# Theoretical possibilities for the existence of tri- and tetraneutron systems

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# The main task:

- To investigate 2-, 3- and 4-neutron systems with different central pairwise $n$-$n$ potentials and to find such potentials, that can at the same time describe all known two-nucleon data and bind a $^4n$ system or to show that it is impossible.

# Motivation:

- The problem of multineutrons stands for more than 40 years and still there are no final answer

- Possible observation of free tetraneutron clusters at LPC-Caen in experiments of $^{14}Be$ breakup

$$^{14}Be + ^{12}C \rightarrow ^{12}C + ^{10}Be + ^{4}n$$

(*Marqués et al., Phys. Rev. C*, **65**, *2002*)

- to obtain deeper theoretical knowledge of 3- and 4-fermion systems.

# Basic equations (3 fermions)

The total antisymmetric wave function of the 4-fermion system

$$\Psi^a(1,2,3) = \frac{1}{\sqrt{2}}\left(\Phi'(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\xi'' - \Phi''(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\xi'\right)$$

where $\Phi'$ and $\Phi''$, spatial components with symmetry [2,1] ($^3n$ system) are defined as

$$\Phi'(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) \equiv \Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3),$$
$$\Phi''(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) \equiv \frac{1}{\sqrt{3}}\left(2\Phi(\mathbf{r}_1,\mathbf{r}_3,\mathbf{r}_2) - \Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\right),$$

Schrödinger equation for one spatial component of the wave function

$$\Bigg\{\sum_{i=1}^{3}\frac{\mathbf{p}_i^2}{2m} + \frac{1}{2}\sum_{i>j=1}^{3}\left(V_s^+(r_{ij}) + V_t^-(r_{ij})\right) +$$
$$+\frac{1}{2}\sum_{(ij)\neq(23)}(-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right) -$$
$$-\frac{1}{2}\sum_{(ij)\neq(12)}(-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right)\hat{P}_{23}\Bigg\}\Phi = E\Phi$$

# Angular momentum of 3-fermion system

In a system of harmonic oscillators we know that the lowest energy state with symmetry [2,1] is the state with angular momentum $L = 1$. Then, to be sure, that we got a lowest state we need to calculate both states with $L = 0$ and $L = 1$ or choose variational function without explicitly set angular momentum.

We use trial variational functions as combination of Gaussians. Variational function for angular momentum $L = 0$:

$$\Phi_{L=0}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \hat{A} \sum_{k=1}^{N} C_k \exp\left(-\sum_{i>j=1}^{3} u_{ij}^k r_{ij}^2\right)$$

and for angular momentum $L = 1$ and odd parity:

$$\Phi_{L=1}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \hat{A} \left(q_{1z} \sum_{k=1}^{N} C_k \exp\left(-\sum_{i>j=1}^{3} u_{ij}^k r_{ij}^2\right) + q_{2z} \sum_{k=1}^{M} D_k \exp\left(-\sum_{i>j=1}^{3} v_{ij}^k r_{ij}^2\right)\right)$$

$q_{1z}$ and $q_{2z}$ — are z-components of Jacobi relative coordinates.

# Basic equations (4 fermions)

The total antisymmetric wave function of the 4-fermion system

$$\Psi^a(1,2,3,4) = \frac{1}{\sqrt{2}}\left(\Phi'(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\xi'' - \Phi''(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\xi'\right)$$

where $\Phi'$ and $\Phi''$, spatial components with symmetry [2,2] ($^4n$ system) are defined as

$$\Phi'(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) \equiv \Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4),$$
$$\Phi''(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) \equiv \frac{1}{\sqrt{3}}\left(2\Phi(\mathbf{r}_1,\mathbf{r}_3,\mathbf{r}_2,\mathbf{r}_4) - \Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right),$$

Schrödinger equation for one spatial component of the wave function

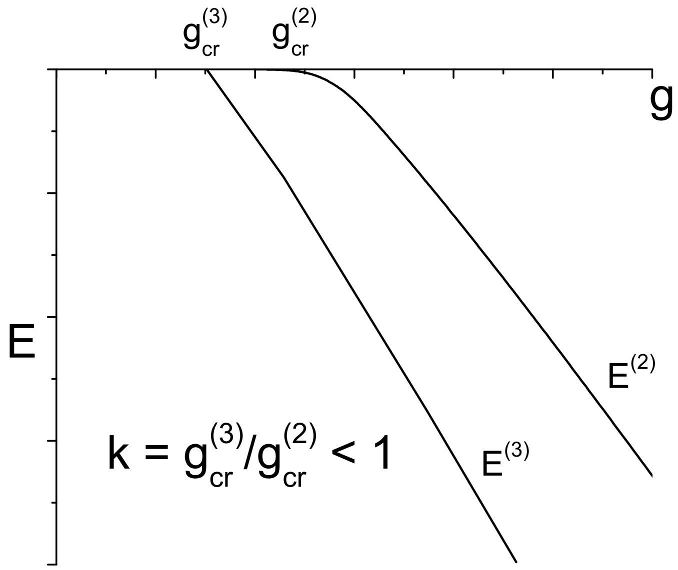
$$\left\{\sum_{i=1}^{4}\frac{\mathbf{p}_i^2}{2m} + \frac{1}{2}\sum_{i>j=1}^{4}\left(V_s^+(r_{ij}) + V_t^-(r_{ij})\right) + \right.$$
$$+\frac{1}{2}\sum_{(ij)\neq(14),(23)}(-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right) -$$
$$\left. -\frac{1}{2}\sum_{(ij)\neq(12),(34)}(-1)^{i+j}\left(V_s^+(r_{ij}) - V_t^-(r_{ij})\right)\hat{P}_{23}\right\}\Phi = E\Phi$$

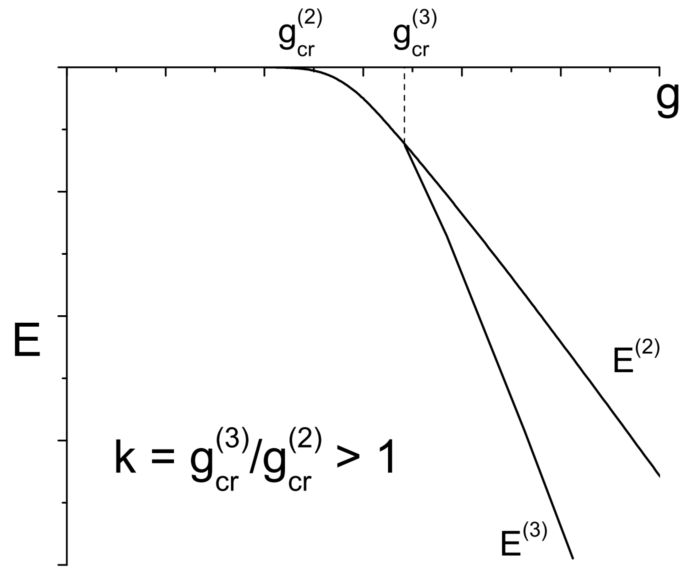
# Critical coupling constants

Let $g$ — is a coupling constant, that characterizes intensity of attractive potential. Than $g^{(2)}_{cr}$, $g^{(3)}_{cr}$ and $g^{(4)}_{cr}$, are critical coupling constants where bound state appears in 2-, 3- and 4-fermion system respectively.

If $k = g^{(3)}_{cr}/g^{(2)}_{cr} > 1$, than $^3n$ may exist only in the region, where $^2n$ is bound.

If $k = g^{(3)}_{cr}/g^{(2)}_{cr} < 1$, than $^3n$ may be bound, while $^2n$ is unbound.



# Criteria to choose the potential

If we want to describe bound tetraneutron, we must choose the potential, that can satisfy these conditions:

- The ratio $k^{(4)} = g^{(4)}_{cr}/g^{(2)}_{cr}$ must be lower than 1 (or $k^{(3)} = g^{(3)}_{cr}/g^{(2)}_{cr}$ for trineutron)
- Low-energy neutron-neutron scattering parameters:
  scattering length ($a_{s(nn)} = -18.9$ fm),
  effective radius ($r_{0s(nn)} = 2.75$ fm).
- Binding energies of light nuclei ($^3H$, $^4He$ ...)
- ... other experimental data

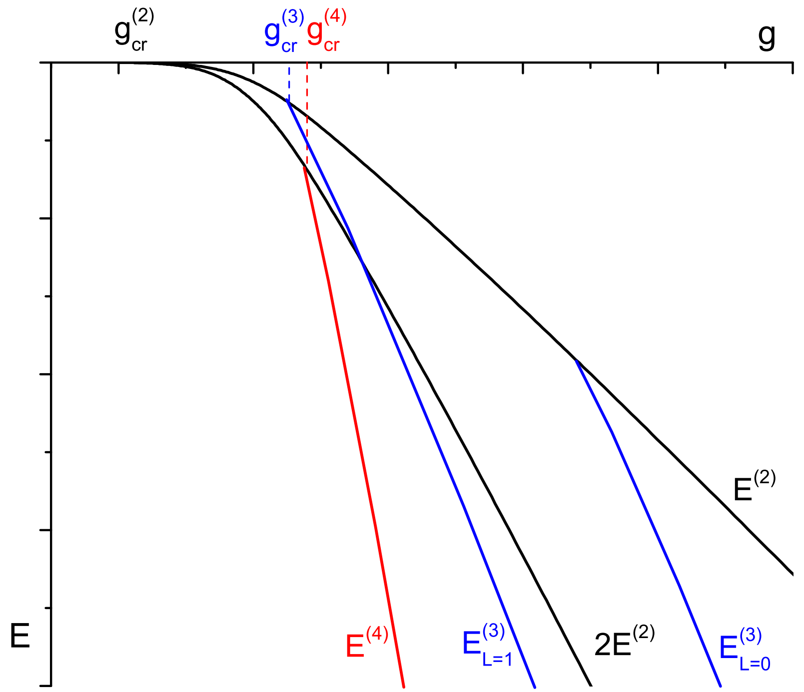
If no potential can satisfy all these conditions, than bound tri- or tetraneutron can not be described with pairwise potentials.

# Standard potentials (spinless case)

- Simplest potential (Gaussian well)

$$V_{ij}(r) = -g\, e^{-r^2}$$

$k^{(3)} = 1.43$
$k^{(4)} = 1.46$




This behavior of energy levels is similar for all standard nuclear potentials

# Standard potentials (spinless case)

- Simplest potential (Gaussian well)

$$V_{ij}(r) = -g\, e^{-r^2}$$

$k^{(3)} = 1.43$
$k^{(4)} = 1.46$

- More realistic nuclear potentials with short range repulsion

– Volkov potential:

$$V_{ij}(r) = g\left(144.86\, e^{-(r/0.82)^2} - 83.34\, e^{-(r/1.6)^2}\right)$$

$k^{(3)} = 1.39$
$k^{(4)} = 1.44$

– Best achieved value of $k$:

$$V_{ij}(r) = g\left(1.5\, e^{-(r/0.9)^2} - e^{-r^2}\right)$$

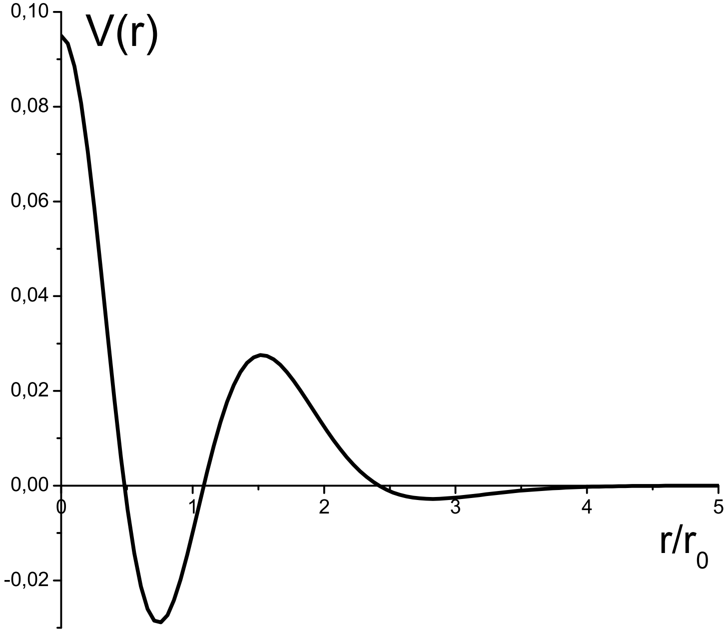
$k^{(3)} = 1.278$
$k^{(4)} = 1.27$

There are no possibilities to bind the $^3n$ or $^4n$ system with standard or near-standard nuclear pairwise potentials.

# Specially constructed potential (spinless case)

- Potential with two attractive regimes

$$V_{ij}(r) = g\left(0.43\, e^{-(r/0.6)^2} - e^{-r^2} + 1.085\, e^{-(r/1.3)^2} - 0.42\, e^{-(r/1.5)^2}\right)$$

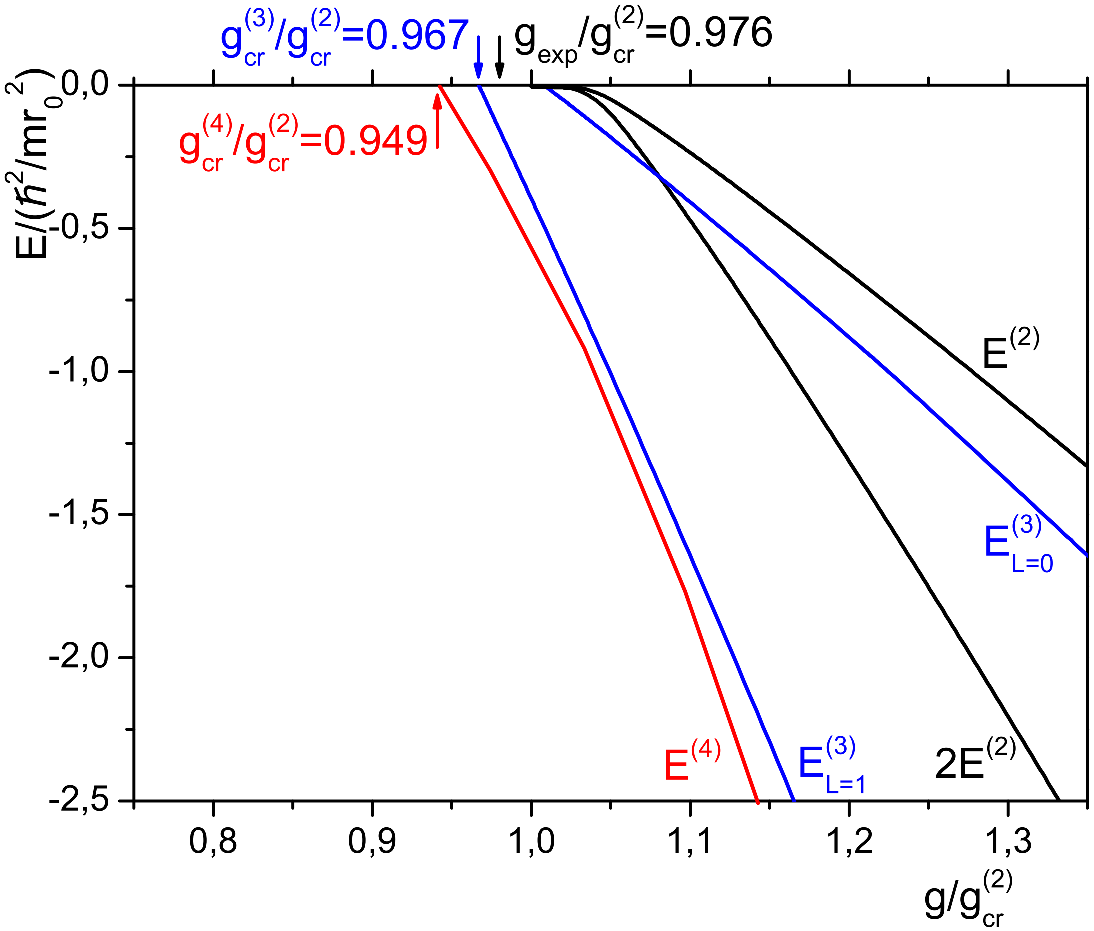



This potential gives
$k^{(3)} = 0.967 < 1$ and
$k^{(4)} = 0.949 < 1$.

Low-energy $n$-$n$ parameters are reproduced by proper change to dimensional quantities ($r \to \frac{r}{r_0}$, $V \to V\,\frac{2\,m\,r_0^2}{\hbar^2}$, were $r_0 = 0.488515$) and putting $g = 322.4 = 0.976 g_{cr}^{(2)}$.

This is the only type of $n$-$n$ potential, that meets criteria above, and hence the only possibility to describe bound $^4n$ with pairwise potentials.

# Behavior of energy levels




Energies of the 4-fermion, 3-fermion (with L=0) and 2-fermion systems versus the coupling constant of potential.

- On the graph $g_{exper}$ is a value of coupling constant where low-energy $n$-$n$ parameters are reproduced. At this point 3- and 4-fermion systems are bound.

- Energy of the 4- and 3-fermion systems are almost parallel to the corresponding thresholds (clustering) in a wide range of coupling constants.

- Slope of the $E^{(4)}$ curve is lower than of $E^{(3)}_{L=1}$ near critical values. It results in that there is a point where these curves are closest to each other.

# Spin-dependent potential

Let's choose potential described above as $V_s^+(r)$.

Then putting $V_t^-(r) \equiv 0$ gives no bound ${}^3n$ and ${}^4n$, while we know, that it must be repulsive.

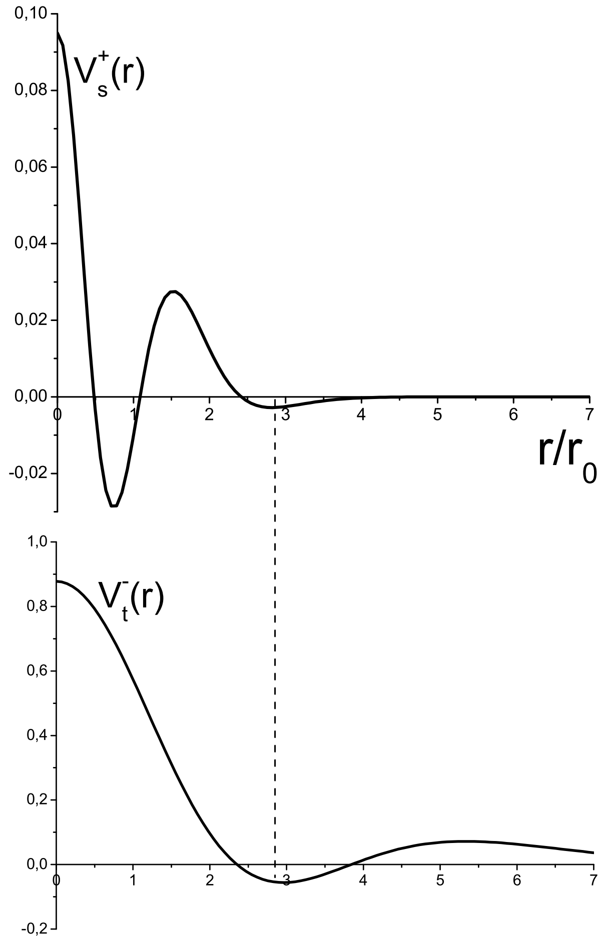
The only way out is to choose repulsive triplet potential with small attractive well, correlated with outer well in singlet.

Full spin-dependent potential:

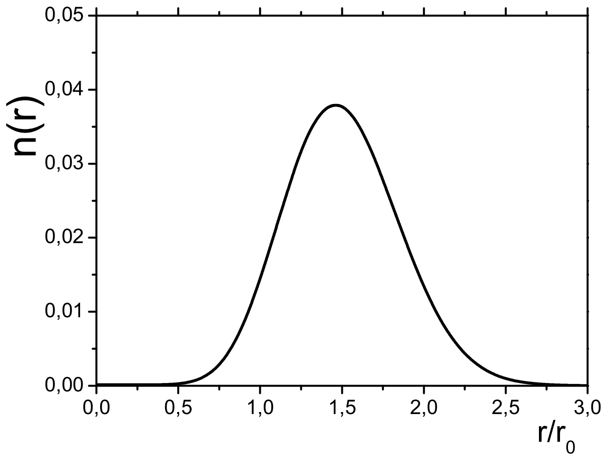
$$V_s^+(r) \;=\; g\left(0.43\,e^{-(r/0.6)^2} - e^{-r^2} + 1.085\,e^{-(r/1.3)^2} - 0.42\,e^{-(r/1.5)^2}\right)$$

$$V_t^-(r) \;=\; 30.968\,e^{-(r/2)^2} - 32.676\,e^{-(r/3)^2} + 14.0\,e^{-(r/4)^2}$$

This potential gives bound ${}^4n$ with energy about 0.5 MeV and no bound ${}^3n$ ($k^{(3)} = 4.27$). There is no possibility to bind ${}^3n$ with "normal" triplet potential.

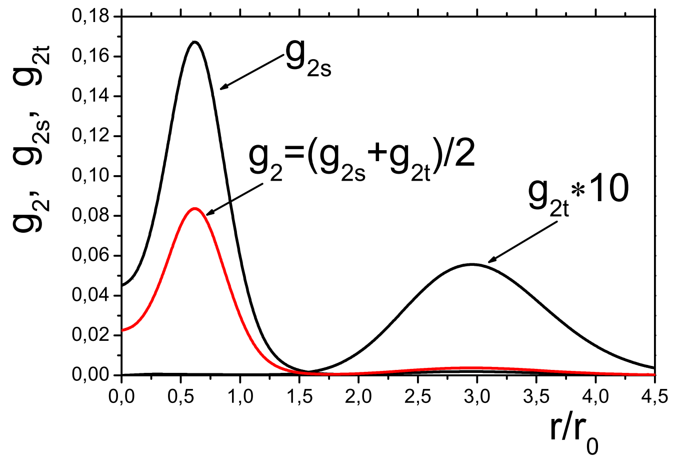


# Structure functions of hypothetical $^4n$




Profile of the density distribution in hypothetical $^4n$ system

- Anomalously small size of 4-neutron system ($\langle r^2 \rangle^{1/2} = 0.83$ fm)
- A "bubble" structure of the system.




Profiles of the singlet, triplet, and total pair correlation functions of the hypothetical $^4n$ system.

- Singlet correlation function is much larger than triplet
- Singlet correlation function has high peak correlated with inner well in potential
- Triplet correlation function is nonzero only in the region near attractive well it triplet potential. It means that triplet potential acts as an effective attraction.

# Conclusion

- We develop precise variational schemes and perform reliable calculations of 3- and 4-neutron system. We show, that bound tetraneutron is impossible with standard pairwise potentials.

- We discovered a wide class of $n$-$n$ potentials with two attraction regimes that possess following properties:

  – negative value of 4-fermion energy at $g_{exper}$ (bound tetraneutron with $B \simeq 0.5$ MeV)

  – small ($\sim 2$ MeV) underbinding of $^3H$ and $^4He$. Exact energies of such systems can be obtained by slight variation of potentials

  – anomalously small size ($\langle r^2 \rangle^{1/2} = 0.83$ fm) and "bubble" structure of the 4-neutron system

  – high maximum in the singlet scattering phase shift $\delta_s \approx 160^0$ at the energies of neutrons near $100 - 150$ MeV.

- Trineutron system can not be bound with any reasonable spin-dependent interaction.

- More realistic results can not be obtained using pairwise potentials and better description may be achieved only using more complicated types of interactions.

**THANK YOU**