Spectrum of relative momenta of the neutron and proton at deuteron peripherical breakup in the limit of very low momentum transfer

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1 Basic formula for the momentum spectrum.

In the framework of the impulse approach $\Rightarrow$ general formula for the spectrum of relative momenta of the neutron and proton at the deuteron peripherical breakup:

$$d^3W = N\left|\int \psi_d(r)\,\psi_{\vec{k}}^{*(-)}(\vec{r})\,e^{i\frac{\vec{q}\vec{r}}{2}}\,d^3\vec{r}\right|^2 \frac{d^3\vec{k}}{(2\pi)^3}$$

Here: $\psi_d(r) \to$ normalized wave function of the deuteron ($4\pi\int_0^\infty \psi_d^2(r)r^2dr = 1$);

$N \to$ normalization constant;

$\vec{k} \to$ momentum of one of the nucleons in the rest frame of the deuteron;

$\vec{q} \to$ momentum transferred to one of the nucleons in the deuteron in the same frame

$\psi_{\vec{k}}^{(-)}(\vec{r}) \to$ wave function of relative movement of the neutron and proton, corresponding to the scattering problem and having the asymptotics in the form of a superposition of a plane wave and a converging spherical wave.

2) The case of very small transferred momenta.

Very small transferred momenta

$\left(q \ll \frac{1}{\rho}, \rho - \text{deuteron radius}\right) \Rightarrow$ we obtain:

(relative momentum $\to 2\vec{k}$)

$$d^3W = \frac{1}{4} N \left| \int \psi_d(r) (\vec{q}\vec{r}) \psi_{\vec{k}}^{*(-)}(\vec{r}) d^3\vec{r} \right|^2 \frac{d^3\vec{k}}{(2\pi)^3}$$

(only the first-order term in $e^{\frac{i\vec{q}\vec{r}}{2}}$ contributes to the spectrum at very low transferred momenta; the zero-order term gives the zero contribution identically: $\int \psi_d(r) \psi_{\vec{k}}^{*(-)}(\vec{r}) d^3\vec{r} \equiv 0 \to$ mutual orthogonality of wave functions)

Normalization constant $N \to$ determined as follows:

a) integration of d^3W over $d^3\vec{k}$ gives (taking into account the completeness condition $\int \psi_{\vec{k}}^{(-)}(\vec{r}) \psi_{\vec{k}}^{*(-)}(\vec{r}') d^3\vec{k} = (2\pi)^3 \delta(\vec{r}-\vec{r}')$)

$$W = \frac{1}{4} N \int \psi_d^2(r) (\vec{q}\vec{r})^2 d^3\vec{r} = \frac{1}{12} N \vec{q}^2 \langle r^2 \rangle,$$

where $\langle r^2 \rangle = 4\pi \int \psi_d^2(r) r^4 dr \to$ the mean square of the distance between the neutron and proton in the deuteron

b) normalization of the relative momentum spectrum to unity: $W = 1 \Rightarrow N = \dfrac{12}{\vec{q}^{\,2}\langle r^2\rangle}$

Neutron and proton in the deuteron → zero orbital momentum ($l = 0$)

(small contribution of the D-wave state is neglected)

⇒ only P-states of the final neutron-proton system ($l = 1$) give the contribution into the integral determining the spectrum of relative momenta d^3W.

Low relative momenta → role of neutron-proton interaction in the P-states is negligibly small
($k = |\vec{k}| < 1/r_0$, r_0 → radius of action of nuclear forces)

⇒ wave function $\psi^{(-)}_{\vec{k}}(\vec{r})$ may be replaced by the plane wave $e^{i\vec{k}\vec{r}}$.

Thus, we obtain:

$$d^3W = \frac{1}{4} N \left| \vec{q} \int \psi_d(r)\, \vec{r}\, e^{-i\vec{k}\vec{r}} d^3\vec{r} \right|^2 \frac{d^3\vec{k}}{(2\pi)^3},$$

or, introducing the function $F(\vec{k}) = \int \psi_d(r) e^{-i\vec{k}\vec{r}} d^3\vec{r}$,

$$d^3W = \frac{1}{4} N \left| \vec{q} \cdot \frac{d}{d\vec{k}} F(\vec{k}) \right|^2 \frac{d^3\vec{k}}{(2\pi)^3} .$$

$F(\vec{k}) \to$ real function having no dependence on the direction of $\vec{k}$

(since $\psi_d(r)$ is a real function, and due to spherical symmetry)

$\Rightarrow$ expression for d^3W can be rewritten in the form:

$$d^3W = \frac{1}{4} \frac{N(\vec{q}\vec{k})^2}{k^2} \left(\frac{d}{dk} F(k) \right)^2 \frac{d^3\vec{k}}{(2\pi)^3}$$

Integration over the directions of $\vec{k}$ $\Rightarrow$ final expression for the spectrum of relative momenta $dW(k)$, normalized to unity:

$$dW(k) = \frac{1}{\langle r^2 \rangle} \left(\frac{d}{dk} F(k) \right)^2 \frac{k^2 dk}{2\pi^2}$$

$$\left(\int dW(k) = \frac{1}{2\pi^2 \langle r^2 \rangle} \int_0^\infty \left(\frac{d}{dk} F(k) \right)^2 k^2 dk = 1 \right)$$

Namely this formula for $dW(k)$ → describes the spectrum of relative momenta of the neutron and proton at the deuteron disintegration in the Coulomb field of a nucleus. Analogous formulas → describe the spectrum of relative momenta of final clusters at the Coulomb dissociation of weakly bound relativistic nuclei and hypernuclei

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3) Calculation of the spectrum of relative momenta with the S-wave Hulthen deuteron wave function

Well-known Hulthen expression for the S-wave deuteron wave function:

$$\psi_d(r) = \frac{1}{\sqrt{2\pi(\rho-d)}}\,\frac{e^{-\frac{r}{\rho}} - e^{-\frac{d r}{\rho}}}{r}$$

Here: $\rho = 4.31$ Fm → deuteron radius;

$d = 1.7$ Fm → effective radius of low-energy n-p interaction in the triplet state;

$\alpha = 6.25$ → constant determined from the normalization condition for $\psi_d(r)$

$$\left(4\pi \int_0^\infty \psi_d^2(r)\, r^2\, dr = 1\right)$$

In the considered case the function $F(k)$, determining the spectrum of relative momenta, takes the following form:

$$F(k) = \sqrt{\frac{8\pi}{\rho - d}}\, \rho^2 \left[\frac{1}{1+(k\rho)^2} - \frac{1}{\alpha^2 + (k\rho)^2}\right]$$

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JINR Communication E1-99-280, Dubna, 1999.
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Proceedings of XVI International Baldin Seminar on High Energy Physics Problems (ISHEPP XVI), JINR E1,2-2004-76, vol. I, Dubna, 2004, p. 199

So, the spectrum of relative momenta $dW(k)$ is described by the expression:

$$dW(k) = \frac{1}{\langle r^2 \rangle} \left(\frac{d}{dk} F(k) \right)^2 \frac{k^2 dk}{2\pi^2} =$$

$$= \frac{16}{\pi} \frac{\rho}{\rho - d} \frac{\rho^7}{\langle r^2 \rangle} \left[\left(\frac{1}{1+(k\rho)^2} \right)^2 - \left(\frac{1}{\alpha^2 + (k\rho)^2} \right)^2 \right]^2 k^4 dk$$

Meantime, we have for the Hulthen wave function:

$$\langle r^2 \rangle = \frac{\rho^3}{\rho - d} \left(\frac{1}{2} + \frac{1}{2\alpha^3} - \frac{8}{(1+\alpha)^3} \right) \Rightarrow$$

$\Rightarrow$ finally, introducing the dimensionless variable

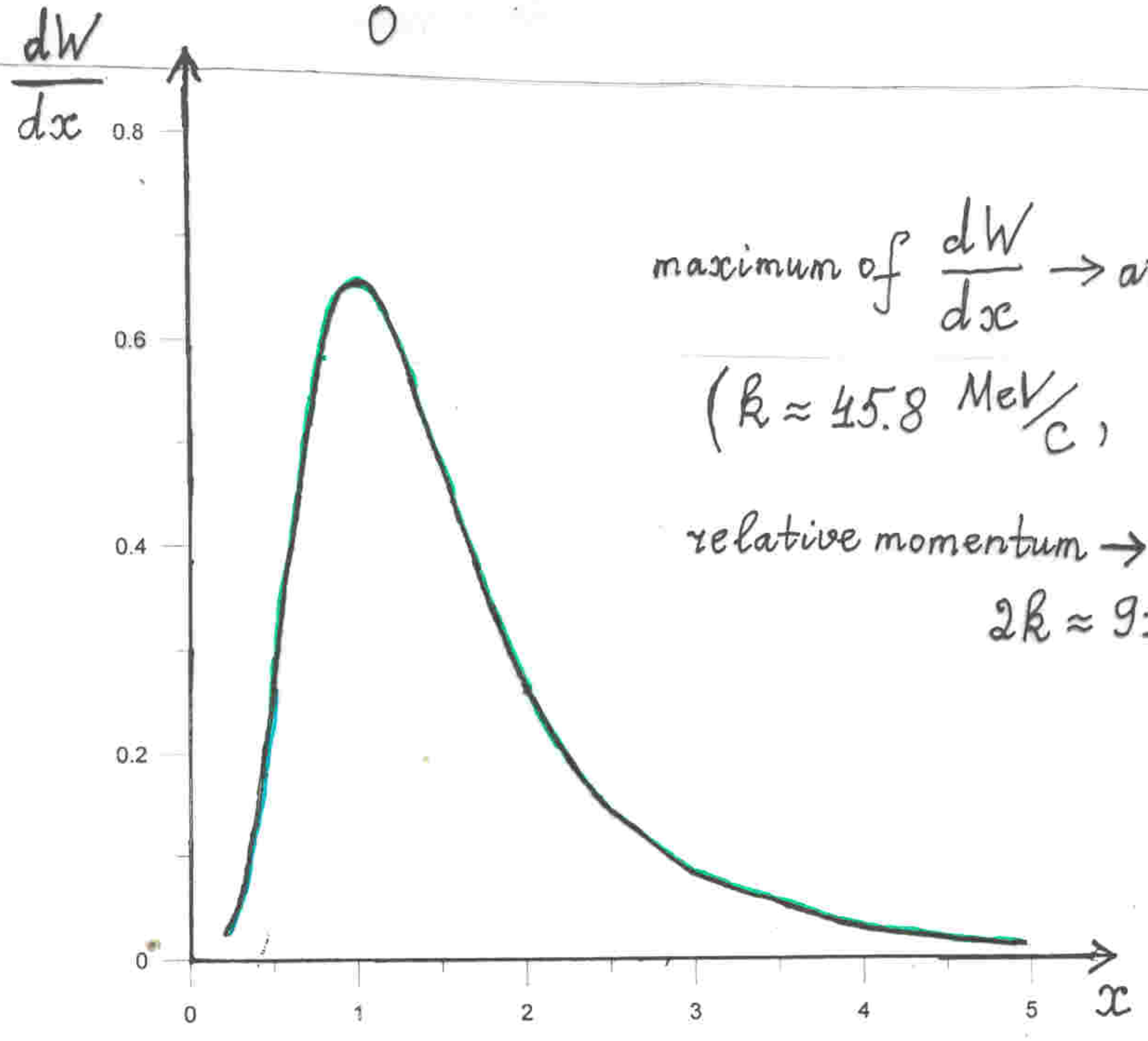
$x = k\rho$ ($k = 45.8x$ MeV/c), we obtain the following expression for the distribution of relative momenta of the neutron and proton in the deuteron rest frame:

$$dW(x) = \frac{32}{\pi}\beta\left[\left(\frac{1}{1+x^2}\right)^2 - \left(\frac{1}{\alpha^2+x^2}\right)^2\right]^2 x^4\,dx,$$

where $\beta = \left[1 + \frac{1}{\alpha^3} - \frac{16}{(1+\alpha)^3}\right]^{-1} \approx 1.04$

This distribution satisfies the normalization condition

$$\int dW(x) = \int_0^\infty \frac{dW(x)}{dx}\,dx = 1$$



maximum of $\frac{dW}{dx} \to$ at $x \approx 1$

($k \approx 45.8$ MeV/c,

relative momentum $\to$

$2k \approx 91.6$ MeV/c)

Contribution of the D-wave state of the deuteron

In the case of an unpolarized deuteron the contribution of the D-wave function of the deuteron to the spectrum of relative momenta of the neutron and proton is summed incoherently with the contribution of the S-wave function:

$$dW(k) = \frac{1}{2\pi^2 \langle r^2 \rangle}\left[(1-a_D)\left(\frac{d}{dk}F^{(S)}(k)\right)^2 + a_D\left(\frac{d}{dk}F^{(D)}(k)\right)^2\right]k^2 dk$$

$a_D \to$ relative fraction of the deuteron D-wave state

$(a_D \approx 0.04)$

Here: $F^{(S)}(k) = \int \psi_d^{(S)}(r) e^{-i\vec{k}\vec{r}} d^3\vec{r}$;

$F^{(D)}(k) = \int \psi_d^{(D)}(r) e^{-i\vec{k}\vec{r}} d^3\vec{r}$;

$\psi_d^{(S)}(r) \to$ the normalized wave function of the S-state;

$\psi_d^{(D)}(r) \to$ the normalized radial wave function of the D-state.

$$4\pi \int_0^\infty (\psi_d^{(S)}(r))^2 r^2 dr = 4\pi \int_0^\infty (\psi_d^{(D)}(r))^2 r^2 dr = 1$$

Mean square of the distance between the neutron and proton in the deuteron, taking into account D-wave

$$\langle r^2 \rangle = 4\pi \int_0^\infty \left[(1-a_D)(\psi_d^{(S)}(r))^2 + a_D(\psi_d^{(D)}(r))^2\right] r^4 dr$$

4. Summary

1. In the limit of very small transferred momenta, expressions for the spectrum of relative momenta of the neutron and proton, produced at the deuteron peripherical breakup, are obtained.

2. Calculations of the spectrum of relative momenta are performed for the Hulthen S-wave function of the deuteron.