The process of Coulomb dissociation of weakly bound relativistic nuclei and hypernuclei within the two-cluster model

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International School

„Few-Body Problems in Physics" – FBS 2006

Dubna, Russia, August 7-17, 2006

1. Introduction. Structure of the effective cross-section of Coulomb dissociation of relativistic nuclei.

Excitation and dissociation of nuclei in the Coulomb field

A. I. Akhiezer, I. Ya. Pomeranchuk. Some topics of nuclear theory. M.-L., Gostekhteorizdat, 1950, § 13
C. J. Mullin, E. Guth, Phys. Rev, 82, 141 (1951)
S.T. Butler, C.A. Pirson, Nuovo Cim, 19, 1266 (1961)
V. L. Lyuboshitz, Yad. Fiz. 51, 1013 (1990); S.A. Avramenko et al, Nucl. Phys. A585, 91c (1995)

Process of excitation and disintegration of relativistic nuclei in the Coulomb field ⇒ analogy with the problem of ionization and excitation of atoms at the propagation of relativistic charged particles through matter, application to the systems with large sizes and small binding energies.

Total effective cross-section of excitation and dissociation of a fast nucleus in the Coulomb field of the point-like charge Z (relativistic invariant; $\hbar = c = 1$)

$$\sigma = \frac{4\pi (Z\alpha)^2}{(u_k u)^2 - 1} \sum_{n \neq 0} \int \frac{\overline{|\langle n|j|0\rangle u_k|^2}}{|q^2|^2} d(|q^2|)$$

Here q ⇒ 4-momentum transferred to the nucleus;
$\langle n|j|0\rangle$ ⇒ 4-vector of the current of transition from the ground state $|0\rangle$ of the projectile nucleus to the excited state $|n\rangle$
u_k, u ⇒ 4-velocities of the Coulomb centre and the projectile.

Rest frame of the projectile (anti-laboratory frame)

$$q^2 = \varepsilon_{no}^2 - \vec{q}^{\,2}; \quad (uu_k)^2 - 1 = \gamma_k^2 v_k^2;$$

$$\langle n|j|0\rangle u_k = \gamma_k \langle n|\rho|0\rangle - \langle n|\vec{j}|0\rangle \vec{v}_k$$

(here ε_{no} => excitation energy of the nucleus;

$\vec{v}_k$ => velocity of the Coulomb centre, $\gamma_k = (1-v_k^2)^{-1/2}$ =>

Lorentz-factor).

Using the condition of continuity of the electromagnetic current:

$$\langle n|j|0\rangle q = 0; \quad \langle n|\rho|0\rangle \varepsilon_{no} - \langle n|\vec{j}|0\rangle \vec{q} = 0;$$

$$q u_k = \gamma_k(\varepsilon_{no} - \vec{q}\vec{v}_k) = 0; \quad \varepsilon_{no} = \vec{q}\vec{v}_k; \quad q_{\parallel} = \frac{\varepsilon_{no}}{v_k^2}\vec{v}_k$$

=> after all substitutions, in the laboratory frame:

$$\sigma = \frac{4\pi (Z\alpha)^2}{v^2} \sum_{n\neq 0} \int \frac{\left|\left(\langle n|\vec{j}|0\rangle/\varepsilon_{no},\ \vec{q}_{\perp} - \vec{v}\varepsilon_{no}/v^2\gamma^2\right)\right|^2}{\left(q_{\perp}^2 + \varepsilon_{no}^2/\gamma^2 v^2\right)^2} d(q_{\perp}^2)$$

($\vec{q}_{\perp}$ → transverse momentum; $\vec{v} = -\vec{v}_k$, $\gamma = \gamma_k$ => velocity and Lorentz-factor of the projectile in the laboratory frame).

The above formulas for σ correspond, strictly speaking, to the one-photon exchange ($Z\alpha \ll 1$). But the analysis shows that corrections to these formulas even at $Z\alpha \sim 1$ remain small, and that is connected with the fact that the main contribution into σ

is provided by the region of very small $q_\perp^2$, and the exact cross-section of Coulomb interaction at small transferred momenta coincides with the one obtained within the Born approximation.

Finally, dividing the integration range into two intervals (very small and larger $q_\perp^2$) and performing all the transformations $\Rightarrow$ the following formula for σ emerges, taking into account the finite size of the target nucleus:

$$\sigma = \frac{4\pi (Z\alpha)^2}{3v^2} \langle 0 | (\sum_p \vec{r}_p)^2 | 0 \rangle \left[\ln \left(\frac{\gamma^2 v^2}{\varepsilon_b^2 \langle 0 | (\sum_p \vec{r}_p)^2 | 0 \rangle} \right) - 2A + B - v^2 \right]$$

[V. L. Lyuboshitz, Yad. Fiz. 51, 1013 (1990)]

Here $\varepsilon_b \Rightarrow$ binding energy of the projectile nucleus;

$\vec{r}_p \Rightarrow$ proton coordinates; $|0\rangle \Rightarrow$ ground state of the projectile;

$A \Rightarrow$ constant involving the dependence of the minimal momentum transfer, at the transition to excited states $|n\rangle$, upon the excitation energy $\varepsilon_{n0} > \varepsilon_b$ ($1/R_{pr} \gg \varepsilon/v\gamma$, R_{pr} – radius of projectile,

$$A = \left(\sum_{n \neq 0} |\langle n | \sum_p \vec{r}_p | 0 \rangle|^2 \ln \left(\frac{\varepsilon_{n0}}{\varepsilon_b} \right) \right) \Big/ \langle 0 | (\sum_p \vec{r}_p)^2 | 0 \rangle$$

$\varepsilon = \varepsilon_b e^A$ – effective energy of excitation)

B => constant describing the contribution of comparatively large transfers of transverse momentum:

$$B = -3\int_0^\infty \ln y \frac{d}{dy}\left(\frac{G(y)H(y)}{y}\right) dy,$$

where $y = q_\perp^2 \langle 0 | (\sum_p \vec{r}_p)^2 | 0 \rangle$,

$G(y) = \langle 0 | |\sum_p e^{-i\vec{q}_\perp \vec{r}_p}|^2 | 0 \rangle - |\langle 0 | \sum_p e^{-i\vec{q}_\perp \vec{r}_p} | 0 \rangle|^2$;

$H(y) = |\langle 0' | \sum_p e^{-i\vec{q}_\perp \vec{r}'_p} | 0' \rangle|^2 / Z^2$ => square of the electromagnetic formfactor of the target nucleus

($|0'\rangle$ => ground state of the target nucleus, $\vec{r}_p$ => proton coordinates for the target nucleus).

At very small y ($y \ll 1$) $\to$ $G(y) \approx \frac{1}{3} y$;

for the projectile with the unity charge => $G(y) = 1 - F^2(y)$

($F(y) = F(\vec{q}_\perp^{\,2}) = \langle 0 | e^{-i\vec{q}_\perp \vec{r}_p} | 0 \rangle$ => => electromagnetic formfactor)

$H(y) = 9\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right)^2$ => for the uniform distribution of charge over the volume of the target nucleus

(here $x = \sqrt{y}\,\frac{R}{\langle 0 | (\sum_p \vec{r}_p)^2 | 0 \rangle^{1/2}}$, $R \to$ radius of the target nucleus).

$H(0) = 1$.

Principal contribution into the cross-section σ $\Rightarrow$

the logarythmic term $\sim \ln\left(\dfrac{\gamma^2 v^2}{\varepsilon_b^2 \langle 0|(\sum_p \vec{r}_p)^2|0\rangle}\right)$.

② Weakly bound systems. Two-cluster (deuteron-like) model.

Under consideration: the Coulomb disintegration of a weakly bound nucleus consisting of two clusters (charged and neutral).

Normalized wave function of the bound state:

$$\varphi_0(r) = \frac{1}{\sqrt{2\pi\rho}} \frac{e^{-r/\rho}}{r}$$

($r \Rightarrow$ distance between the clusters;
$\rho = \left(2\dfrac{m_1 m_2}{M}\varepsilon_b\right)^{-1/2}$)

$m_1 \Rightarrow$ mass of the charged cluster;
$m_2 \Rightarrow$ mass of the neutral cluster; $M \Rightarrow$ mass of the nucleus. $(M = m_1 + m_2)$.

Thus, the quantity $\langle 0|(\sum_p \vec{r}_p)^2|0\rangle$ can be explicitly determined:

$$\langle 0|(\sum_p \vec{r}_p)^2|0\rangle = 4\pi \tilde{Z}^2\left(\frac{m_2}{M}\right)^2 \int_0^\infty \varphi_0^2(r)\, r^4\, dr = \tilde{Z}^2\left(\frac{m_2}{M}\right)^2 \frac{\rho^2}{2} =$$

$$= \frac{1}{4}\tilde{Z}^2 \frac{m_2}{m_1 M \varepsilon_b}$$

($\tilde{Z} \to$ number of protons in the charged cluster)

Analytical expression for the function $G(y)$:

$$G(y) = \tilde{Z}^2\left(1 - \left|\int \varphi_0^2(r)\, e^{i\vec{q}_\perp \frac{m_2}{M}\vec{r}}\, d^3\vec{r}\right|^2\right) =$$

$$= \tilde{Z}^2\left(1 - \frac{2\tilde{Z}^2}{y}\left(\operatorname{arctg}\left[\left(\frac{y}{2\tilde{Z}^2}\right)^{1/2}\right]\right)^2\right)$$

(here $y = \frac{1}{4}\vec{q}_\perp^{\,2}\tilde{Z}^2 \frac{m_2}{m_1 M \varepsilon_b}$).

Very small binding energies $\Rightarrow$ effective radius ρ of the projectile nucleus considerably exceeds the target radius R ($\rho \gg R$) $\Rightarrow$ we may, in principle, take $H(y) = 1$, considering the target nucleus as a point-like Coulomb centre $\Rightarrow$

$\Rightarrow$ the following expression for the constant B arises:

$$B = -3\tilde{Z}^2 \int_0^\infty \ln y \frac{d}{dy}\left(\frac{1 - \frac{2\tilde{Z}^2}{y}\left[\operatorname{arctg}\left(\frac{y}{2\tilde{Z}^2}\right)^{1/2}\right]^2}{y}\right) dy =$$

$$= \ln \tilde{Z}^2 + \ln 2 + C; \qquad (C \approx 0.316)$$

Meantime: expression for the constant A for a two-cluster system with the wave function $\varphi_0(r)$:

$$A = \frac{16}{\pi}\int_0^\infty \frac{t^{3/2}}{(1+t)^4}\ln(1+t)\,dt \approx 1.218$$

(at small binding energies all excited states belong to the continuous spectrum).

Finally, the formula for the total cross-section σ of Coulomb dissociation of the weakly bound nucleus under consideration is as follows:

$$\sigma = \frac{\pi}{3}(Z\alpha)^2\tilde{Z}^2\frac{m_2}{v^2 m_1 M \varepsilon_B}\left[\ln\left(\frac{8\gamma^2 v^2 M m_1}{m_2\varepsilon_B}\right)-(2A-C)-v^2\right]$$

$(2A-C=2.12)$.

So, it is well seen that the cross-section σ depends very essentially upon the binding energy ε_B, and in the limit of very small ε_B its magnitude strongly increases.

Measuring experimentally the cross-section σ of Coulomb dissociation of weakly bound nuclei (hypernuclei), one can determine the value of binding energy ε_B for these nuclei.

3 Corrections due to the finite size of the target nucleus.

For the uniform charge distribution in the target nucleus $\Rightarrow$

$$H(y) = 9\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right)^2,$$

where, in the case of the weakly bound two-cluster system under study,

$$x = \frac{2\sqrt{y}\,R}{\tilde{Z}}\sqrt{\frac{m_1 M}{m_2}\varepsilon_b}.$$

So, taking into account the formfactor of the target nucleus, we obtain a certain decrease of the effective cross-section of Coulomb dissociation:

2.12

$$\sigma = \frac{\pi}{3}(Z\alpha)^2\tilde{Z}^2\frac{m_2}{v^2 M m_1 \varepsilon_b}\left[\ln\left(\frac{8\gamma^2 v^2 M m_1}{m_2 \varepsilon_b}\right) - (2A - C) - v^2 - \Delta B\right],$$

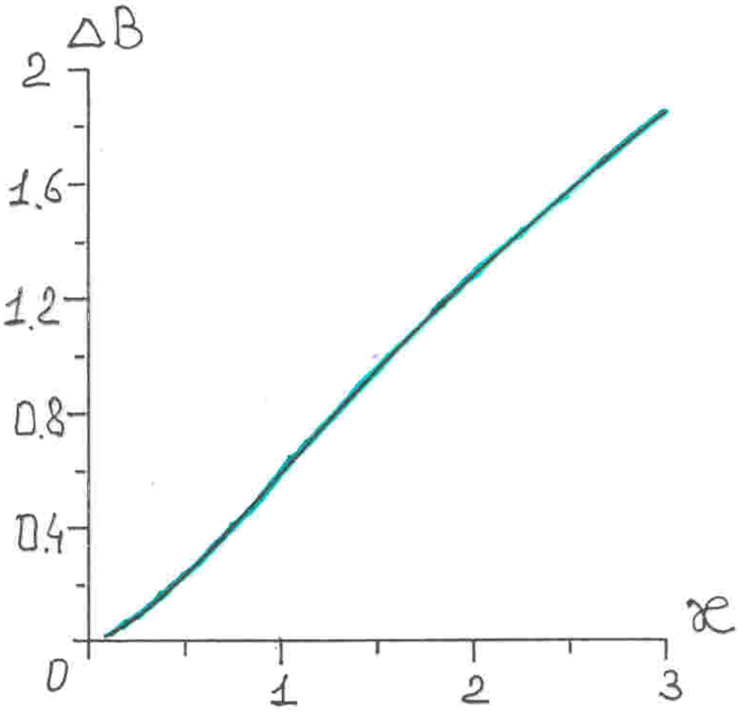
where the quantity ΔB is determined by the integral

$$\Delta B = 3\int_0^\infty \frac{1 - \frac{1}{s^2}(\operatorname{arctg} s)^2}{s^3}\left[1 - 9\left(\frac{\sin x}{x^3} - \frac{\cos x}{x^2}\right)^2\right] ds$$

(here $s = \frac{1}{\tilde{Z}}\sqrt{\frac{y}{2}}$, $x = \varkappa s$, $\varkappa = 2\frac{R}{\rho}\frac{M}{m_2} = 2\sqrt{2}R\sqrt{\frac{M m_1}{m_2}\varepsilon_b}$).

It should be also emphasized that due to the correction ΔB, depending on the target radius $R = 1.1 A^{1/3}$ (for heavy nuclei $R \approx 1.5 Z^{1/3}$), the total cross-section σ of Coulomb dissociation gets a certain deviation from the pure dependence $\sigma \sim Z^2$.

The quantity ΔB as a function of $\varkappa$



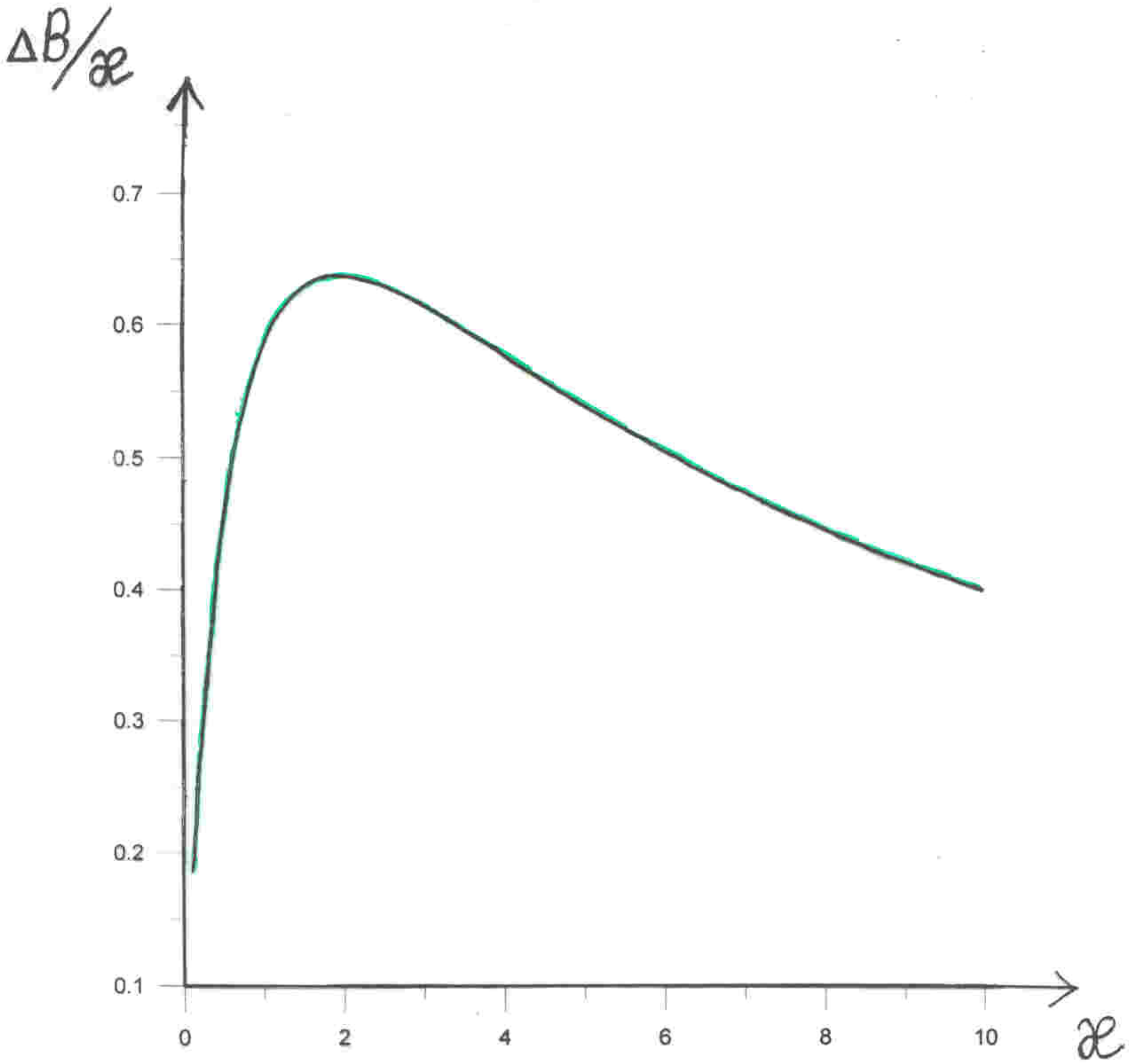
$\varkappa = 4;\ \Delta B \approx 2.31$

$\varkappa = 5;\ \Delta B \approx 2.7$

$\varkappa = 6;\ \Delta B \approx 3.03$

$\varkappa = 7;\ \Delta B \approx 3.31$

$$\Delta\sigma = -\frac{\pi}{3}(Z\alpha)^2 \tilde{Z}^2 \frac{m_2}{v^2 M m_1 \varepsilon_B} \Delta B(Z)$$



For the same projectile nucleus at charges of target nuclei in the interval $Z = 50 \div 100 \Longrightarrow$ almost linear dependence of ΔB on æ ($\Delta B \approx b$æ).

As a result, the quantities ΔB and $\Delta\sigma$ are proportional to the factors $Z^{1/3}$ and $Z^{7/3}$, respectively:

$$\Delta B \sim Z^{1/3}, \quad \Delta\sigma \sim Z^{7/3}.$$

The „effective" charge: $Z^2 \to Z_{eff}^2 = Z^2(1 - a_{pr} Z^{1/3})$;

coefficient a_{pr} depends on the concrete projectile nucleus.

4. Calculations of the cross-section of Coulomb dissociation for the hypernuclei ${}^3H_\Lambda$ and ${}^6He_\Lambda$.

1. ${}^3H_\Lambda + Z \to d + \Lambda + Z$

(Coulomb dissociation of ${}^3H_\Lambda$ into the deuteron and the Λ-particle; $M = M({}^3H_\Lambda) = 2993.6\ MeV/c^2$; $m_1 = m_d = 1878\ MeV/c^2$; $m_2 = m_\Lambda = 1115.7\ MeV/c^2$).

Binding energy of Λ-hyperon in ${}^3H_\Lambda$:

G. Bohm et al. Nucl. Phys. B4, 511 (1968):

$$\varepsilon_B^{(\Lambda)} = (0.01 \pm 0.07)\ MeV$$

M. Juric et al. Nucl. Phys. B52, 1 (1973):

$$\varepsilon_B^{(\Lambda)} = (0.15 \pm 0.08)\ MeV$$

Taking $\gamma = 6$ and $\varepsilon_B = 0.08\ MeV \Rightarrow$ we obtain:

$$\varkappa \approx 0.43\, Z^{1/3};\quad \Delta B \approx 0.63\,\varkappa;\quad Z_{eff}^2 \approx Z^2(1 - 0.02\, Z^{1/3}).$$

a) Uranium target ($Z = 92$).

$\sigma_0 = 6.38\ bn$; $\Delta\sigma = -0.58\ bn$; $\sigma = 5.8\ bn$

б) Tin target ($Z = 50$).

$\sigma_0 = 1.88\ bn$; $\Delta\sigma = -0.14\ bn$; $\sigma = 1.74\ bn$

(here σ_0 is the cross-section calculated without taking into account the finite sizes of the target)

2) $^6He_\Lambda + Z \rightarrow {}^5He_\Lambda + n + Z$

(Coulomb dissociation of $^6He_\Lambda$ into the hypernucleus $^5He_\Lambda$ and the neutron; $\tilde{Z}=2$; $M = M(^6He_\Lambda) = 5.78\ GeV/c^2$; $m_1 = M(^5He_\Lambda) = 4.84\ GeV/c^2$; $m_2 = m_n = 939\ MeV/c^2$)

Analysis of the data on masses of hypernuclei and their nucleon bases $\Rightarrow$ the binding energy of the neutron in the hypernucleus $^6He_\Lambda$: $\varepsilon_B^{(n)} = (0.23 \pm 0.13)\ MeV$.

Taking $\gamma = 6$ and $\varepsilon_B = 0.15\ MeV \Rightarrow$ we obtain:

$\varkappa \approx 1.43\, Z^{1/3}$; $\Delta B \approx 0.5\varkappa$; $Z_{eff}^2 \approx Z^2(1 - 0.05 Z^{1/3})$

a) $Z = 92$;

$\sigma_0 = 2.50\ bn$; $\Delta\sigma = -0.54\ bn$; $\sigma = 1.96\ bn$.

b) $Z = 50$;

$\sigma_0 = 0.74\ bn$; $\Delta\sigma = -0.14\ bn$; $\sigma = 0.60\ bn$.

So, for the considered cases (especially – for the case of $^6He_\Lambda$) the correction $\Delta\sigma$ to the Coulomb dissociation cross-section, connected with the finite size of the target, proves to be rather essential.

5 Summary

1. The process of Coulomb dissociation of weakly bound relativistic nuclei and hypernuclei has been studied within the two-cluster „deuteron-like" model. Explicit expressions for the total effective cross-section of Coulomb disintegration, taking into account the corrections conditioned by the finite size of the target nucleus, are obtained.

2. It is shown that the experimental measurement of Coulomb dissociation cross-section for weakly bound nuclei and hypernuclei enables one to determine the value of the binding energy for these systems.