdense $\pi$ cond. state.

## Yet exotics

- **1979** V.Berezovoy, I.Krive, E.Chudnovsky: idea of $P$ wave kaon condensation, **1995** E.Kolomeitsev, D.V., B.Kampfer: $P$ wave kaon condensation in NS.
- **1986** D.Kaplan, A.Nelson: $S$ wave kaon condensation.
- **1992** N.Glendenning: mixed phases: $npe \leftrightarrow (\pi_c, K_c, q)$ constructed by Gibbs conditions, **1993** H.Heiselberg, C.Pethick, E.Staubo: Coulomb+surface effects, **2002** T.Tatsumi, D.V., M.Yasuhira: shrinking of mixed phases due to screening effects.
- **1978** S.Frautchi, **1984** D.Balin, A.Love, suggested color superconductivity $\Delta_q \lesssim 1$MeV,
- **1995** M.Dyakonov, H. Forkel, M.Lutz, **1998** M.Alford, K.Rajagopal, F. Wilczek; and R.Rapp, T. Schaefer, E. Shuryak, M. Velkovsky: $\Delta_q \lesssim 100$MeV, variety of phases.

# Classification of Observed NS

There are observed $\sim 1500$ pulsars in the Galaxy.
Expected number of NS in the Galaxy $\sim 10^8 \div 10^9$.

Single and binary systems:

- Radio pulsars

  a) $33\,ms\,(Crab) \lesssim P \lesssim 4s$,
  $\dot{P} \sim 10^{-12} \div 10^{-14} s \cdot s^{-1}$.(Clocks of high accuracy),
  $B_{surf} \sim 10^{11} \div 10^{13} G$ ($B_e \sim m_e^2/e \sim 10^{13} G$, $B_{atom} \sim 10^9 G$).
  Magnetospheric activity generates radio signal. Magnetic and rotation axes do not coincide $\rightarrow$ pulses on the Earth.

  b) Millisecond pulsars ($\sim$ 1/2 in binaries).
  $1.56\,ms \lesssim P \lesssim 10^2\,ms$, $\dot{P} \sim 10^{-19} s \cdot s^{-1}$
  $v = 2\pi R/p \sim 1/10c$. $\rightarrow$ NS is relativistic object,
  $B_{surf} \sim 10^8 \div 10^9 G$,

- X ray binaries.

  a) HMXB: companion $M \sim (10 \div 40) M_\odot$,
  NS: $B_{surf} \sim 10^{12} \div 10^{13} G$, $0.1s \lesssim P \lesssim 0.3h$.
  A short life-time ($\lesssim 10^7 yr$).

  b) LMXB: companion: $M \lesssim 1.2 M_\odot$,
  NS: $B_{surf} \lesssim 10^{10} G$, life-time ($\gtrsim 10^8 yr$).

Masses of NS



different types of systems. Working backward to determine the population from the set of observed systems is also difficult. An interesting recent development has been the application of a Bayesian analysis to the observed double-neutron-star binaries, to determine the number of coalescing systems expected to be seen by gravity-wave observatories for different model assumptions, taking into account survey selection effects (*67*, *68*). Such an approach could in principle be used to constrain evolutionary parameters as well, whether for subpopulations or the full population, but more examples of observed systems will likely be needed before this can be truly constraining. This is a strong motivation for carrying out further pulsar searches.

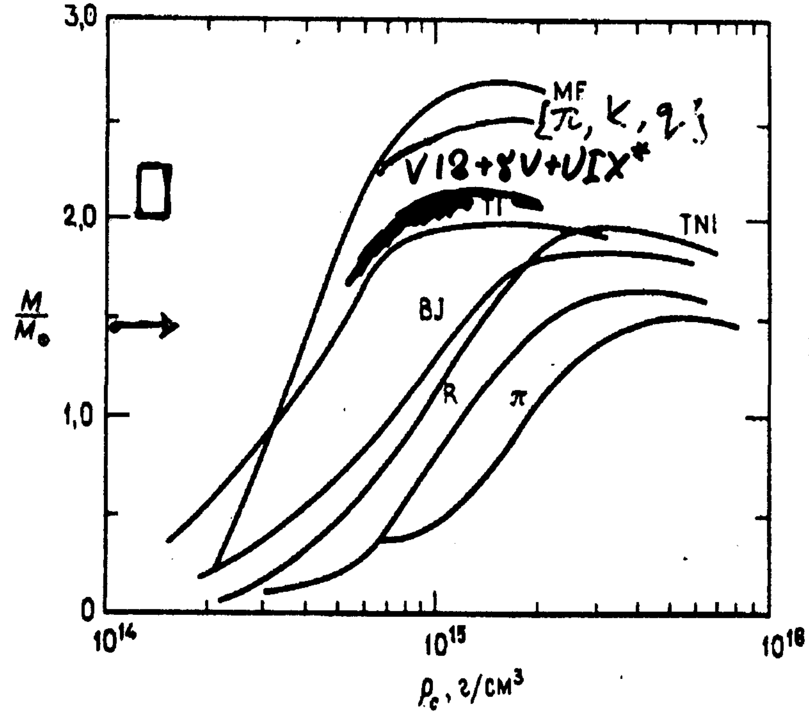
**Table 1.** Parameters for different classes of binary radio pulsars, drawn from the ATNF pulsar catalog (*106*) and primary references. The first four classes are fully listed, but only representative examples are given for intermediate-mass, low-mass, and globular-cluster binary pulsars. However, all systems with measured or well-constrained masses are included. The spin and orbital periods are known to much higher precision than is reported here. The eccentricity is often also known to higher precision and is similarly truncated; where only one significant digit is reported, it implies that there is uncertainty in that digit. The references consist of the most recent published timing reference for each pulsar as well as references for mass determinations. The lower limits on companion masses have been determined assuming pulsar masses of 1.4 $M_\odot$. The uncertainties reported on all mass measurements are 68% confidence levels, whereas upper limits on masses and eccentricities are 95% confidence levels. For the double-neutron-star binaries J1811–1736 and J1849+2456, the likely mass ranges have been calculated following the method described for J1518+4904 in (*10*).

| Name | Spin period (s) | Orbital period (days) | Orbital eccentricity | Companion mass ($M_\odot$) | Pulsar mass ($M_\odot$) | Remarks | References |
|---|---|---|---|---|---|---|---|
| *Young pulsars with B- or Be-star companions* | | | | | | | |
| J0045–7319 | 0.926 | 51.2 | 0.808 | $10^{+1}_{-1}$ | $1.58^{+0.34}_{-0.34}$ | | (*10*, *18*) |
| B1259–63 | 0.0478 | 1236.7 | 0.870 | >3.13 | | | (*107*) |
| J1740–3052 | 0.570 | 231.0 | 0.579 | >11.0 | | | (*13*) |
| *Young pulsars in eccentric orbits with massive white dwarf companions* | | | | | | | |
| J1141–6545 | 0.394 | 0.198 | 0.172 | $0.986^{+0.020}_{-0.020}$ | $1.30^{+0.02}_{-0.02}$ | | (*94*) |
| B2303+46 | 1.066 | 12.3 | 0.658 | $1.3^{+0.10}_{-0.10}$ | $1.34^{+0.10}_{-0.10}$ | | (*14*, *49*) |
| *Double-neutron-star binaries* | | | | | | | |
| J0737–3039A | 0.0227 | 0.102 | 0.088 | $1.250^{+0.005}_{-0.005}$ | $1.337^{+0.005}_{-0.005}$ | Double pulsar | (*17*) |
| J0737–3039B | 2.77 | 0.102 | 0.088 | $1.337^{+0.005}_{-0.005}$ | $1.250^{+0.005}_{-0.005}$ | Double pulsar | (*17*) |
| J1518+4904 | 0.0409 | 8.63 | 0.249 | $1.05^{+0.45}_{-0.11}$ | $1.56^{+0.13}_{-0.45}$ | | (*10*, *108*) |
| B1534+12 | 0.0379 | 0.421 | 0.274 | $1.3452^{+0.0010}_{-0.0010}$ | $1.3332^{+0.0010}_{-0.0010}$ | | (*98*) |
| J1811–1736 | 0.104 | 18.8 | 0.828 | $1.11^{+0.53}_{-0.15}$ | $1.62^{+0.22}_{-0.55}$ | | (*109*) |
| B1820–11 | 0.280 | 357.8 | 0.795 | >0.65 | | May have MS companion | (*92*) |
| J1829+2456 | 0.0410 | 1.18 | 0.139 | $1.36^{+0.50}_{-0.17}$ | $1.14^{+0.28}_{-0.48}$ | | (*110*) |
| B1913+16 | 0.0590 | 0.323 | 0.617 | $1.3873^{+0.0003}_{-0.0003}$ | $1.4408^{+0.0003}_{-0.0003}$ | | (*99*) |
| B2127+11C | 0.0305 | 0.335 | 0.681 | $1.36^{+0.04}_{-0.04}$ | $1.35^{+0.04}_{-0.04}$ | M 15 | (*111*) |
| *Pulsars with planets* | | | | | | | |
| B1257+12 | 0.00622 | 66.5 | 0.0183 | | | Three planets | (*112*) |
| B1620–26 | 0.0111 | 191.4 | 0.0253 | $0.34^{+0.04}_{-0.04}$ | | M 4; WD + 1 planet | (*74*, *113*) |
| *Representative "intermediate-mass" systems: mildly recycled pulsars with massive white dwarf companions* | | | | | | | |
| J0621+1002 | 0.0289 | 8.32 | 0.0025 | $0.97^{+0.27}_{-0.15}$ | $1.70^{+0.32}_{-0.29}$ | | (*114*) |
| B0655+64 | 0.196 | 1.03 | <0.00003 | >0.66 | | | (*92*) |
| J1157–5112 | 0.0436 | 3.51 | 0.00040 | >1.18 | | | (*56*) |
| J1904+0412 | 0.0711 | 14.9 | 0.0002 | >0.22 | | | (*58*) |
| *Representative "low-mass" systems: millisecond pulsars with low-mass white dwarf companions* | | | | | | | |
| J0034–0534 | 0.00188 | 1.59 | | >0.14 | | | (*115*) |
| J0218+4232 | 0.00232 | 2.03 | | $0.21^{+0.17}_{-0.04}$ | | | (*116*, *117*) |
| J0437–4715 | 0.00576 | 5.74 | 0.000019 | $0.236^{+0.017}_{-0.017}$ | $1.58^{+0.18}_{-0.18}$ | | (*39*) |
| J0751+1807 | 0.00348 | 0.263 | <0.000003 | $0.188^{+0.012}_{-0.012}$ | $2.2^{+0.2}_{-0.2}$ | | (*118*) |
| B0820+02 | 0.865 | 1232.5 | 0.012 | >0.19 | | | (*92*) |
| J1012+5307 | 0.00526 | 0.605 | <0.0000013 | $0.16^{+0.02}_{-0.02}$ | $1.64^{+0.22}_{-0.22}$ | | (*46*, *119*) |
| J1640+2224 | 0.00316 | 175.5 | 0.00080 | >0.25 | | | (*112*) |
| J1713+0747 | 0.00457 | 67.8 | 0.000075 | $0.33^{+0.04}_{-0.04}$ | $1.60^{+0.24}_{-0.24}$ | | (*40*) |
| J1732–5049 | 0.00531 | 5.26 | 0.00001 | >0.18 | | | (*56*) |
| B1855+09 | 0.00536 | 12.3 | 0.000022 | $0.267^{+0.010}_{-0.014}$ | $1.58^{+0.10}_{-0.13}$ | | (*120*) |
| J1909–3744 | 0.00295 | 1.53 | <0.0000006 | >0.20 | | | (*121*) |
| B1957+20 | 0.00161 | 0.382 | | >0.02 | | Eclipsing | (*122*) |
| J2019+2425 | 0.00393 | 76.5 | 0.00011 | >0.31 | <1.51 | | (*48*) |
| *Sample of binary pulsars in globular clusters* | | | | | | | |
| B0021–72H | 0.00321 | 2.38 | 0.071 | $0.180^{+0.086}_{-0.016}$ | $1.41^{+0.04}_{-0.08}$ | 47 Tuc | (*123*) |
| B0021–72J | 0.00210 | 0.121 | <0.00004 | >0.021 | | 47 Tuc; eclipsing | (*123*) |
| J0514–4002A | 0.0499 | 18.8 | 0.89 | >0.90 | | NGC 1851 | (*124*) |
| B1516+02B | 0.00795 | 6.86 | 0.14 | >0.11 | | M 5 | (*125*) |
| B1639+36B | 0.00353 | 1.26 | | >0.16 | | M 13 | (*126*) |
| B1718–19 | 1.004 | 0.258 | | >0.11 | | NGC 6342; eclipsing | (*127*) |
| J1740–5340 | 0.00365 | 1.354 | <0.0001 | >0.18 | | NGC 6397; eclipsing | (*128*) |
| B1744–24A | 0.0116 | 0.0756 | | >0.09 | | Ter 5; eclipsing | (*129*) |
| B1802–07 | 0.0231 | 2.62 | 0.21 | >0.29 | <1.43 | NGC 6539 | (*10*, *92*) |
| J2140–2310A | 0.0111 | 0.174 | <0.00012 | >0.10 | | M 30; eclipsing | (*130*) |

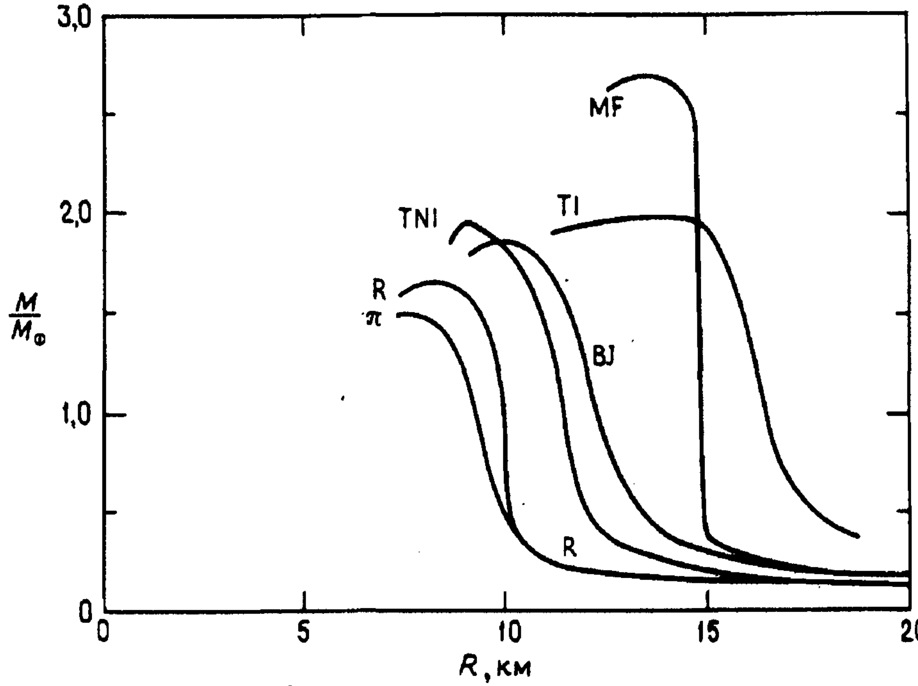
Masses of NS are different, massive 2,2±0,2 NS?

# Neutron Star Structure for various EoS

masses




radius




**MF** mean field Walecka model (hard EoS)

**R** Reid potential **BJ** Bethe-Jonson potential

**TNI** three nucleon forces **TI** tensor inteaction

π pion condensate

Problem for very soft EoS.
Any ph. tr. yields softening

Table 1: Isolated neutron stars which show thermal surface emission

| Source | $P$ [ms] | $D$ [kpc] | $B[10^{12}$ G] | Comments |
|---|---|---|---|---|
| PSR J0205+6449 | 65 | $\sim 3.2$ | 3.6 | PSR in SN1181 |
| Crab | 33 | $\sim 2.5$ | 3.8 | PSR in SN1054 |
| RX J0822–4300 | 75 | 1.9–2.5 | 6.8 | CCO in Puppis A |
| 1E 1207.4–5209 | 424 | $2.1^{+1.8}_{-0.8}$ | 4 | CCO in G296.5+10.0 |
| Vela | 89 | $0.293^{+0.019}_{-0.017}$ [a] | 3.4 | PSR |
| PSR B1706–44 | 102 | $\sim 2.3$ | 3 | PSR |
| Geminga | 237 | $0.159^{+0.059}_{-0.034}$ [a] | 1.6 | Musketeer |
| RX J1856.4–3754 | | $117 \pm 12$ [a] | | Dim object |
| PSR B1055–52 | 197 | $\sim 0.9$ | 1.1 | Musketeer |
| RX J0720.4–3125 | 8391 | $\sim 0.2$ | 9.3 | Dim object |

[a] parallax measured

Table 2: Observational limits on surface temperatures of isolated neutron stars

| Source | $t$ [kyr] | $T_s^\infty$ [MK] | Confid. | References |
|---|---|---|---|---|
| PSR J0205+6449 | 0.82 | <1.1 [b] | | Slane et al. (2002) |
| Crab | 1 | <2.0 [b] | 99.7% | Weisskopf et al. (2004) |
| RX J0822–4300 | 2–5 | 1.6–1.9 [a] | 90% | Zavlin et al. (1999) |
| 1E 1207.4–5209 | $\gtrsim 7$ | 1.1–1.5 [a] | 90% | Zavlin et al. (1998) |
| Vela | 11–25 | 0.65–0.71 [a] | 68% | Pavlov et al. (2001) |
| PSR B1706–44 | ~17 | $0.82^{+0.01}_{-0.34}$ [a] | 68% | McGowan et al. (2004) |
| Geminga | ~340 | $0.56^{+0.07}_{-0.09}$ [b] | 90% | Halpern & Wang (1997) |
| RX J1856.4–3754 | ~500 | <0.5 | – | Pavlov & Zavlin (2003) |
| PSR B1055–52 | ~530 | ~0.7 [b] | – | Pavlov (2003) |
| RX J0720.4–3125 | $\sim 1300$ | $\sim 0.5$ [a] | – | Motch et al. (2003) |

[a] Inferred using a hydrogen atmosphere model
[b] Inferred using the black-body spectrum

1209–51/52). They belong to the class of radio silent compact central objects (CCOs) in supernova remnants. CCOs have recently been reviewed by Pavlov et al. (2002b) and Pavlov & Zavlin (2003). 1E 1207.4–5209 is the first isolated neutron star found to exhibit pronounced spectral features (spectral lines) in its radiation spectrum (Sanwal et al. 2002), although the interpretation of these features seems ambiguous. The next member of the list, the Vela pulsar (PSR

# Surface temperatures

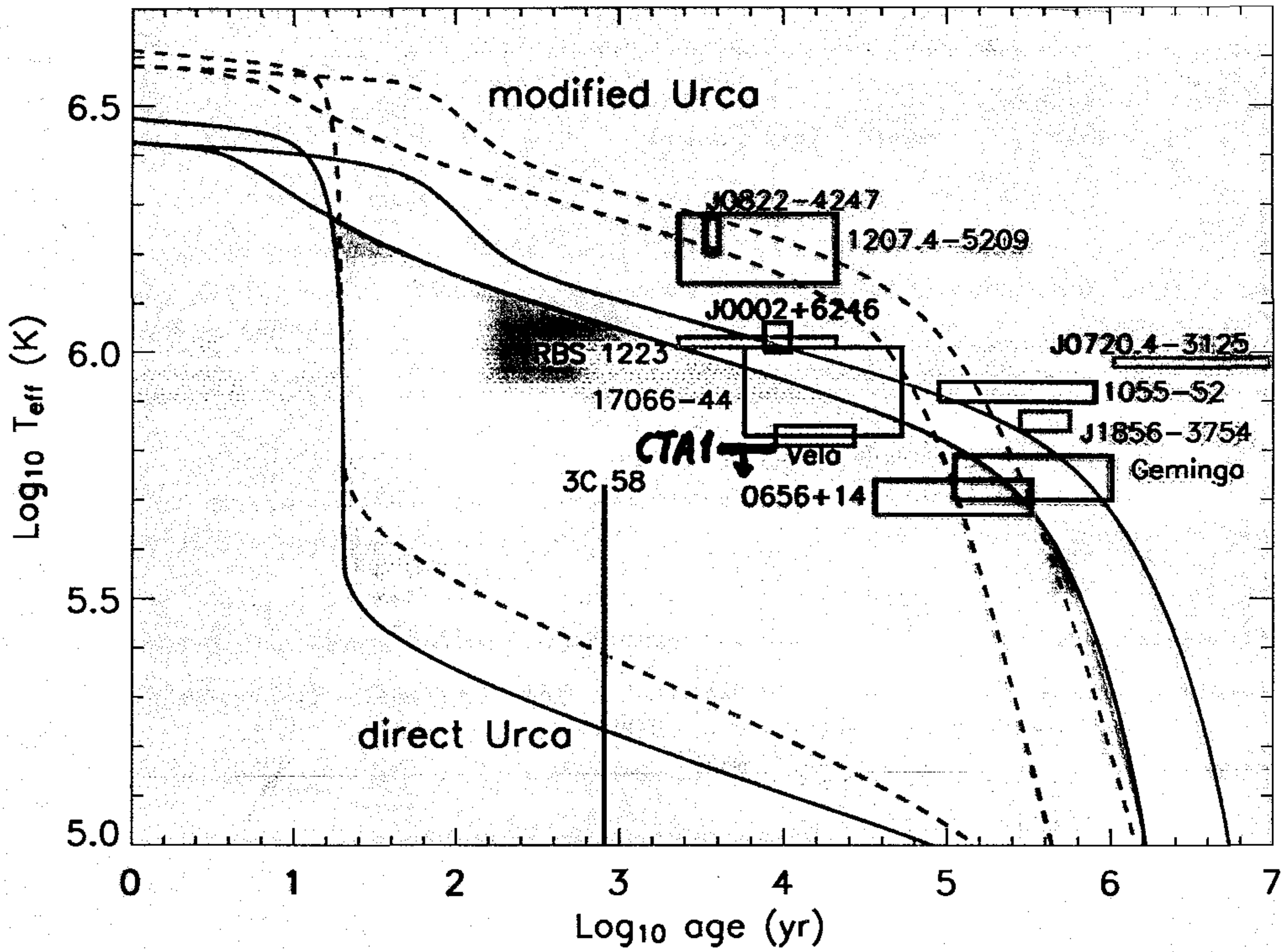



FIG. 4: Observational estimates of neutron star temperatures and ages together with theoretical cooling simulations for $M = 1.4\ M_\odot$. Models (solid and dashed curves) and data with uncertainties (boxes) are described in [43]. The green error boxes indicate sources from which thermal optical emissions have been observed in addition to thermal x-rays. Simulations with Fe (H) envelopes are displayed by solid (dashed) curves; those including (excluding) the effects of superfluidity are in red (blue). The upper four curves include cooling from modified Urca processes only, the lower two curves allow cooling with direct Urca processes and neglect the effects of superfluidity. Models forbidding direct Urca processes are relatively independent of $M$ and superfluid properties. The yellow region encompasses cooling curves for models with direct Urca cooling including superfluidity. (D. Page)

MU $nn \to npe\bar{\nu}$ $\varepsilon_\nu \sim 10^{21} (T/10^9 K)^8$ $\gtrsim 10^4$

DU $n \to pe\bar{\nu}$ $\varepsilon_\nu \sim 10^{27} (T/10^9 K)^6$

sional estimate of the characteristic time during which the neutrinos carry heat to the surface layer of width $\sim\lambda_\nu$, from which they are radiated outside the star, is

$$t_0 \sim \lambda_{\nu R}^{-1} R^2 C_V \sigma^{-1} T^{-3}. \tag{6.2}$$

The neutrino mean free path $\lambda_\nu$ can be determined from the simple relation[19]

$$[1+\exp(\omega_\nu/T)]dL/d\omega_\nu = 2^{-1}\pi^{-2}\lambda_\nu^{-1}(\omega_\nu)\,\omega_\nu^3 T^4, \tag{6.3}$$

where $dL/d\omega_\nu$ is the derivative of the luminosity per unit volume with respect to the neutrino energy $\omega_\nu$.

Making the calculations, we obtain from (6.3) and (4.7) for the neutrino mean free path in the URCA process

$$\lambda^{-1}_{\nu\,\mathrm{URCA}}$$

$$= \frac{G^2 f^4}{24\pi^7}(m_n^*)^3(m_p^*)\gamma^6(p_F^n)^4 p_F^e T^4 f(y)\, I^{\mathrm{exch}}_{\mathrm{URCA}}/\tilde{\omega}_{\pi^*}^4(p_F^n),$$

$$f(y) = y^4 + 10\pi^2 y^2 + 9\pi^4, \quad y = \omega_\nu/T. \tag{6.4}$$

Since $\lambda_\nu^{-1} \sim \varepsilon_\nu$, we can find from (6.4) and the relations obtained above for the luminosities of the various processes the corresponding values of $\lambda_\nu^{-1}$. The neutrino mean free path is a minimum for the generalized URCA process. The numerical estimate (6.4) gives

$$\frac{\lambda_\nu}{R} \approx 1.8\cdot 10^8 T_9^{-4} f^{-1}(y)\gamma^{-6}\left[\frac{m_\pi}{\tilde{\omega}_{\pi^*}(p_F^n)}\right]^{-4}(I^{\mathrm{exch}}_{\mathrm{URCA}})^{-1}(\rho/\rho_0)^{-2}, \tag{6.5}$$

where the radius $R$ of the neutron star is taken to be $\sim 10$ km. Equation (6.5) shows that neutrinos of mean energy $y = \bar{y} = \overline{\omega}_\nu/T \approx 4.7$, where

$$\overline{\omega}_\nu = \int f(y)(e^y+1)^{-1} y\, dy \Big/ \int f(y)(e^y+1)^{-1} dy,$$

produced in the process $nn \to npe\bar{\nu}$, will have a mean free path less than the radius of the star if $T_9$ exceeds $T_9^{op} \approx 6\tilde{\omega}^2(p_F^n)(\rho_0/\rho)^{5/6}\gamma^{-2}$. (Here, we have assumed that the main contribution to the luminosity of the $nn \to npe\bar{\nu}$ process is made by reactions with the emission of neutrinos in intermediate particle-hole states.) The analogous quantity $T_9^{op}$ obtained in Ref. 27 for the modified URCA process was $\sim 22$, and in Ref. 19 it was $T_9^{op} \sim 50$. Thus, the minimal value of $\lambda_\nu$ can be less than was expected in Refs. 27 and 19. According to the estimate of (6.2), this can lead to additional confinement of the neutrinos during the first minutes in the life of the neutron star. Therefore, there may be basic changes in the description of the initial stage of cooling of the neutron star, when the neutrino radiation is trapped. In the first place, this will be manifested in the description of interesting physical phenomena such as the ejection of the supernova shell, vibrations of the neutron star about the equilibrium position, and associated neutrino pulsations, which, in principle, could be detected on the earth if a supernova explodes in our Galaxy.

Leaving aside these important questions, which relate to the description of the initial stage in the evolution of the neutron star (at times $t \lesssim t_0$, on the order of minutes or an hour), we turn to the time evolution of the neutron star luminosity during times $t \gg t_0$, when neutrino absorption is no

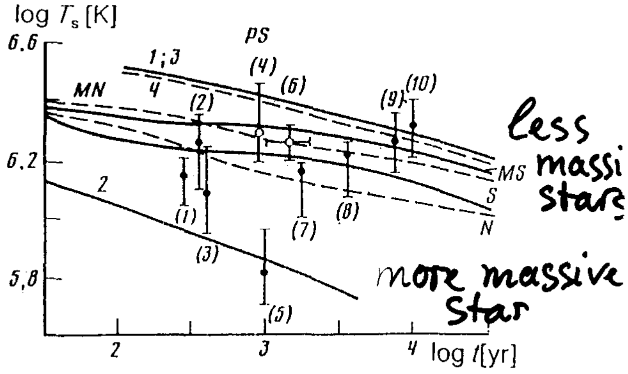



FIG. 6. Comparison of theoretical calculations with data of the Einst observatory. The abscissa is the logarithm of the time, the ordinate logarithm of the surface temperature. The equation of state is that Pandharipande and Smith ($M \approx 1.3\,M_\odot$, $R \approx 8$ km, $R^\infty = 11$ km). experimental data (see Ref. 7): 1) Cas A, 2) Kepler, 3) Tycho, 4) Cra 5) SN1006, 6) RCW103, 7) RCW86, 8) W28, 9) G350, 0-18, 10) G22 02. The open circles represent observed sources, the black circles are per limits. The curves are as follows: $S$ for superfluid neutron stars, $N$ normal stars, $MS$ for magnetic superfluid stars, and $MN$ for magne normal stars.[7] Curves 1 and 2 represent our calculations with mean d sity $\bar{\rho} = \rho_0$, $\bar{\omega}_\pi = m_\pi$, $\gamma = 0.47$ and $\bar{\rho} = 2\rho_0$, $\bar{\omega}_\pi = 0.5m_\pi$, $\gamma = 0.47$, spectively. Curves 3 and 4 are our calculations for the same values of $\bar{\rho}$ with luminosity from Ref. 27.

longer important. It is determined by the equation

$$C_V \dot{T} = -L, \quad L = \sum_i \int \varepsilon_\nu\, d\mathbf{r}, \tag{6.}$$

Here, $C_V \approx C_V^n + C_V^\pi$, $C_V^n$ is the specific heat of the bary subsystem, and $C_V^\pi \sim C_V^n$ is the contribution to the speci heat of the pion fluctuations.[23,24] As can be seen from I (6.6), the cooling time of the star is shortened by as ma times as the luminosity is increased.

In Fig. 6, which is taken from Ref. 7, we give cooli curves calculated in different scenarios (with and withc allowance for superfluidity and a magnetic field) with t equation of state of Pandharipande and Smith in conjui tion with the experimental data of the Einstein observato All curves of Ref. 7 lie appreciably above some of the expe mental data.

In Fig. 6, to demonstrate qualitatively the influence the collective effects of the nucleon medium on the time ev lution of the cooling of a neutron star, we give $L(t)$ cur calculated for different mean values of the nuclear mat density. The following values were taken: $\bar{\rho} = \rho_0$ (curve and $\bar{\rho} = 2\rho_0$ (curve 2). The dependence $T_{in}(T_s)$ was tak from Ref. 9. We assumed

$$\tilde{\omega}_\pi(p_F^n(\rho_0)) = m_\pi, \quad \gamma(\rho_0) = 0.47;$$

$$\tilde{\omega}_\pi(p_F^n(2\rho_0)) = 0.5m_\pi, \quad \gamma(2\rho_0) = 0.47.$$

The specific heat $C_V$ was taken equal to the specific heat an ideal neutron gas. Curves 3 and 4 were also calculated $\bar{\rho} = \rho_0$ and $\bar{\rho} = 2\rho_0$ but with the luminosity of Ref. 27 a $C_V \approx C_V^{n\,id}$. As can be seen from Fig. 6, our curves 1 an can be reconciled with all the experimental data under t assumption that different densitites are attained in the ne tron stars (with different masses). The results of other stu ies cannot be reconciled with the experimental data withc recourse to the hypothesis of pion condensation (under t assumption that neutron stars exist in the objects studiec

# Other helpful info

**X ray bursters and mass to radius relation:**

SAX-J1808.4-3658, U1728-34, HerX1, RXJ1856-37, 4U1820-30, GRO J1744-28 have may be too small radii → candidates to hybrid stars or even self-bound objects, cf. Bombaci et al.

**NS kicks and asymmetry of SN:**

NS receive $v > 10^2 \div 10^3 km/s$, why so high?

**GRB:**

Once per life – energy release $\gtrsim 10^{53} erg$. Collision of NS? Strong phase tr. to $\pi$, $K$ condensate states, to quarks, to self-bound objects?

**Magnetars:**

$B_{surf} \lesssim 10^{15} G$, the strongest magnet of Universe. $BR^2 = const$, $B_{surf}(\rho_{surf}/\rho)^{2/3} = const$, $\rightarrow B_{in} \sim (10^3 \div 10^5) B_{surf}$, if up to $10^{18} G$, may affect EoS.

**Glitches, post-glitch relaxation and superfluidity:**

$\Delta P/P \sim 10^{-9} \div 10^{-8}$ for Crab and

$\Delta P/P \sim 10^{-6}$ for Vela,

$\Delta P/P \simeq 4.4 \cdot 10^{-6}$, $\Delta \dot{P}/\dot{P} \simeq 10^{-1}$ for PSR0355+54

Many very accurate data.

Explained by two component (normal-superfluid) liquid.

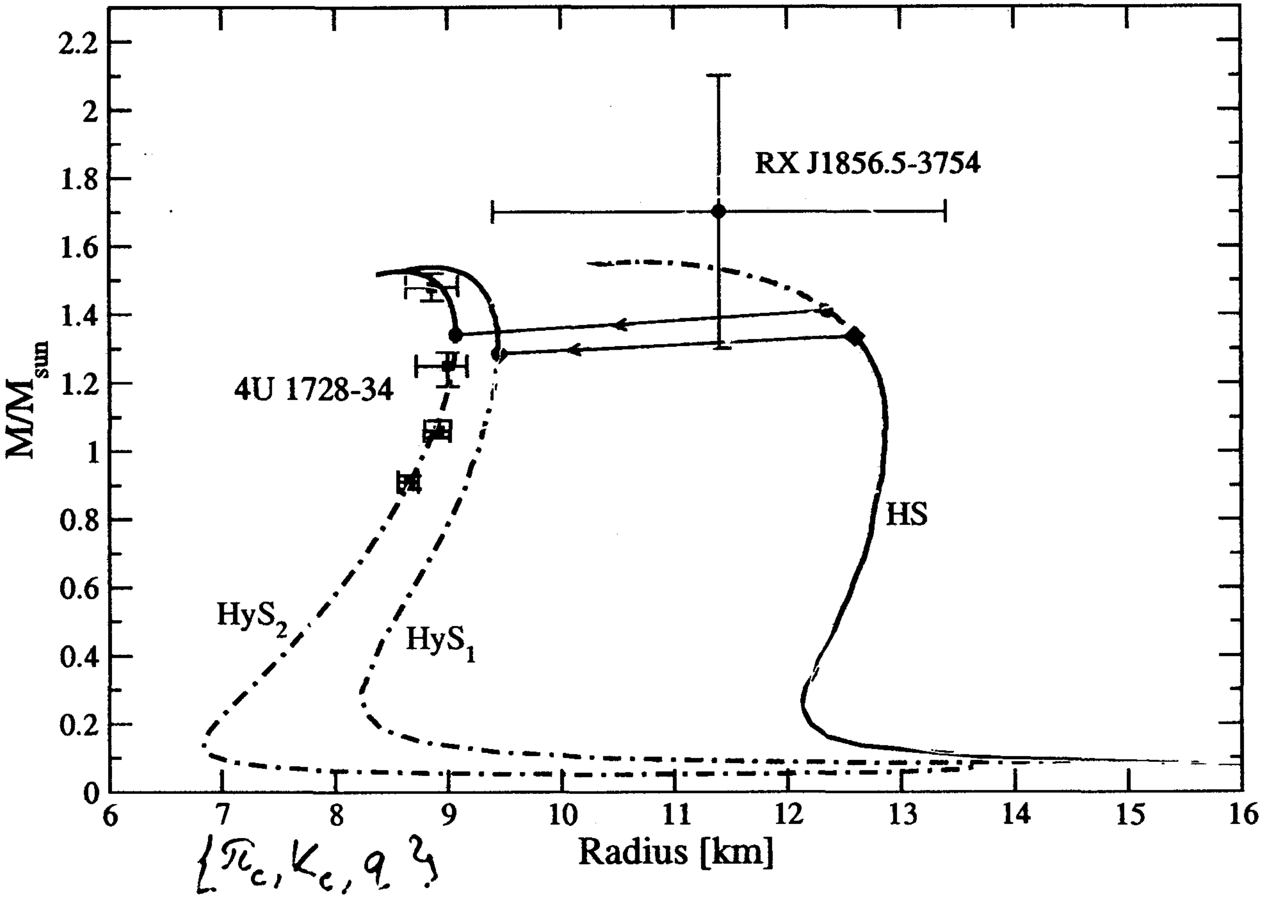



Fig. 10.— The radius and the mass for RX J1856.5-3754 (full circle with error bars labelled RX J1856.5-3754) obtained by Walter & Lattimer (2002) from fitting the multi-wave lenght spectral energy distribution. The radius and mass for 4U 1728-34, extracted by Shaposhnikov *et al.* ((66)) for different best-fits of the X-ray burst data, is shown by the filled circles with error bars (error contour for 90% confidence level). The curves labeled HS reprersents the MR relation for pure hadronic star with the GM3 equation of state. The curves labeled $HyS_1$ and $HyS_2$ are the MR curves for hybrid stars with the GM3+Bag model EOS, for $B = 85.29 \mathrm{MeV/fm}^3$ and $m_s = 150$ MeV ($HyS_1$), and $B = 100 \mathrm{MeV/fm}^3$ and $m_s = 0$ MeV ($HyS_2$). The full circles and diamonds on the MR curves represent the critical mass configuration (symbols on the HS curve) and the corresponding hybrid star configurations after the stellar conversion process (symbols on the $HyS_1$ and $HyS_2$ curves).

# Evolution of single NS

NS are formed in SN II explosions of massive ($M \gtrsim 10 M_\odot$) stars.

After formation of iron core of Chandrasekhar mass $M_{ch} \simeq 1{,}5 M_\odot$ $\Rightarrow$

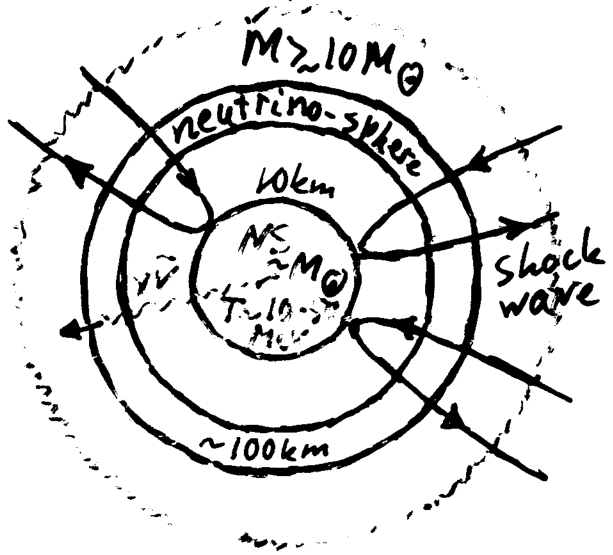
$\sim 10$ sec collapse with formation of rather incompressible ($\rho \sim \rho_0$) NS core $\Rightarrow$ shock wave + $\nu\bar{\nu}$ support at $R \sim 10^2$ km (neutrinosphere) $\Rightarrow$ explosion of most part of the mass

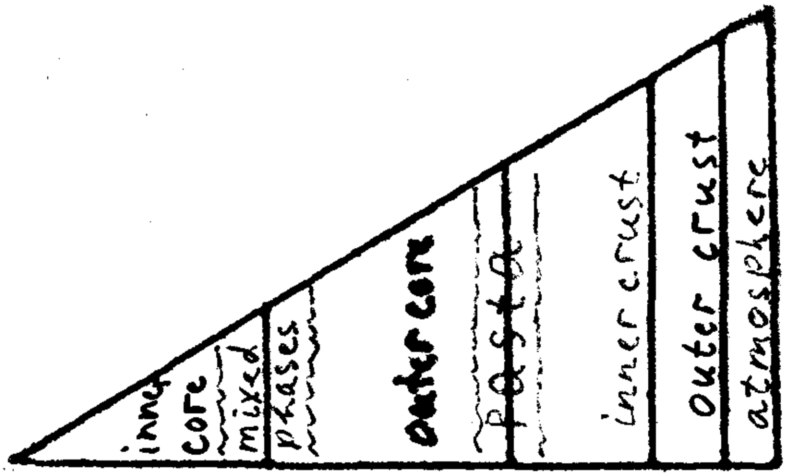
What remains is a hot ($T \sim (10-50)$ MeV) NS

$\varepsilon_F(\rho_0, N=Z) \simeq 40$ MeV

in envelope $E > 10^{51}$ erg

in $\nu\bar{\nu}$ $E \sim 10^{53}$ erg for $\sim 10$ s






## Structure of NS

- **atmosphere**, $\sim mm \div 10\ cm$, $\rho \lesssim 10^6\ g/cm^3$, plasma: determines photon radiation, $T_{surf}, B_{surf}$ affect EOS.

- **outer crust**, $\sim 10^2\ m$, $\rho < \rho_d = 4 \cdot 10^{11}\ g/cm^3$, solid of heavy nuclei + rel. electrons;

- **inner crust**, $\sim km$, $\rho \lesssim 0.5 \div 0.7\rho_0$, neutronized nuclei + neutron gas + rel. electrons.

  $0.3\rho_0 \lesssim \rho \lesssim 0.5 \div 0.7\rho_0$, nuclear pasta = mixed phase: nuclear drops, rods, slabs, etc.

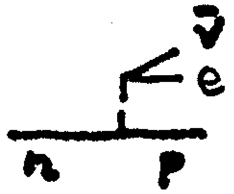
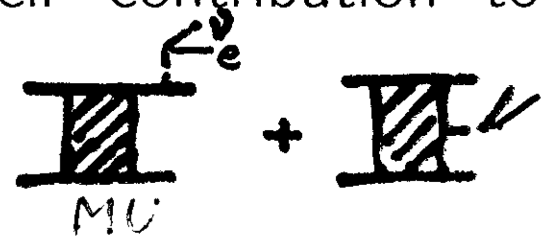
- **outer core**, $\rho \lesssim 2 \div 4\rho_0$, $\gtrsim$ several km., m.b. up to center, superfluid (at $T \lesssim MeV$) of $nn$, $pp$ + normal electrons.

- **inner core**, up to center (larger for massive NS),

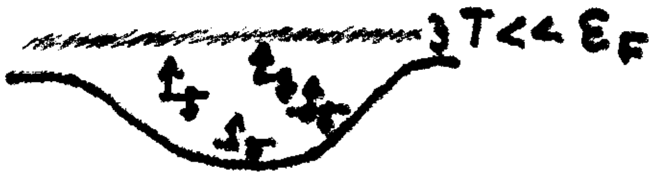
  Kingdom of Exotics: possible mixed phases between $npe$, $\pi_c$, $K_c$, $H$, $q$-CSC? and pure phases $\pi_c$, $K_c$, $H$, $q$-CSC?

# Ideas of Fermi liquid theory

**We need** $NN$ **interaction amplitude for** $\rho \gtrsim \rho_0$ to describe excitations and their contribution to reactions

when DU  forbidden, 


Couplings are strong → perturbation theory does not work.




1956 L.Landau, Sov. JETP

Application to nuclei, see A.B.Migdal, Theory of finite Fermi systems, Willey,N.Y. 1967.

Idea: Separation of short and long scales.

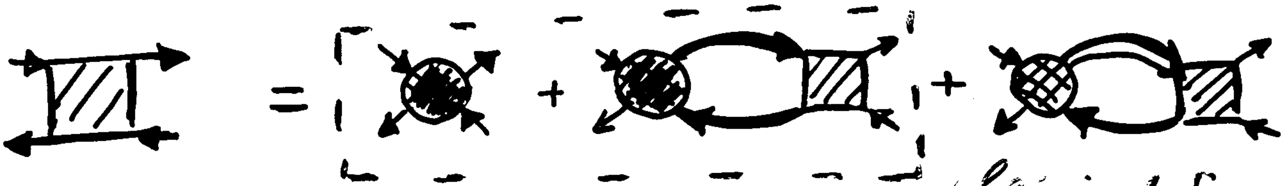
Short scale relates to $r_\Lambda \simeq 0.2\, fm$, also to $\sigma$, $\rho$, $\omega$, $N$, $r_\sigma$, $r_\rho$, $r_\omega$, $r_N$.

→ local quantities = constants in momentum space.

Long scale relates to low-lying excitations: $NN^{-1}$, $\pi$, $\Delta N^{-1}$ ($m_\Delta - m_N \simeq 2m_\pi$).

All processes for $\omega, k \lesssim (2 \div 3) m_\pi$ are treated explicitly.

Particular role of pion: $\rho_0 = 0.5$ in units $m_\pi = 1$, i.e. of order of one, $f_{\pi NN} = 1.01$ (strong coupling) → strong medium effects for $\rho_0 \gtrsim \rho_0$

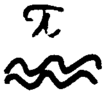


classical Fermi liquid by Landau

→ $N = (n, p)$ q.p. Green funct.

⇉ $\Delta(1232)$, $m_\Delta - m_N \simeq 2{,}1\, m_\pi$, $f_{\pi N \Delta} \simeq 2 m_\pi^{-1}$

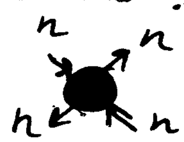
⇇ give small contrib. for $T \ll m_\pi$

 $= \left[\omega^2 - \vec{k}^2 - m_\pi^2 - \Pi^{\text{irr}}\right]^{-1}$

└ irreducible in part-hole, Δ-hole channels

All(!) diagrams

 irreducible in part-hole, Δ-hole, π channels

└ local:  $= f_{nn} + g_{nn}\, \vec{\sigma}_1 \vec{\sigma}_2$

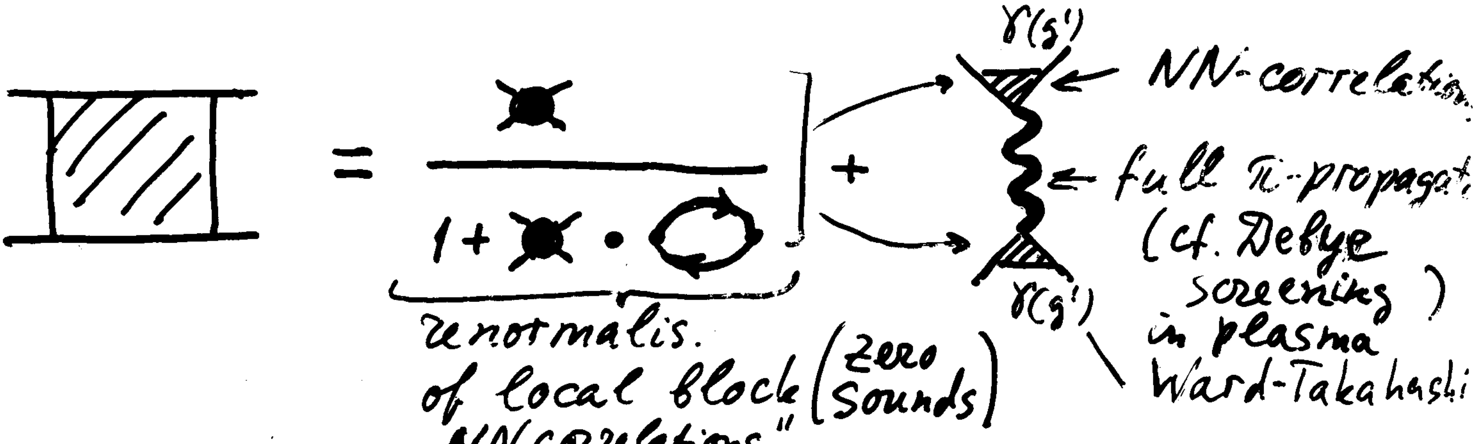
approximations { are supposed to be constants extracted from at. nucl. exp.

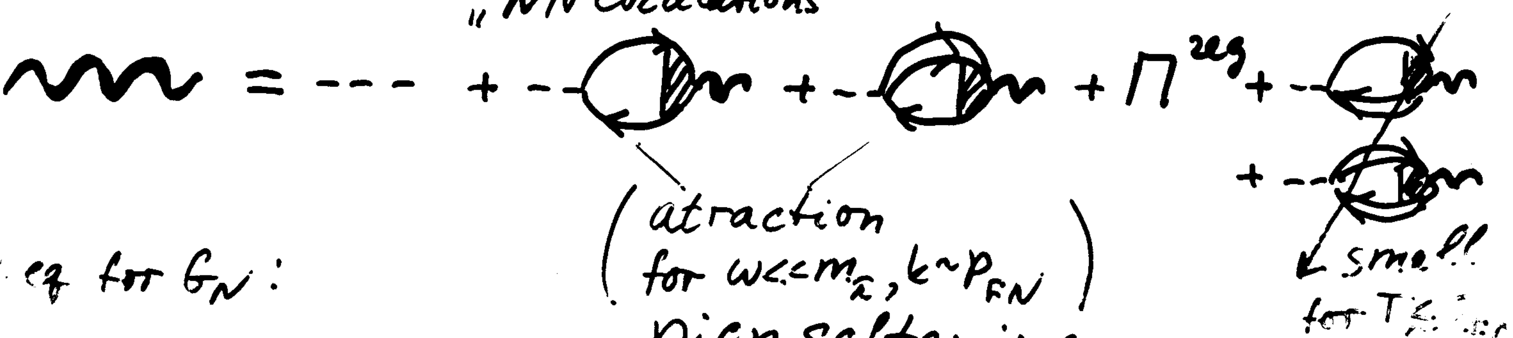
is small

Except for specific cases heavy particles like $N, \Delta$ can be treated in QPA, with widths estim from QP analysis, self-cons. as π is treated explicitly.

$m_N^* = m_N^*(\varphi)$.

After straightforward resummation:



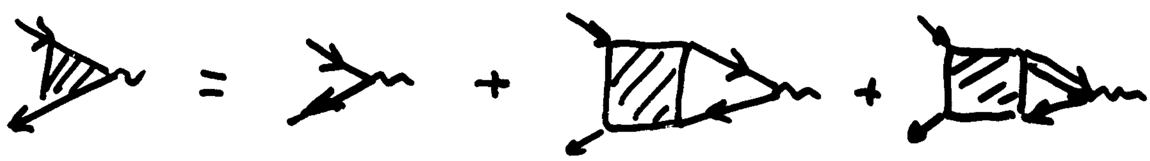




eq for $G_N$:

**pion softening**

N in many situations is a good QP ($m_N \to m_N^*(\rho)$)

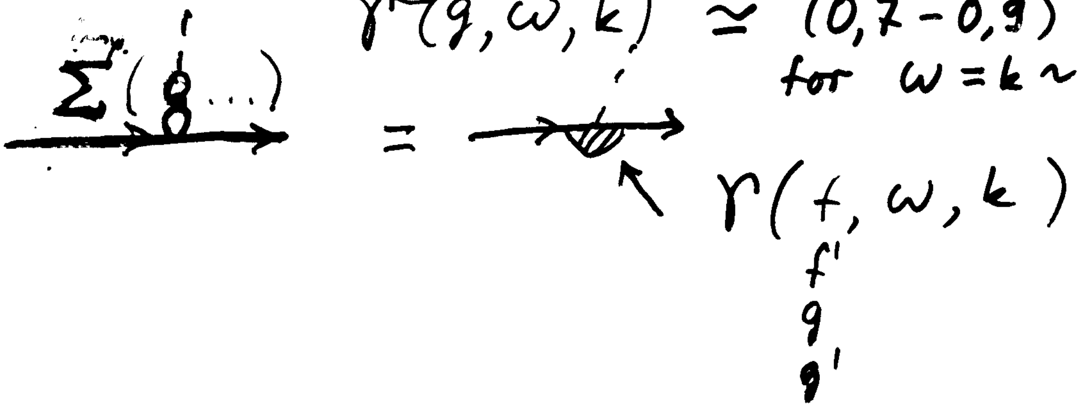
+ eq for vertex



$\Rightarrow$ NN correlation suppression of vertices

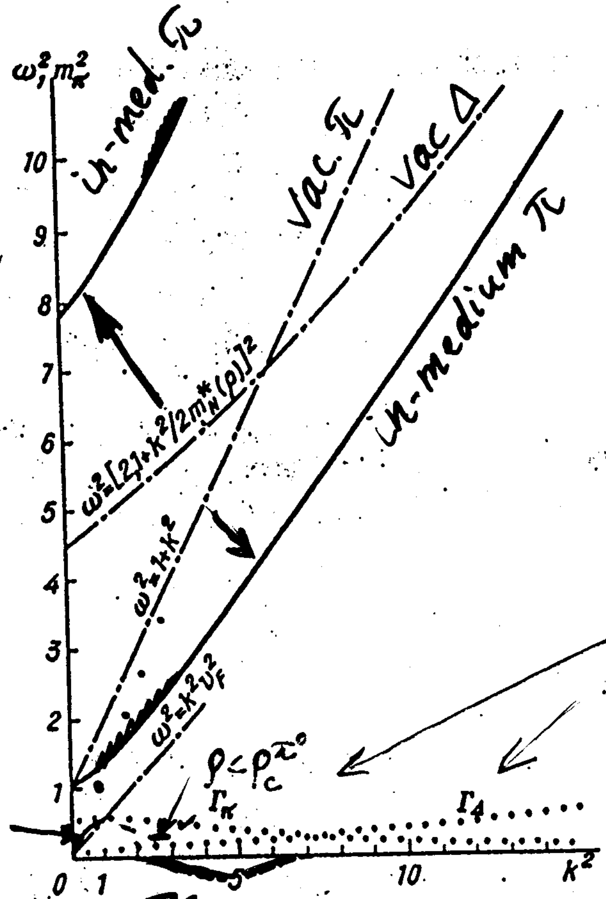
$$\gamma(g', \omega, k) = \frac{1}{1 + g' \bigcirc} \overset{\rho \simeq \rho_0}{\simeq} (0,3-0,5) \quad \text{for } \omega \ll m_\pi,\ k \sim p_F$$

$$\gamma^{-1}(g', \omega, k) \simeq (0,7-0,9) \quad \text{for } \omega = k \sim T \ll \varepsilon_F$$



# Pion spectra and $\pi$ condensation

$\{\pi^0\}$




in-med. $\pi$

$\Gamma_\pi$, $\Gamma_\Delta$ show branch populations by free pions
$\Gamma_S \simeq 0$

analogy: $N+\pi$ –
(ions + phonons)

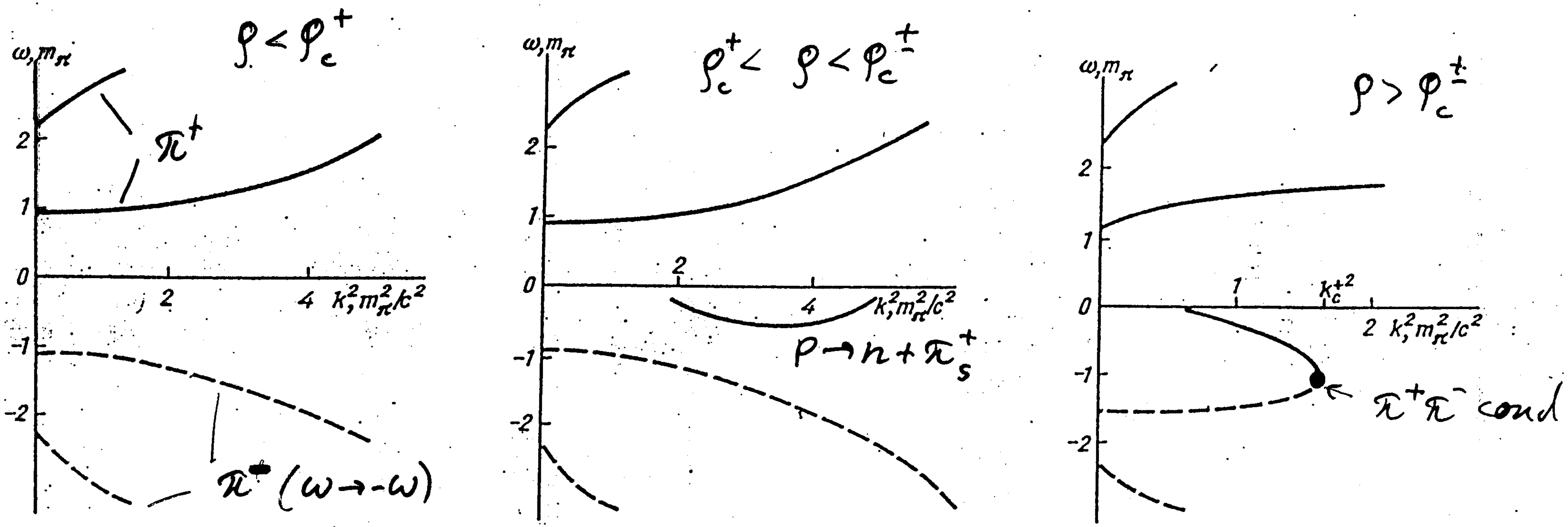
for $\rho > \rho_c^{\pi^0}$ $\omega^2 < 0$

$k \neq 0$, $\varphi \sim e^{-i\omega t} \sim e^{+|\tilde{\omega}|t} \to \infty$

$\Rightarrow$ pion condensation

# Pion spectra and $\pi$-cond.

$\{\pi^{\pm}\}$




$\rho_c^+ \lesssim \rho_0$ , $\rho_c^{\pm} \sim (1-3)\rho_0$ , $\rho_c^{\pi^0} \sim (1-3)\rho_0$ see Migdal Rev. Mod. Ph. 50 (1978) 107

in variational th: $\pi_c^{\pm}$ : $\rho_c = 2\rho_0$ $N = Z$, Migdal, Saperstein, Troitsky, D.V. Ph. Rep 192 (1990) 179

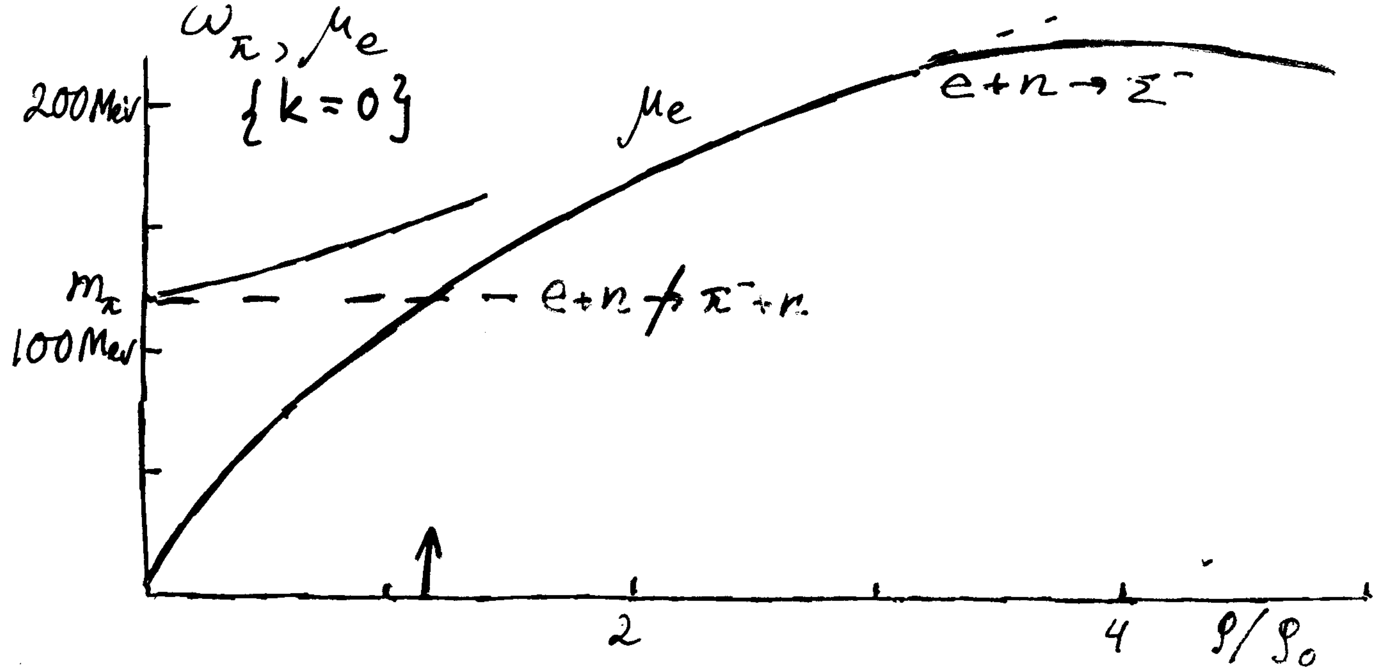
$\pi_c^0$ : $\rho_c = 2\rho_0$ $N = Z$ , $\rho_c = 1{,}3\rho_0$ $N \gg Z$ Akmal, Pandharipande, Ravenhall. Ph. Rev C58 (1998) 1804

Recent estimates of $\rho_{c\pi}(T=0)$:

| | P-wave $\pi^0$ | $\pi^\pm$ cond.: |
|---|---|---|
| $N=Z$ | $\simeq 1{,}4\,\rho_0\ (2\rho_0)$ | $\simeq 1{,}4\,\rho_0\ (2\rho_0)$ |
| $N \gg Z$ | $\simeq 2{,}2\rho_0\ (1{,}3\rho_0)$ | $\simeq 1{,}5\rho_0$ |

- Suzuki, Sakai, Tatsumi 1999
- Akmal, Pandharipande, Ravenhall 1998

S-wave $\pi$-cond. does not occur (medium ef.!)

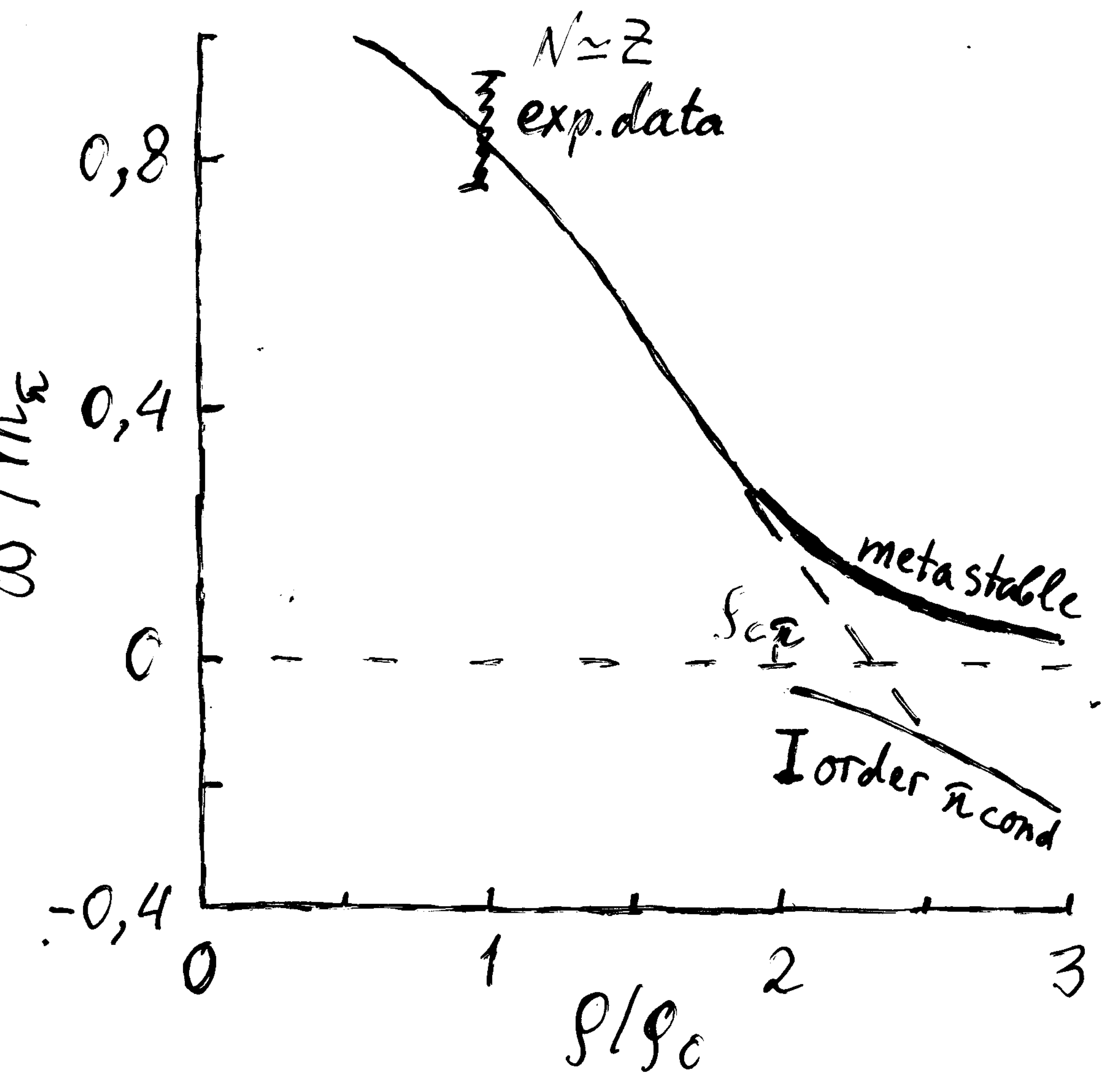


# Pion softening

$$\pi^0 : \ \tilde{\omega}^2 \equiv m_\pi^2 + k_0^2 + \Pi(\omega=0, k_0) \equiv -G_\pi^{-1}(\omega=0, k_0)$$

$k_0 \simeq p_{Fn}$ corresponds to min $\tilde{\omega}^2(k)$

$\tilde{\omega}^2$ - effective pion gap




I order ph.tr. to $\pi$-cond at $\rho_{c\pi} \simeq (1.5-3)\rho_0$

T=0 Dyugaev JETP Lett 22 (1975)

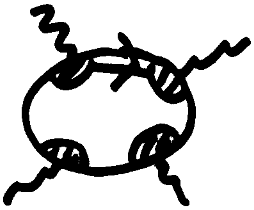
T≠0 D.V, Mishustin JETP Lett 22

# First order phase tr.

$\delta\Pi_{reg} =$  — has different behaviour above and below $\rho_{c\pi}$.

$\Lambda_{ef}$.

$$\delta\Pi_{reg} \propto \frac{T}{|\tilde{\omega}|}\Lambda_{ef}(k_0)$$

$\Lambda_{ef} =$  $+$  $+$  $+ \ldots \overset{k_0 \lesssim p_F}{\simeq}$ 

$\Rightarrow$  $+$  $+$  $+ \ldots \overset{k_0 < p_F}{\simeq}$ 

Energy gain due to $\pi$-cond.

$$E_{\pi^0} = \frac{\overbrace{[m_\pi^2+k_0^2+\Pi(\omega=0,k_0)]}^{\tilde{\omega}^2}\varphi^2}{2} + \frac{\lambda(\omega=0,k_0)\varphi^4}{4}$$

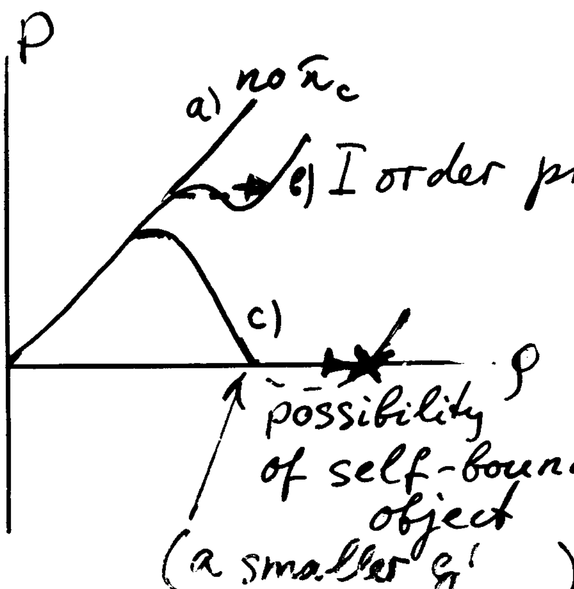
$$E_{\pi^0} \simeq -\frac{\tilde{\omega}^4}{6\lambda} \sim m_\pi^4 \frac{(\rho-\rho_c)^2}{\rho_c^2}, \quad \rho > \rho_c^{\pi^0}$$

? type of crystallin structure; $k_0 \simeq p_{Fn}$.

Analogy: energy gain for $\pi_c^\mp$:

{liquid crystallin structure, }
{superconductivity
{$\pi_c^0$ - three dim. crystall } Variety of phases

Consequences for EOS:

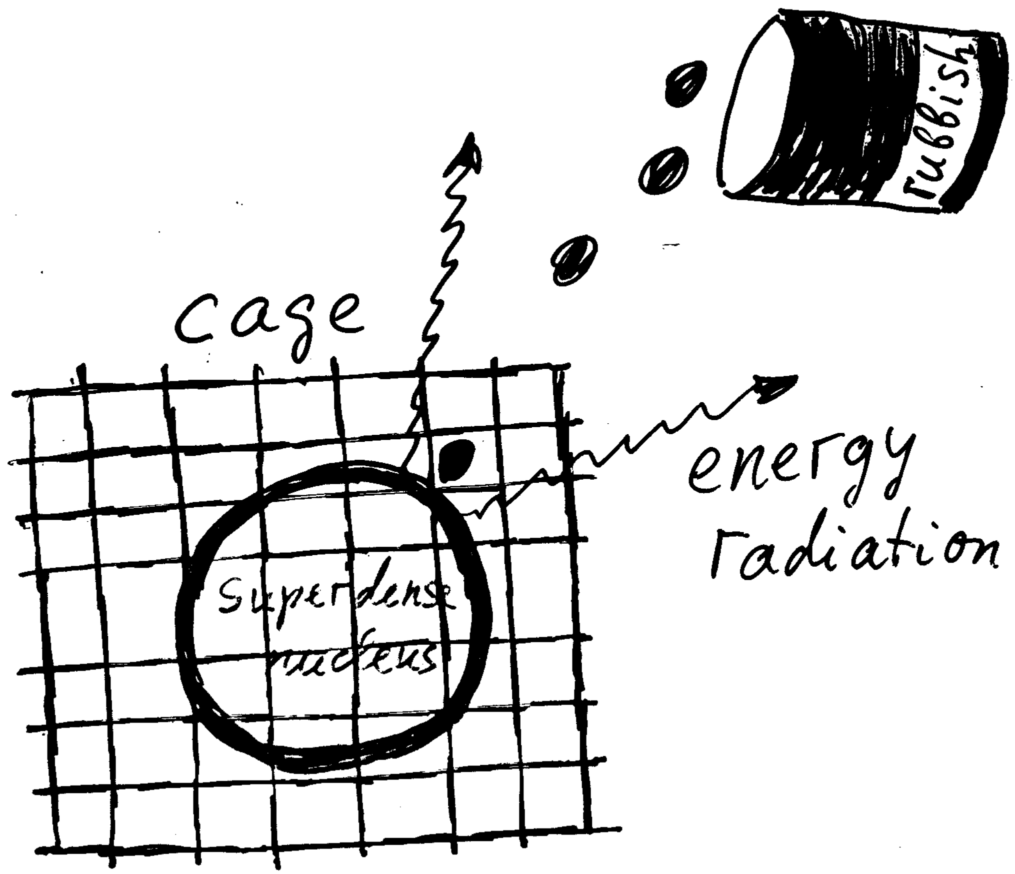



for typical time ~10 s governing weak processes N star achieves superdense state with $\nu$-burst and $\lesssim 10^{53}$ erg energy release, peculiarity in grav wave

L $\lambda M$ parameter

With $\pi^\mp$ cond – proton-neutron stars ($N \sim Z$)

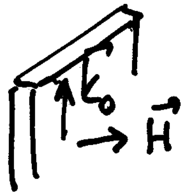
# Solution of energetic problems :)



Pion condensation ($\pi^{\pm}_{c}$) in magnetic field and in rotation field

N. Anisimov, D.V. 1980, JETP,
A.B. Migdal et al 1990, Ph. Rep.

Specifics is due to $k_0 \neq 0$




instead of threads

$H < H_{c1}$ Meissner effect

$H_{c1} < H < H_{c2}$

plane layers of $\pi^{\mp}$ condensate and normal phase

$H_{c1} \sim 10^{16} - 10^{17}\,G$, $H_{c2} \sim 10^{19}\,G$.

$\Omega_{c1} < \Omega < \Omega_{c2}$

$\Omega_{c1} \sim 10^{-14}\,s^{-1}$, $\Omega_{c2} \sim 10^{21}\,s^{-1}$

rotating NS are always in mixed state.

Consequences:

- possible explanation of strong glitches $\begin{cases} \Delta_n \lesssim \text{MeV} & \text{Crab} \\ \varepsilon_\pi \lesssim 10^2\,\text{MeV} & \text{Vela} \end{cases}$
- evolution of magnetic field $B(t)$.

K - condensation ($K^-$, $\bar{K}^0$)

S and P-wave kaon-baryon inter. and kaon cond. in NS

kaon is SU(3) partner of pion

→ constr. of interaction is analogous to π:

low-lying excitations relate to

$\Lambda(1116)$, $\Sigma(1195)$, $\Sigma^*(1385)$ − $n^{-1}$, $p^{-1}$,

$\Lambda^{-1}$, $\Sigma^{-1}$, $\Xi^{-1}$ holes in corresponding Fermi seas


$\rho < \rho_{cH}$

includes BB-correl. when hyperon Fermi seas are not filled.

H takes analogous role as Δ in π-case

$\Pi_{reg}$ contains a stronger S-wave term being attractive

Different possibilities

a) s-wave $K^-$-cond., $e \to K^-$ cf. G. Brown, V. Thorsson, K. Kubodera, M. Rho Ph. Lett 1992

b) p-wave K-cond, cf. E. Kolomeitsev, D.V., B. Kämpfer, Nucl. Ph 1995, E. Kolomeitsev, D.V. Ph. Rev C, 2003, T. Muto 2003,2

$K^-$ ($k_0 \sim p_{Fn}$) and $\bar{K}^0$

Conclusion:

most probably $K^-$ cond. arises by first order ph. tr. to a proton enriched matter at $\rho_c \simeq (3\div 4)\rho_0$, in P-wave state, as for $\pi_c^-$.

Consequences are analogous to those in $\pi_c^-$ case, with m. b. a stronger energy release $(m_K/m_\pi)$

# Quark core (CSC)

$u, d, s$ $\quad d \leftrightarrow u + e$, $d \leftrightarrow s$

$\mu_d = \mu_u + \mu_e$, $\mu_d = \mu_s$

Energy gain is due to extra Fermi seas and depends on bag const. $B$ and $m_s$.

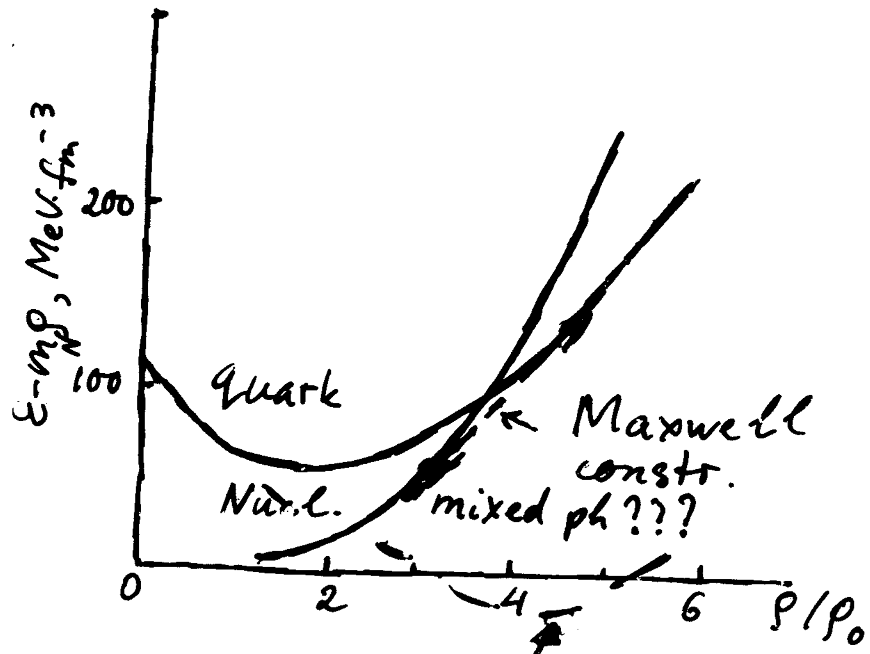



Fig. for $\alpha_c \simeq 0{,}4$, $B = 120$ MeV fm$^{-3}$.

With decrease of $\alpha_c$, $m_s$, $B$, energy decreases. Uncertainty allows possibility of self-bound strange quark matter!

Consequences of Color supercond.
are analogous to those of $\pi^\pm$ cond.

- Superconductivity + Meissner effect
- Reaction on rotation – vortices
  relating to possibility of strong glitches
- Energy release in I order ph. tr.

  etc.
- Specificity of the $\nu$-cooling etc.
- variety of phases