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Three-body decay of nuclear resonances

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CONTENT

1. Theoretical formulation
2. Energies and widths, ex. ^{12}C
3. Short-range interaction, ex. ^6He
4. Two large scattering lengths (Efimov effect), ex. ^{11}Li
5. Short-range plus Coulomb interaction, ex ^6Be
6. Isospin mixing, ex. ^6Li
7. Conclusions

Theoretical formulation:

Coordinates: (\mathbf{x}, \mathbf{y}) are mass scaled Jacobi coordinates

\mathbf{r}_{ij} is the vector connecting particle i and j

$$x^2 = r_{ik}^2 \frac{m_i m_k}{m(m_i + m_k)}$$

$$y^2 = r_{j,ik}^2 \frac{m_j(m_i + m_k)}{m(m_i + m_k + m_j)}$$

$$\rho^2 = x^2 + y^2 = \frac{1}{m(m_i + m_k + m_j)} \sum_{i < j} m_i m_j r_{ij}^2$$

$\Omega = \{\Omega_x, \Omega_y, \alpha\}$, i.e. directions of (\mathbf{x}, \mathbf{y}) and $\tan \alpha = \frac{x}{y}$

Complex scaling: $\rho \rightarrow \rho \exp(i\theta)$

Adiabatic hyperspherical expansion:

Choose interactions and solve Faddeev equations for each ρ

Compute angular eigenvalues λ_n and eigenfunctions $\{\Phi_n(\rho, \Omega)\}$

The three-body bound state or resonance wave function Ψ is:

$$\begin{aligned} \Psi(\mathbf{x}, \mathbf{y}) &= \sum_n f_n(\rho) \Phi_n(\rho, \Omega) \\ &= \sum_n f_n(\rho) \left(\phi_1^{(n)}(\rho, \Omega) + \phi_2^{(n)}(\rho, \Omega) + \phi_3^{(n)}(\rho, \Omega) \right) \end{aligned}$$

Solve radial equations: $f_n(\rho)$ and complex energy eigenvalues

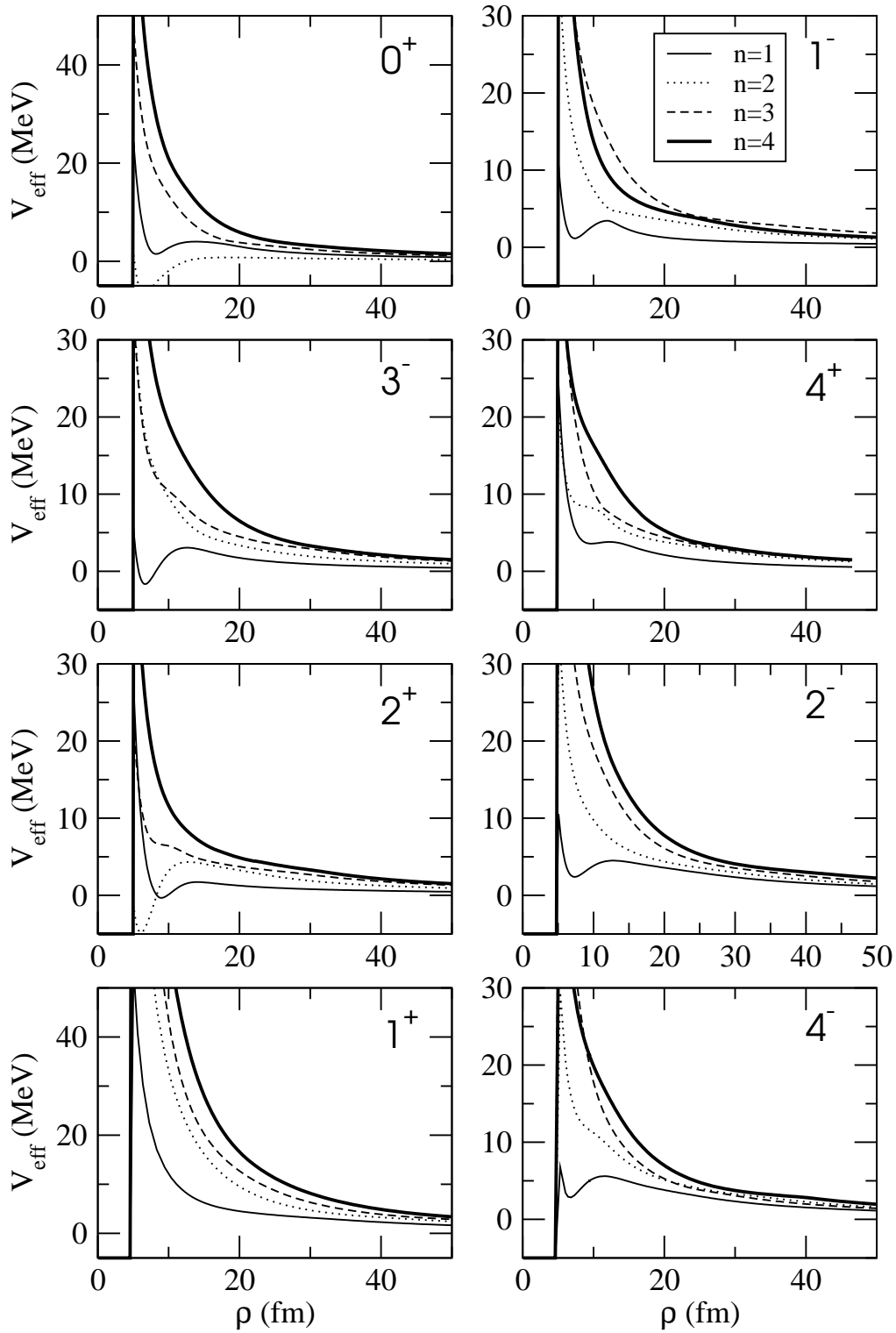


Figure 1: The real parts of the four lowest adiabatic effective potentials as functions of ρ for the ^{12}C resonances with J^π as indicated on the figure.

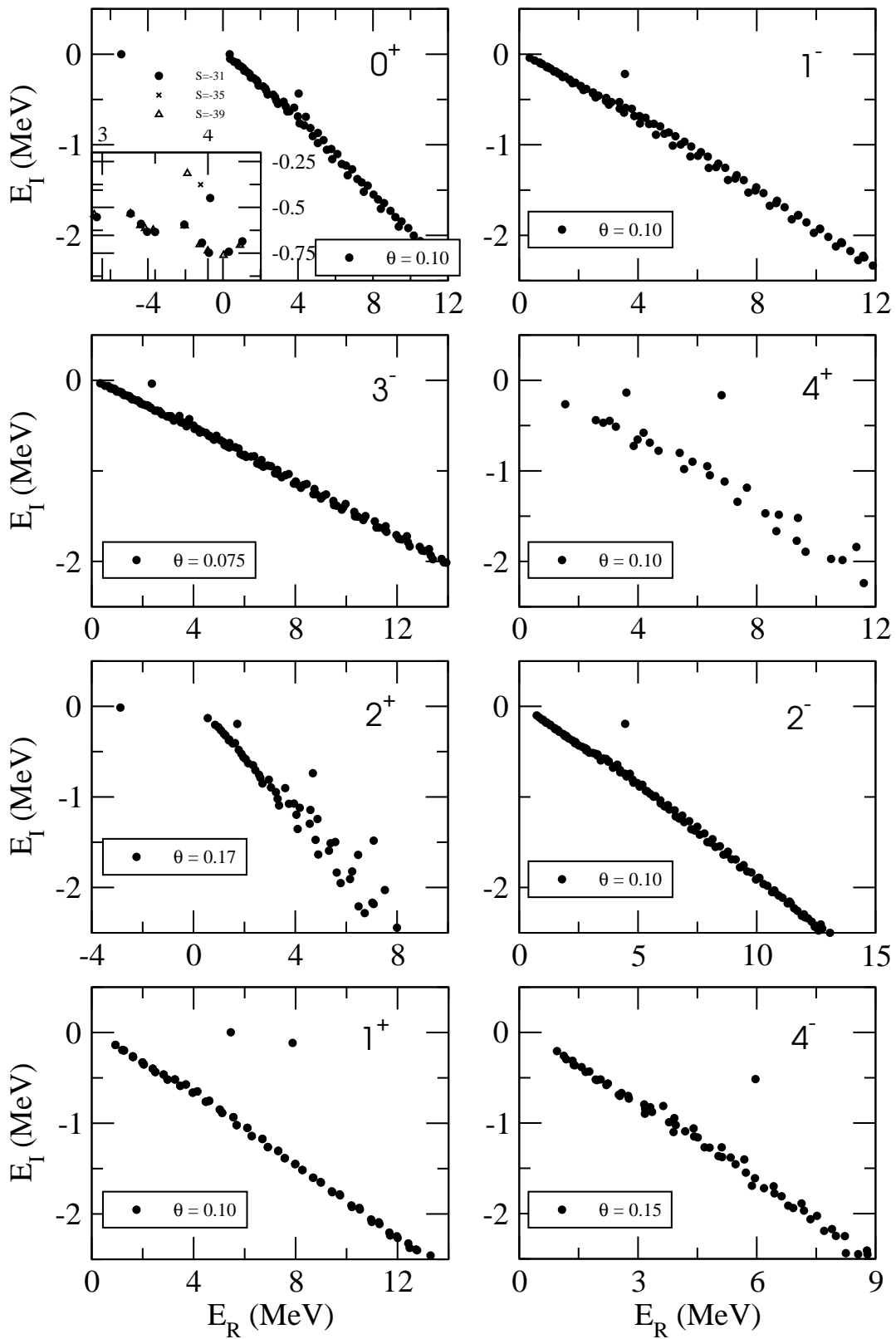


Figure 2: The real and imaginary parts of the ^{12}C resonances after rotation by the angle θ .

Table 1. J^π ; $E_{R,exp}$ (MeV); $\Gamma_{R,exp}$ (keV); $E_{R,th}$ (MeV); $\Gamma_{R,th}$ (keV); 3-body potential parameters: S (MeV); b (fm); rotation angle (rad).

J^π	$E_{R,exp}$	$\Gamma_{R,exp}$	$E_{R,th}$	$\Gamma_{R,th}$	S	b	θ
0^+	-7.25	bound	-6.113	bound	-39	5	0.0005
	0.38	8.5×10^{-3}	0.318	5.97×10^{-3}	-39	5	0.0005
			4.037	0.922	-31	5	0.1
1^-	3.57	315	3.569	432.8	-6.8	5	0.1
3^-	2.37	34	2.363	71.6	-1.7	5	0.075
4^+	6.81	258	6.808	361.3	-26	5	0.1
			3.633	300.0	-26	5	0.1
2^+	-2.875	bound	-2.875	bound	-21.5	5	0.17
	3.88	430	1.730	387.7	-21.5	5	0.17
			4.690	1.459	-21.5	5	0.17
			7.057	3.051	-21.5	5	0.17
2^-	4.55	260	4.464	464.1	-7.3	5	0.1
	6.08	375					
1^+	5.43	18.1	-	-	-154.5	5	
	7.84	43.6	-	-	-141.5	5	

Figure 3: Experimental and computed energies and widths of ^{12}C resonances with J^π .

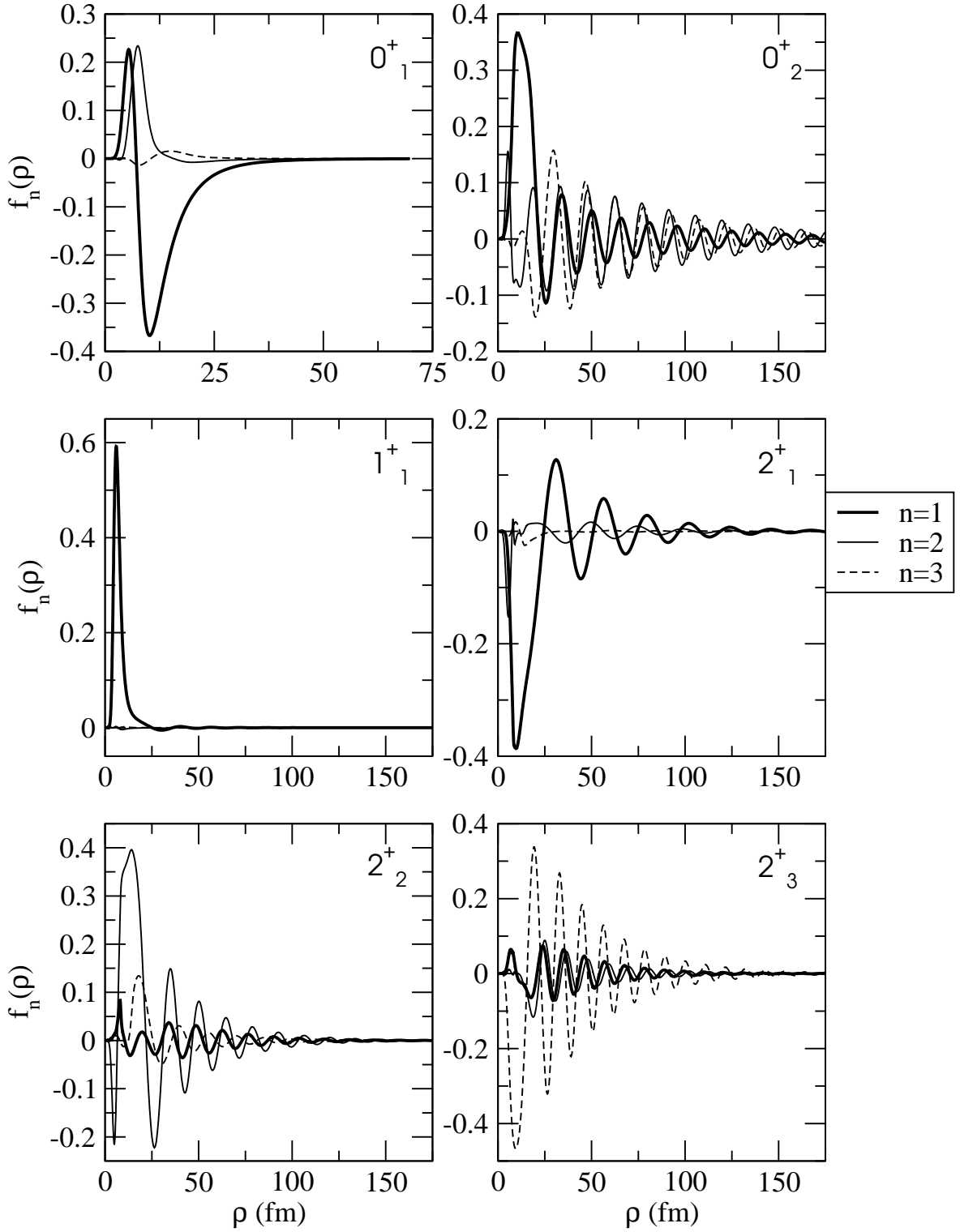


Figure 4: The real parts of the three lowest adiabatic radial wavefunctions as functions of ρ for the ^{12}C resonances with J^π .

$$\Psi(\mathbf{x}, \mathbf{y}) = \sum_n f_n(\rho) \Phi_n(\rho, \Omega) \quad 6$$

Each fall off exponentially while oscillating around zero

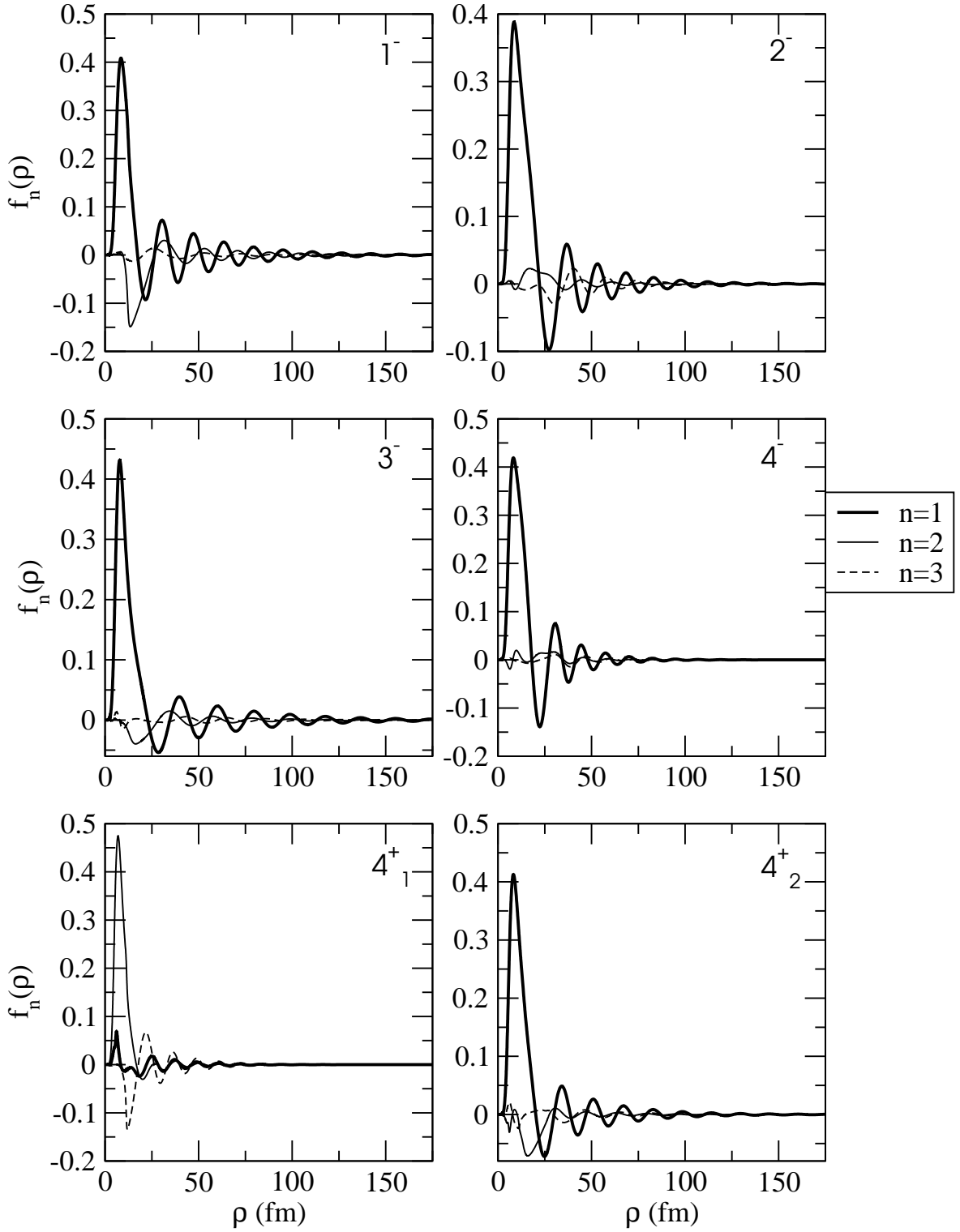


Figure 5: The real parts of the three lowest adiabatic radial wavefunctions as functions of ρ for the ^{12}C resonances with J^π .

$$\Psi(\mathbf{x}, \mathbf{y}) = \sum_n f_n(\rho) \Phi_n(\rho, \Omega)$$

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Each fall off exponentially while oscillating around zero

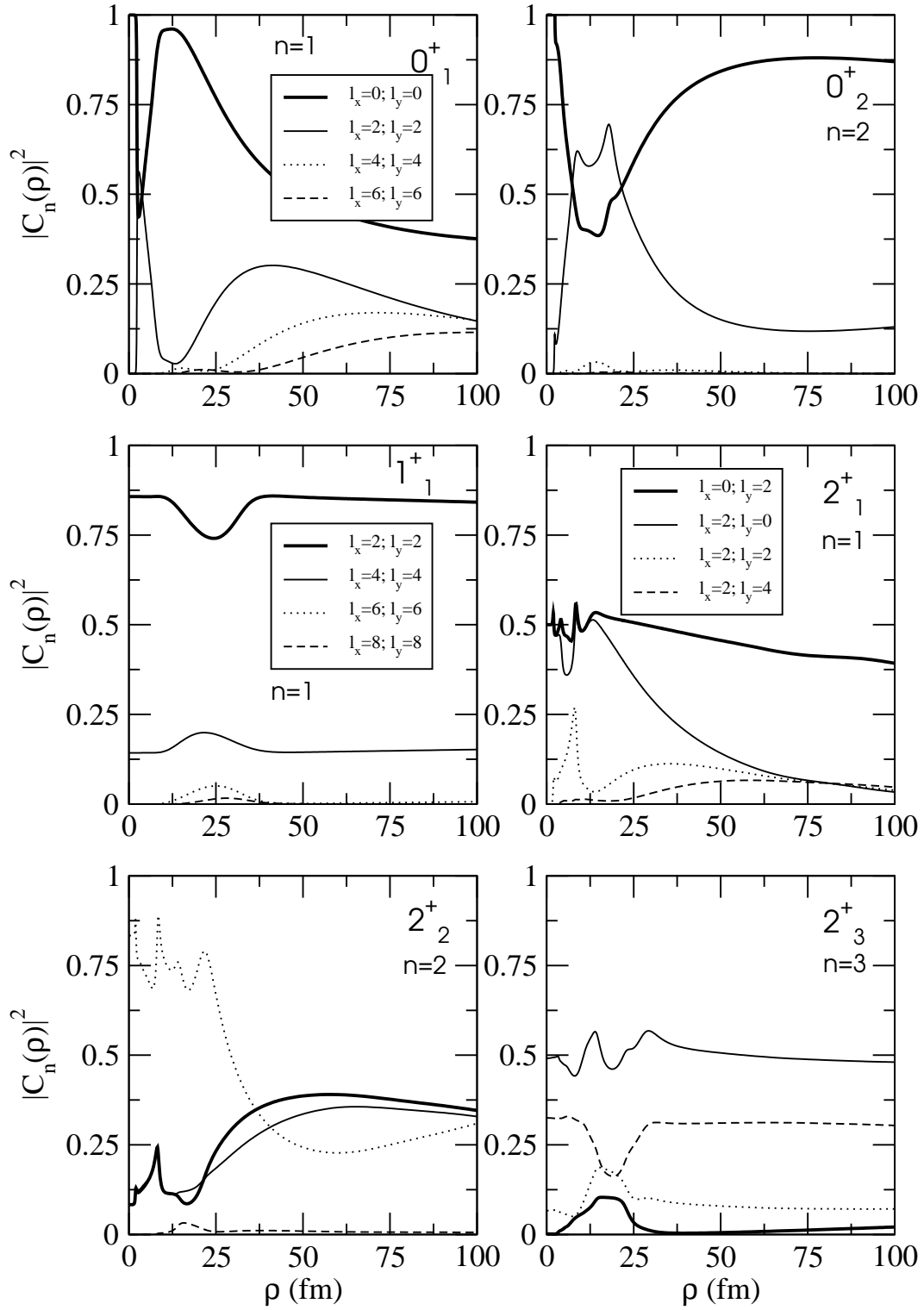


Figure 6: The partial wave decomposition of the ^{12}C resonances with J^π as indicated on the figure shown as function of ρ for the dominating adiabatic eigenvalue.

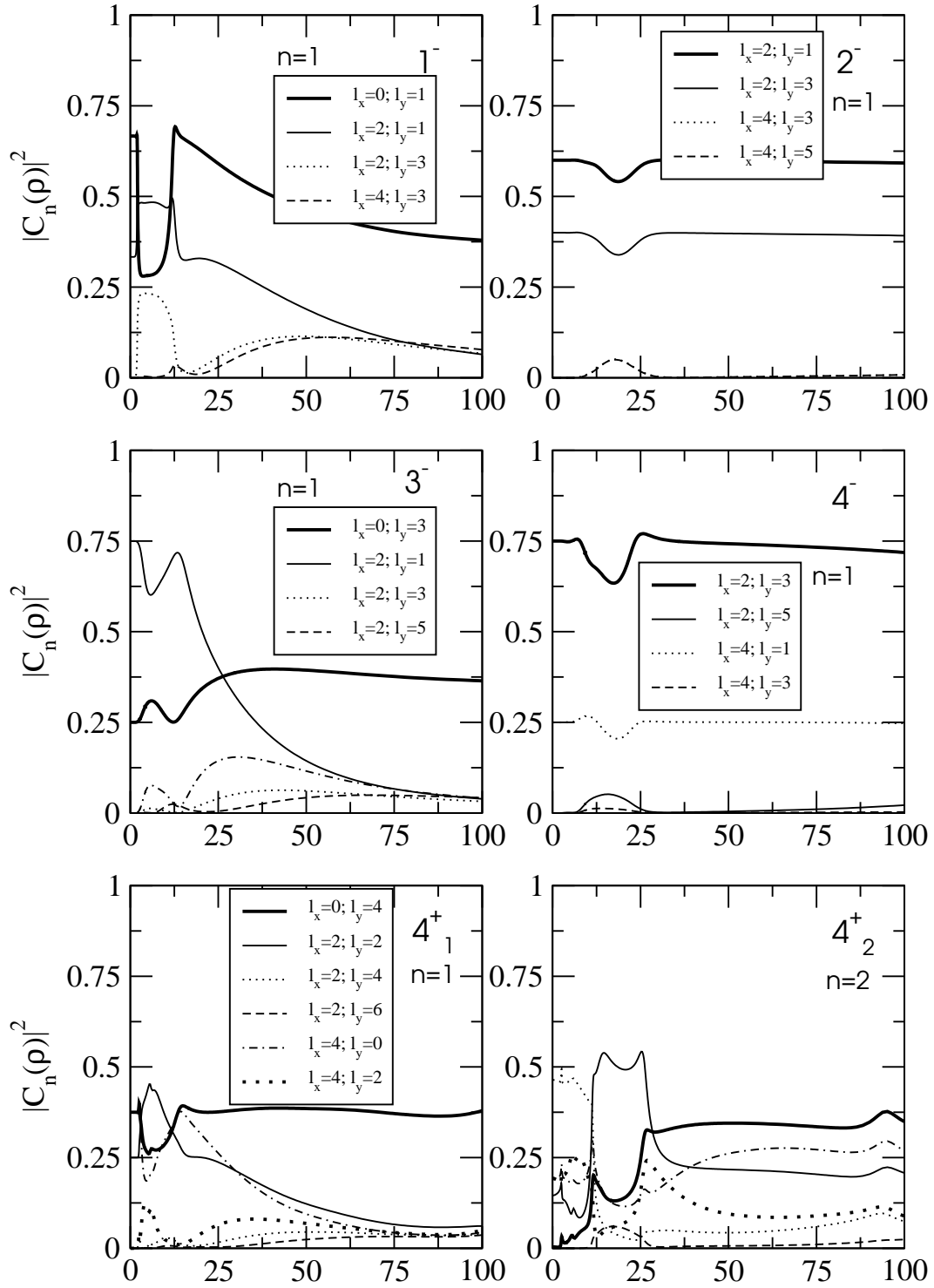
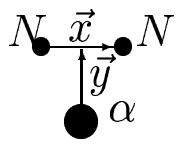
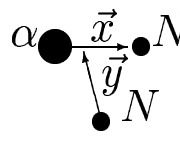


Figure 7: The partial wave decomposition of the ^{12}C resonances with J^π as indicated on the figure shown as function of ρ for the dominating adiabatic eigenvalue.

Table 1: Components included in the three-body calculations have $K_{max} = 20$ except those specified here. The left part refers to the components in the first Jacobi set (\mathbf{x} connecting the two nucleons), and the right part to the ones in the second and third Jacobi sets (\mathbf{x} connecting the alpha-particle and one of the nucleons).

1 st Jacobi set						2 nd and 3 rd Jacobi sets					
											
ℓ_x	ℓ_y	L	s_x	s_y	K_{max}	ℓ_x	ℓ_y	L	s_x	s_y	K_{max}
0	2	2	0	0	180	0	2	2	1/2	0	44
2	0	2	0	0	180	0	2	2	1/2	1	44
1	1	1	1	1	180	2	0	2	1/2	0	70
1	1	2	1	1	64	2	0	2	1/2	1	44
2	2	2	0	0	90	1	1	1	1/2	1	240
1	3	2	1	1	42	1	1	2	1/2	0	240
3	1	2	1	1	42	1	1	2	1/2	1	44
2	4	2	0	0	54	2	2	1	1/2	1	32
4	2	2	0	0	54	2	2	2	1/2	0	50
4	4	2	0	0	68	2	2	2	1/2	1	42
						1	3	2	1/2	0	42
						1	3	2	1/2	1	42

Notice Jacobi coordinates

Each of the Faddeev components are partial wave expanded

Rather large K_{max} in each of these many partial waves

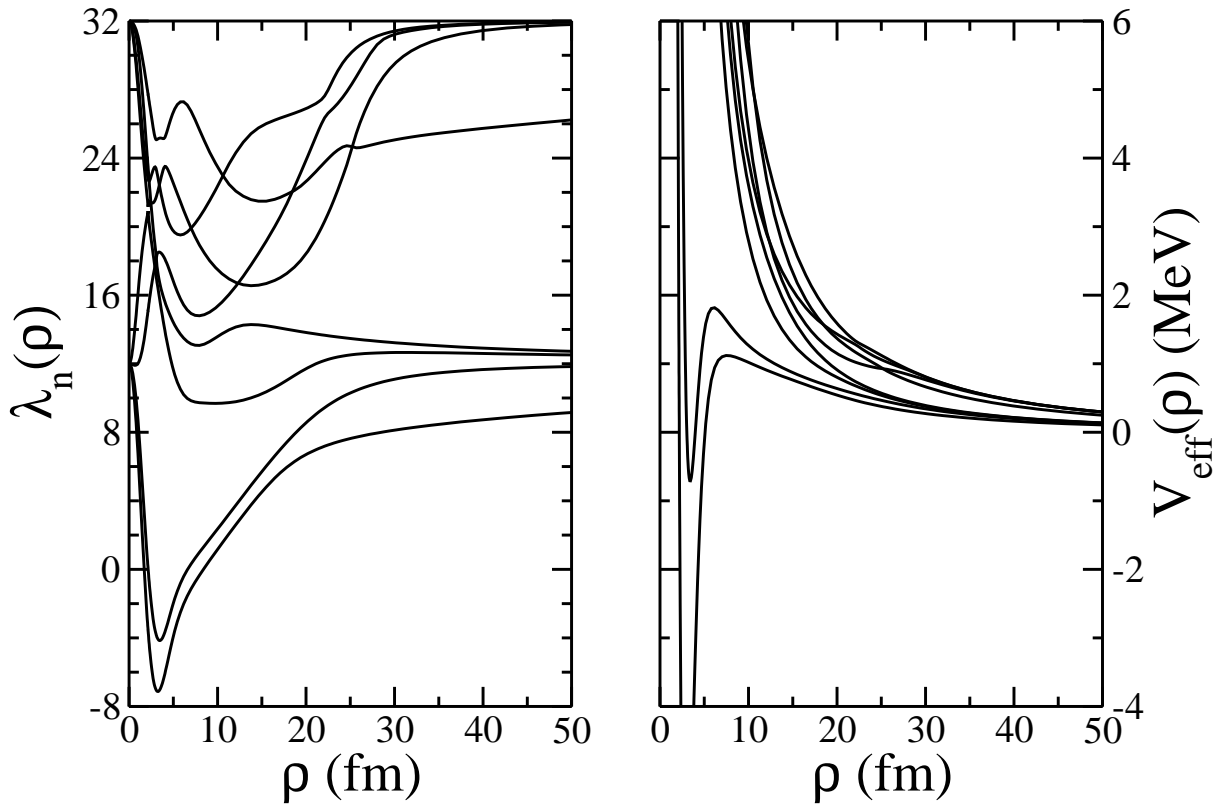


Figure 8: The real parts of the lowest 8 angular eigenvalues (left) and corresponding adiabatic potentials (right) as functions of ρ for the 2^+ states in ${}^6\text{He}$ (${}^4\text{He} + n + n$). The scaling angle is $\theta = 0.10$.

Effective hyperradial potentials
 Attractive region and a barrier

Resonance properties:

Energy determined by attractive pocket

Width determined by the barrier

Large-distance behavior determines final state energy distribution

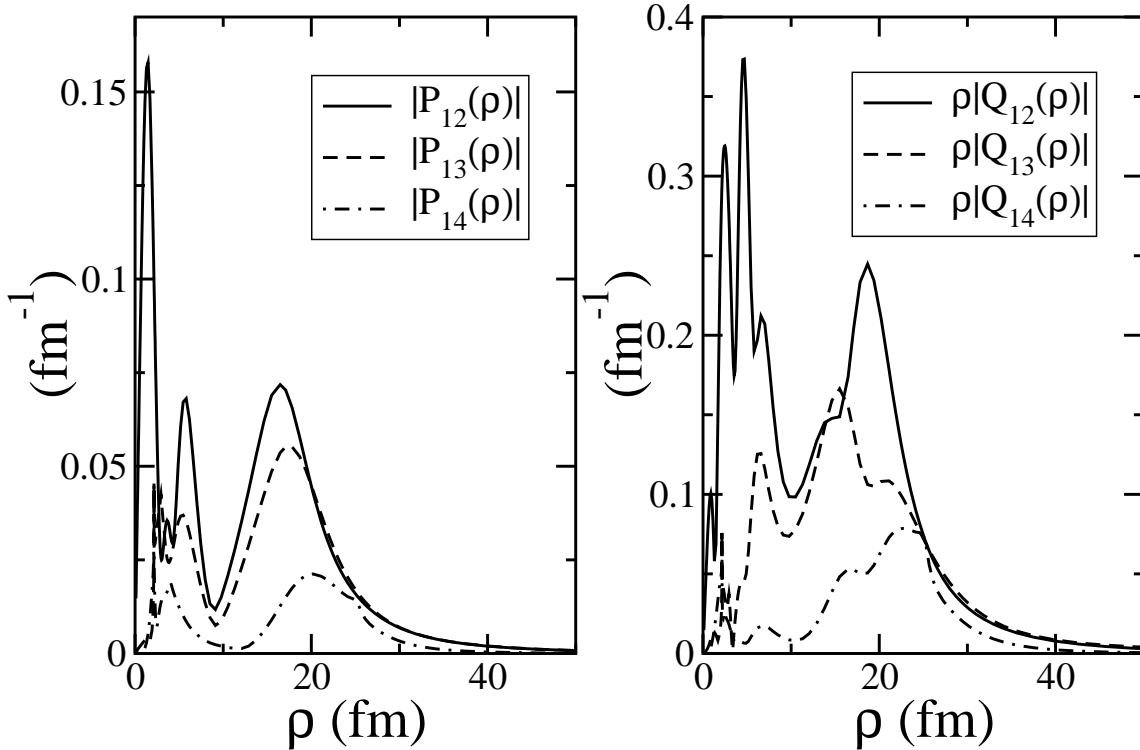


Figure 9: The coupling potentials between the four lowest adiabatic levels for $\theta = 0.10$ shown as functions of ρ for ${}^6\text{He}(2^+)$. The first and the fourth levels have similar quantum numbers but approach the $K = 2$ and 4 levels, respectively. To show the first (P) and second (Q) order coupling potentials in the same units (fm^{-1}) we multiply Q by ρ . (The energy unit is restored in the coupling potentials by including the omitted factor, i.e. $\hbar^2 Q/(2m)$, $\hbar^2 P/(2m)\partial/\partial\rho$).

Couplings determine relative size of radial wavefunctions
 Fall off at intermediate distance
 Numerical stability at large distance
 Compromise between:
 lowest (adiabatic) state and maintaining the structure

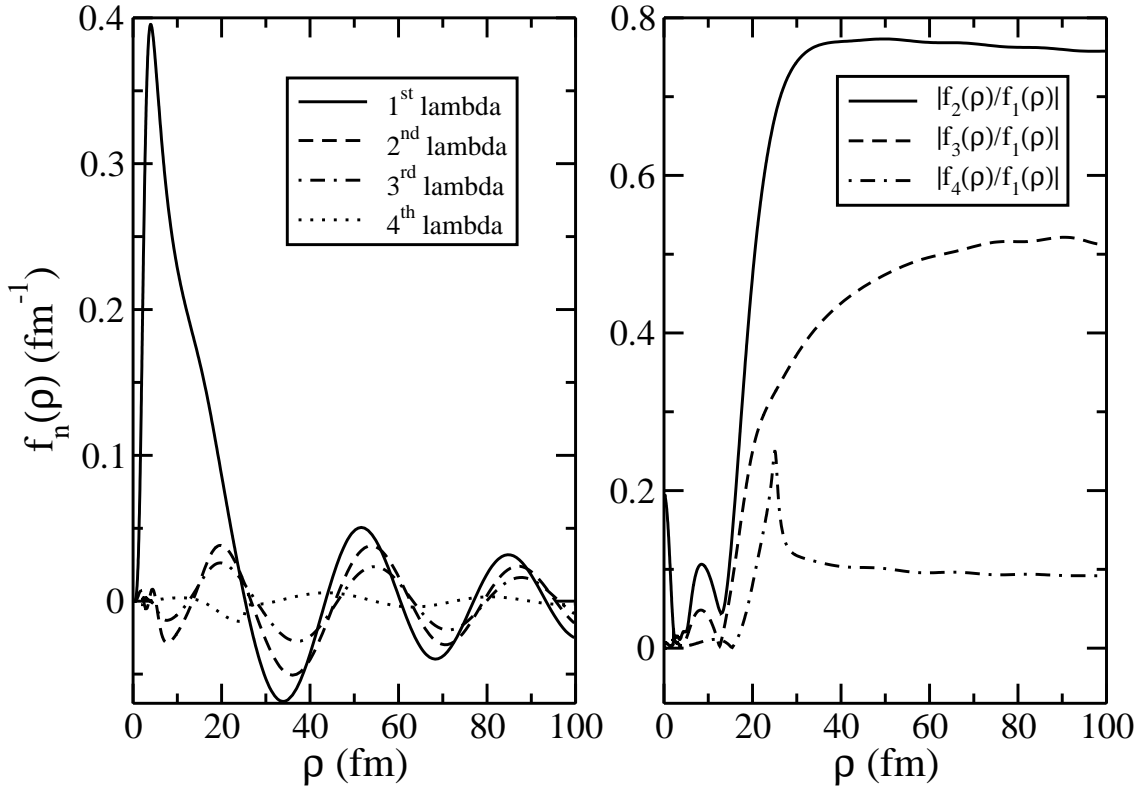


Figure 10: The lowest four radial wavefunctions (left) and their relative sizes (right) for $\theta = 0.10$ as functions ρ for the ${}^6\text{He}(2^+)$ resonance.

$$\Psi(\mathbf{x}, \mathbf{y}) = \sum_n f_n(\rho) \Phi_n(\rho, \Omega)$$

Each fall off exponentially while oscillating around zero
 Relative size at large distance is stable
 Determine energy distribution

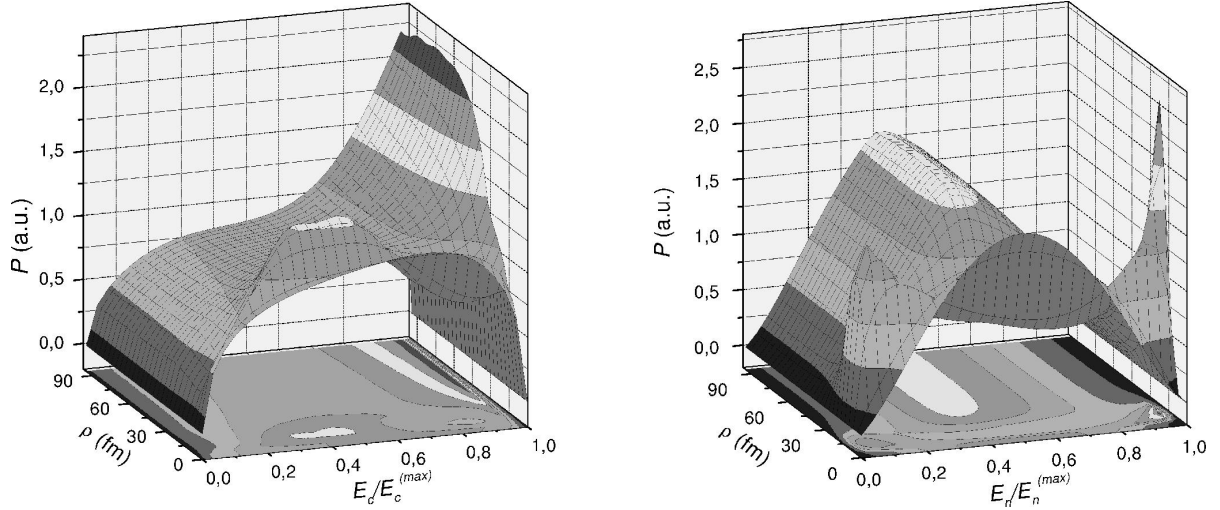


Figure 11: The energy distributions of neutrons and α -particles after decay of ${}^6\text{He}(2^+)$ for $\theta = 0.10$. The three-dimensional plot show the dependence on ρ with inclusion of 8 adiabatic wavefunctions. The maximum energies are $(m_\alpha + m_n)/(m_\alpha + 2m_n)E_0$ and $2m_n/(m_\alpha + 2m_n)E_0$ for the neutron and the α -particle, respectively. Here E_0 is the energy of the decaying resonance.

Kinetic energy distribution of third particle:

$$P(k_y^2) \propto P(\cos^2 \alpha) \propto \sin(2\alpha) \int d\Omega_x d\Omega_y |\Psi(\rho, \alpha, \Omega_x, \Omega_y)|^2$$

Neutrons peak at intermediate energy

α -particles peak at large energy

Two neutron go together, not α -neutron against neutron

Virtual neutron-neutron state is essential

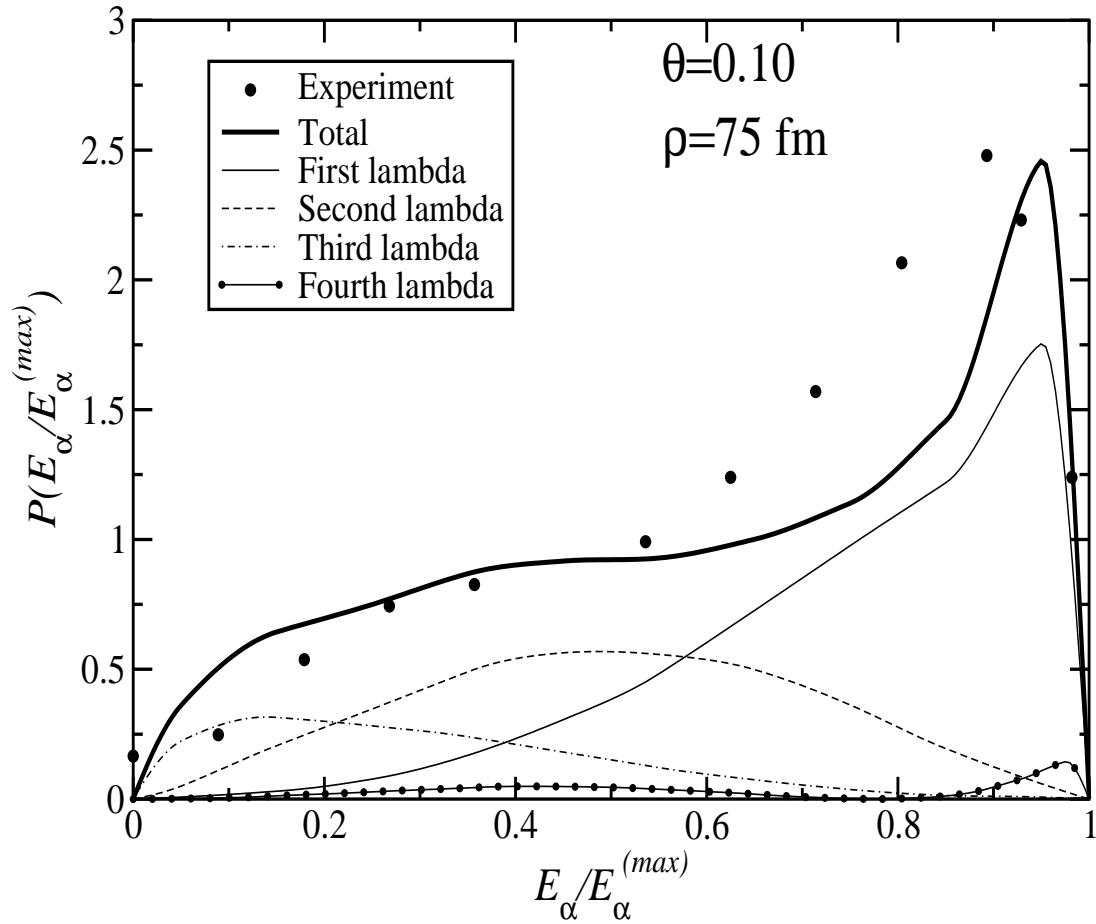


Figure 12: The energy distribution of the α -particle after decay of the 2^+ -resonance in ${}^6\text{He}$. The scaling angle is $\theta = 0.10$ and $\rho = 75 \text{ fm}$ where convergence is reached. The points are extracted from the measurements in [8]. Contributions from the lowest 4 adiabatic potentials are shown individually.

Old data
 Contributions from several adiabatic potentials
 Interference is important

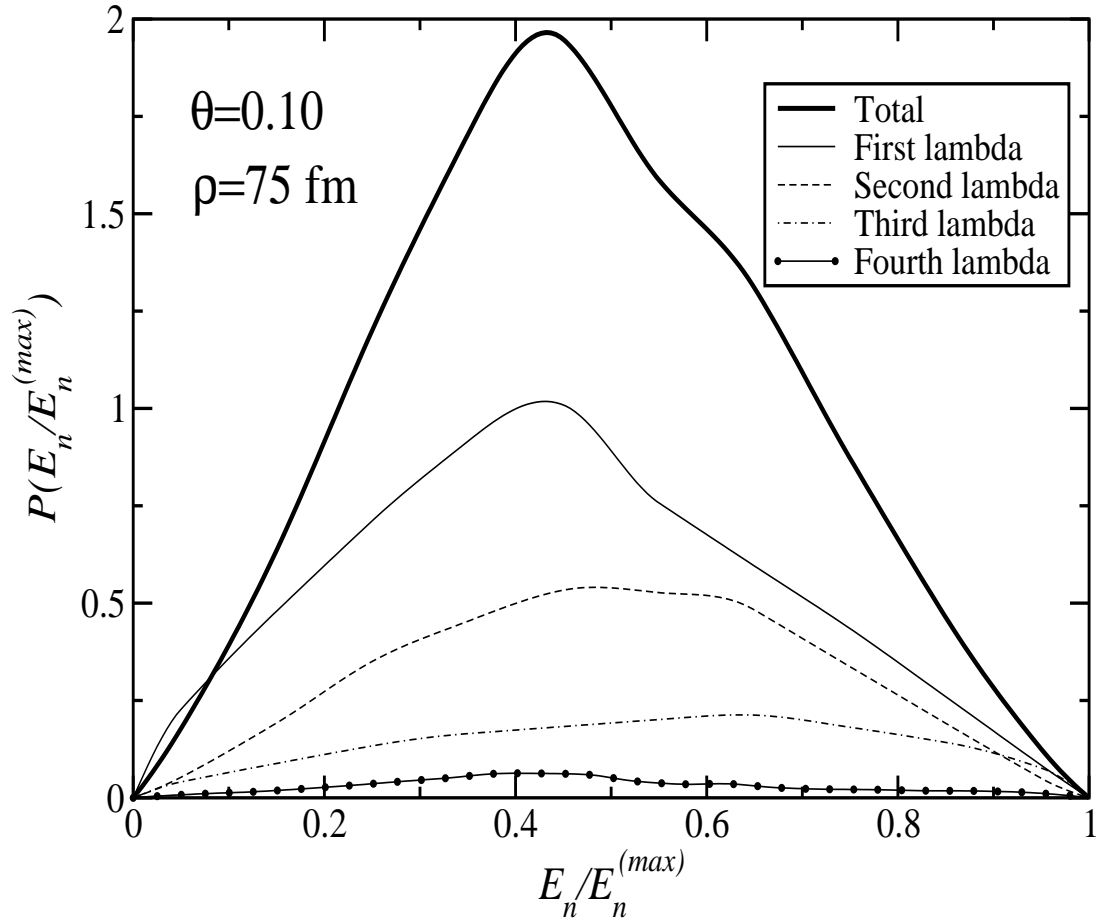


Figure 13: The energy distribution of the neutrons after decay of the 2^+ -resonance in ${}^6\text{He}$. The scaling angle is $\theta = 0.10$ and $\rho = 75$ fm where convergence is reached. The points are extracted from the measurements in [8]. Contributions from the lowest 4 adiabatic potentials are shown individually.

No data

Same resonance wavefunction as for α -particle

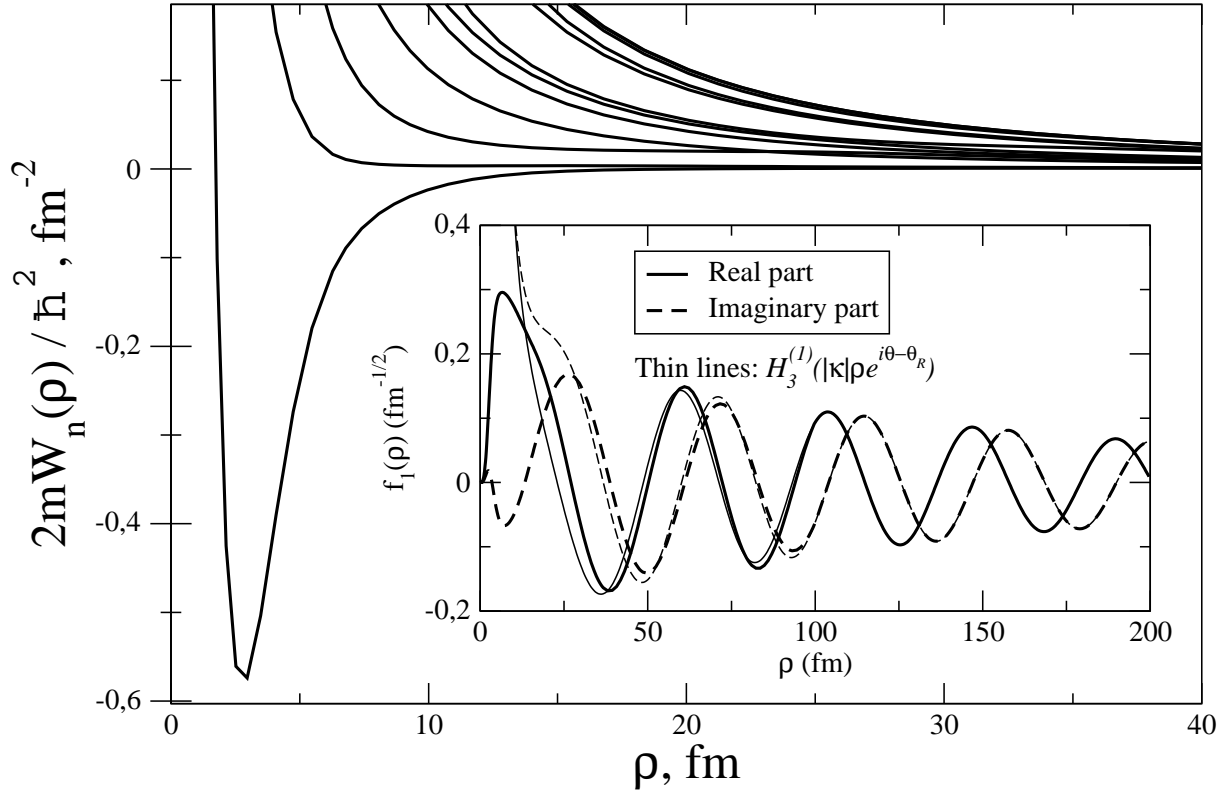


Figure 14: The lowest adiabatic potentials $W_n(\rho)$ for the $^{11}\text{Li}(1^-)$ halo nucleus within the three-body $^9\text{Li}+n+n$ model with interactions from [7]. The n -core and n - n scattering lengths are $a_{nc} \approx a_{nn} \approx 20$ fm. The inset shows the lowest hyper-radial resonance function with its large distance asymptotics – the Hankel function. The complex scaling angle $\theta = 0.15$, the resonance angle $\theta_R = 0.12$ corresponds to the resonance energy of about $0.4 - 0.1i$ MeV.

Large scattering length
Decreasing and oscillating radial function

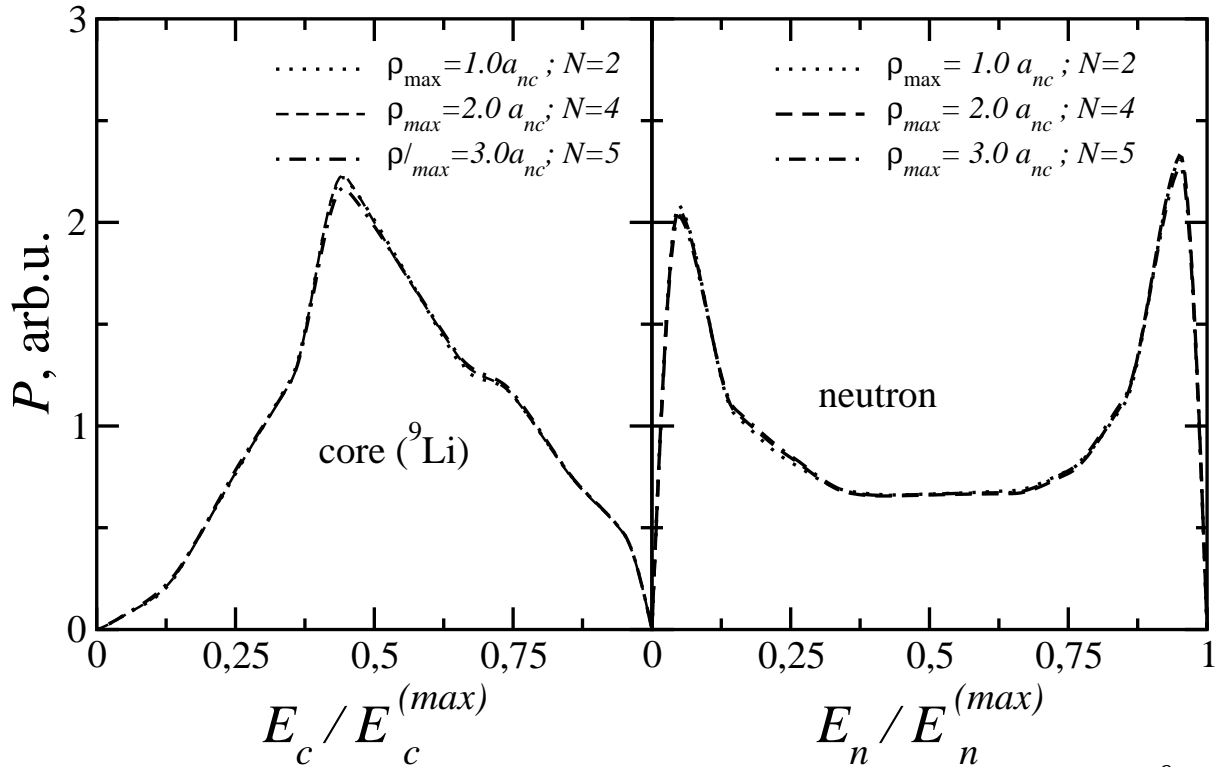


Figure 15: The energy distributions of the fragments - the core, ${}^9\text{Li}$, and the neutrons - in the decay of a three-body resonance ${}^{11}\text{Li}(1^-)$ calculated in the three-body ${}^9\text{Li} + n + n$ model with only s -wave n -core interactions (scattering length $a_{nc} \approx 50$ fm). The different curves are calculated with different ρ_{max} and different numbers of adiabatic channels N to illustrate the convergence.

Schematic model

Only s -waves and large neutron-core scattering length

Lowest adiabatic function is very accurate

Very stable against large variation of ρ_{max}

Core distribution peaks at intermediate energies

Neutron distribution has a low and a high energy peak

Mechanism is neutron emission, high energy

Then neutron-core stick together, low energy neutron

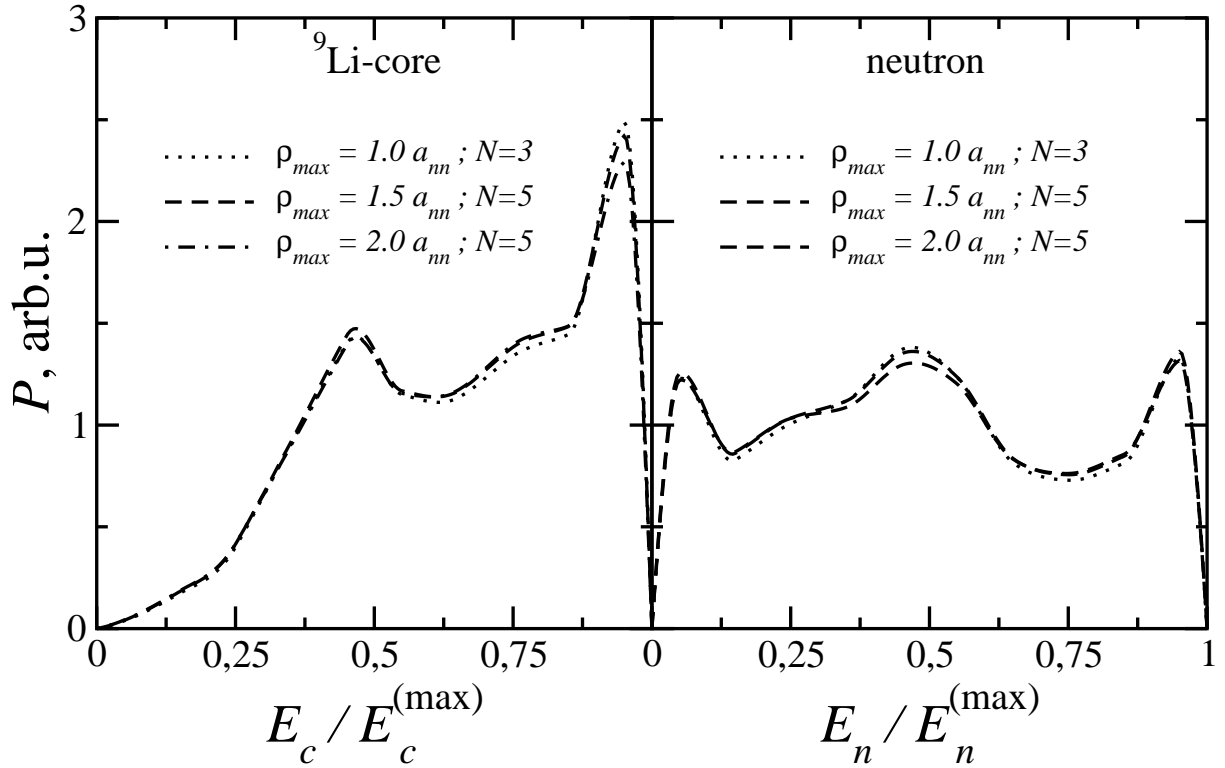


Figure 16: The energy distributions of the fragments - the core, ${}^9\text{Li}$, and the neutrons - in the decay of a three-body resonance ${}^{11}\text{Li}(1^-)$ calculated in the three-body ${}^9\text{Li} + n + n$ model with s -wave n -core interactions (scattering length $a_{nc} \approx 50$ fm), and an s -wave interaction in the $n - n$ subsystem (scattering length $a_{nn} \approx a_{nc} \approx 50$ fm). The different curves are calculated with different ρ_{max} and different numbers of adiabatic channels N to illustrate the convergence.

Schematic model

Only s -waves, and large neutron-neutron and neutron-core scattering length

Lowest adiabatic function is very accurate

Very stable against large variation of ρ_{max}

Core distribution gets new peak at high energy

Neutron distribution gets a new peak at intermediate energy

Two coherent decay mechanisms: Neutron emission, high and low energy neutrons, intermediate core energy

Core emission, two neutrons stick together

Core energy is maximum, neutron energy is intermediate

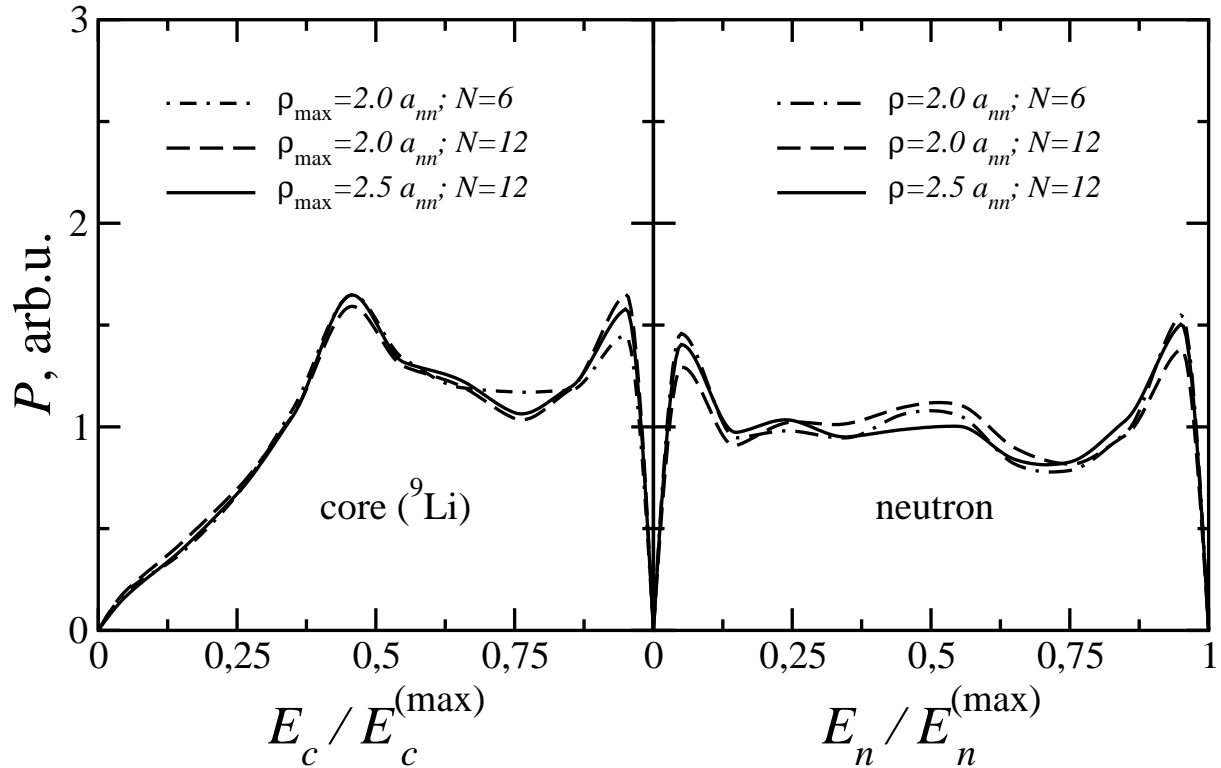


Figure 17: The energy distributions of the fragments - the core, ${}^9\text{Li}$, and the neutrons - in the decay of a three-body resonance ${}^{11}\text{Li}(1^-)$ calculated with interactions from [7] where the n -core and n - n scattering lengths are about 20 fm.

Realistic interactions reproducing all other known ${}^{11}\text{Li}$ properties
 All three scattering lengths now about 20 fm
 All peaks from schematic model remains
 Trace of Efimov effect

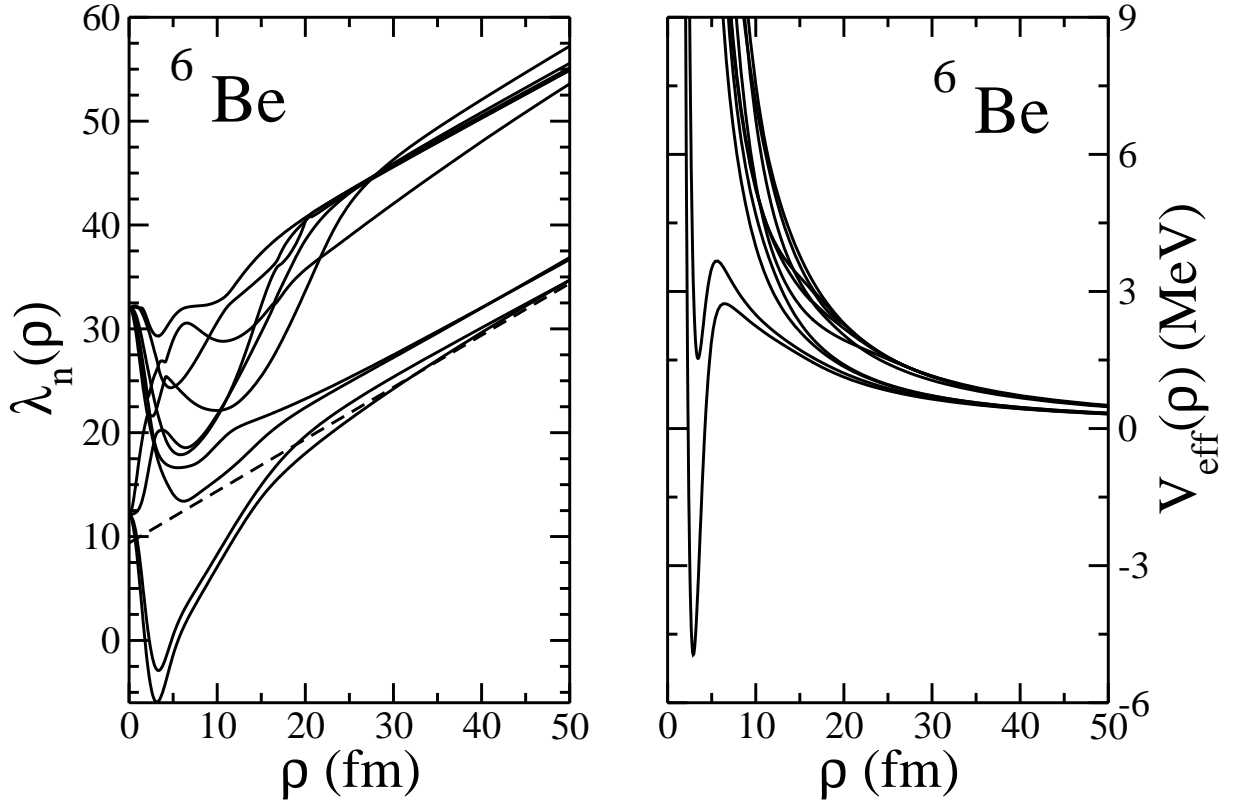


Figure 18: Real parts of the lowest 10 angular eigenvalues (left) and their corresponding adiabatic potentials (right) as functions of ρ for the 2^+ -resonance in ${}^6\text{Be}$. The scaling angle is $\theta = 0.15$. The dashed line is the estimated behaviour at large distances for the lowest angular eigenvalue.

The ${}^6\text{He}$ analog 2^+ -resonance in ${}^6\text{Be}$

Only difference is Coulomb

Small ρ : Same structure as for ${}^6\text{He}$

For large ρ :

Angular eigenvalues: linear in ρ , Potentials: $1/\rho$

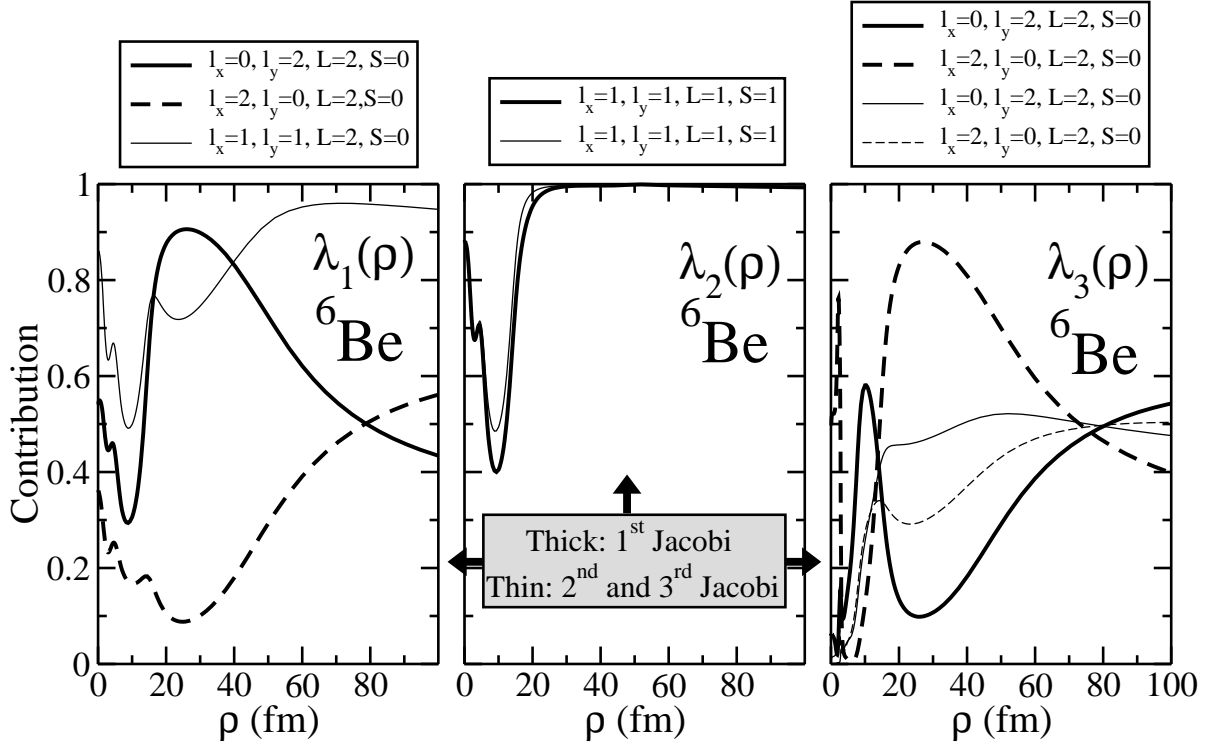


Figure 19: The fraction of the dominating components in the angular eigenfunction for the three lowest adiabatic potentials as function of ρ for ${}^6\text{Be}$ (2^+), see Fig. 1. The quantum numbers are as given in table 1. Thick lines: x refers to the two-proton system and y to its center of mass motion relative to the α -particle. Thin lines: x refers to the proton- α system and y to its center of mass motion relative to the other proton.

Partial wave decomposition of angular wavefunction
 Strong variation from small to large distance
 First eigenvalue:
 Proton-proton s -wave dominates at small distance
 Proton-proton s and d -waves are comparable at large ρ
 α -proton p -waves dominate at large distance

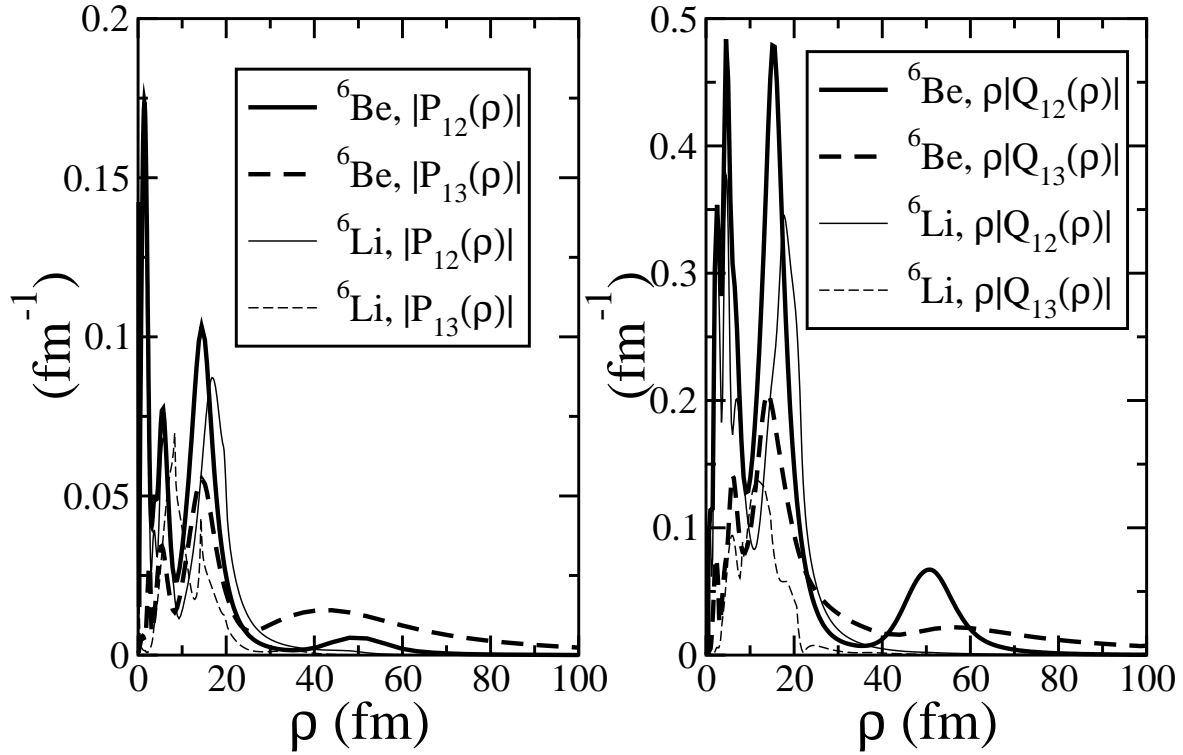


Figure 20: The absolute values of the coupling potentials between the three lowest adiabatic levels for the 2^+ -resonance in ${}^6\text{Be}$ (thick curves $\theta = 0.15$ rads) as functions of ρ , and the corresponding isobaric analog states in ${}^6\text{Li}$ (thin curves $\theta = 0.10$ rads).

Couplings determine relative size of radial wavefunctions
 Fall off at intermediate distance
 Numerical stability at large distance

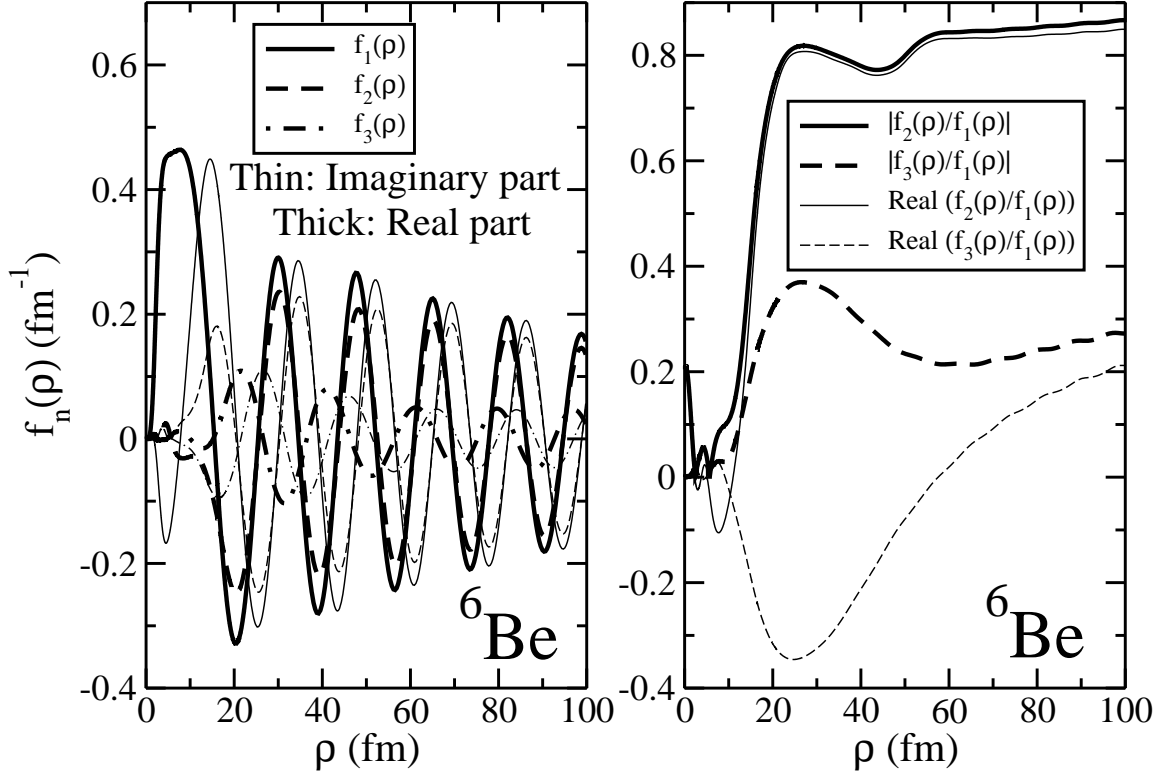


Figure 21: Left: Radial wave functions corresponding to the three first adiabatic potentials for the 2^+ -resonance in ${}^6\text{Be}$. The real and imaginary parts are shown by the thick and thin curves, respectively. Right: Absolute values (thick curves) and the real parts (thin curves) of the ratios between the radial wave functions. The probability distribution has for each ρ been normalized to 1 as function of α .

$$\Psi(\mathbf{x}, \mathbf{y}) = \sum_n f_n(\rho) \Phi_n(\rho, \Omega)$$

Each fall off exponentially while oscillating around zero
 Relative size at large distance determine energy distribution
 Numerical stability is not obvious

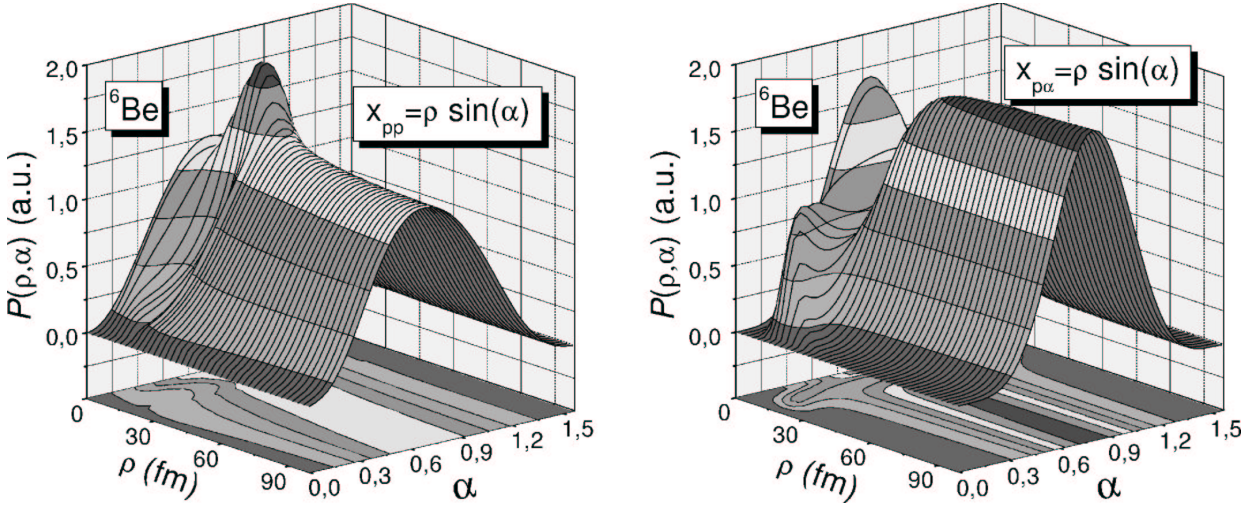


Figure 22: The probability distribution for the 2^+ -resonance in ${}^6\text{Be}$ including the lowest 10 adiabatic potentials as function of the hyper-radius ρ and hyperangle α related to the distance by $r_{ik} \propto \rho \sin \alpha$, i.e. the distance between either the one proton and core r_{pc} (right) or the two protons r_{pp} (left).

Structure of total resonance wavefunction
 Vary strongly from small to large distance
 Stable at larger distance
 Much better than indicated by the first radial wavefunctions

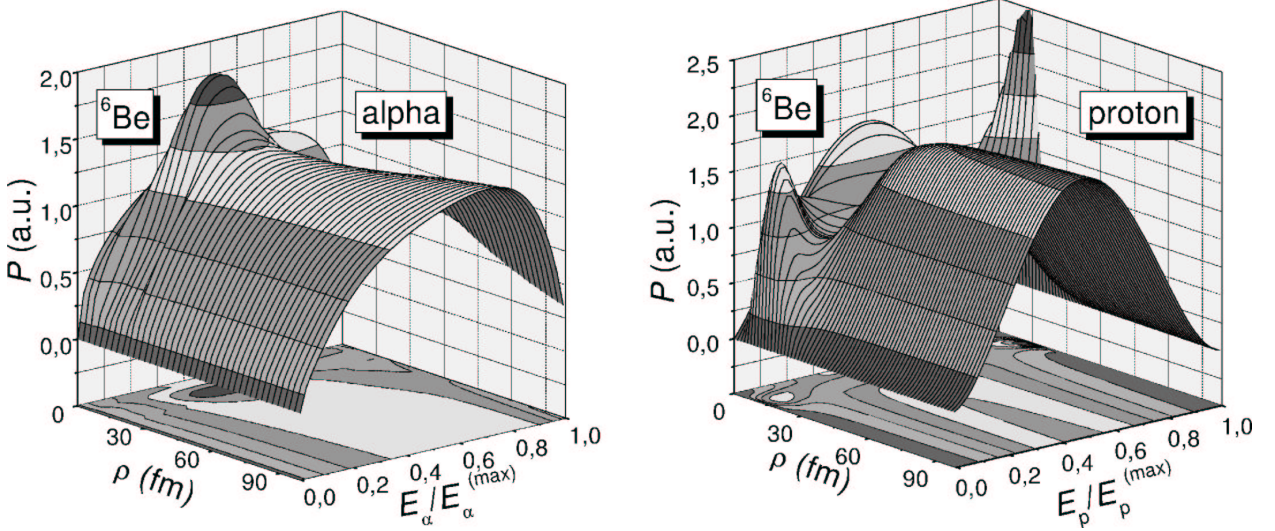


Figure 23: Kinetic energy distributions of protons (right) and α -particles (left) after decay of the 2^+ -resonance in ${}^6\text{Be}$. The three-dimensional plots show the dependence on ρ with inclusion of 10 adiabatic wave functions as function of $\cos^2 \alpha$, i.e. the kinetic energies $E_{\alpha,p}$ are in units of their maximum values $E_{\alpha,p}^{(max)}$ given by $(m_\alpha + m_p)/(m_\alpha + 2m_p)E_R$ and $2m_p/(m_\alpha + 2m_p)E_R$ for the proton and the α -particle, respectively, where E_R is the energy of the decaying resonance.

Kinetic energy distribution of third particle:

$$P(k_y^2) \propto P(\cos^2 \alpha) \propto \sin(2\alpha) \int d\Omega_x d\Omega_y |\Psi(\rho, \alpha, \Omega_x, \Omega_y)|^2$$

Protons peak at intermediate energy

α -particles with broad peak tilted towards large energy

Coulomb is coupling a lot and broadening distributions

Virtual proton-proton state is not present or very weak

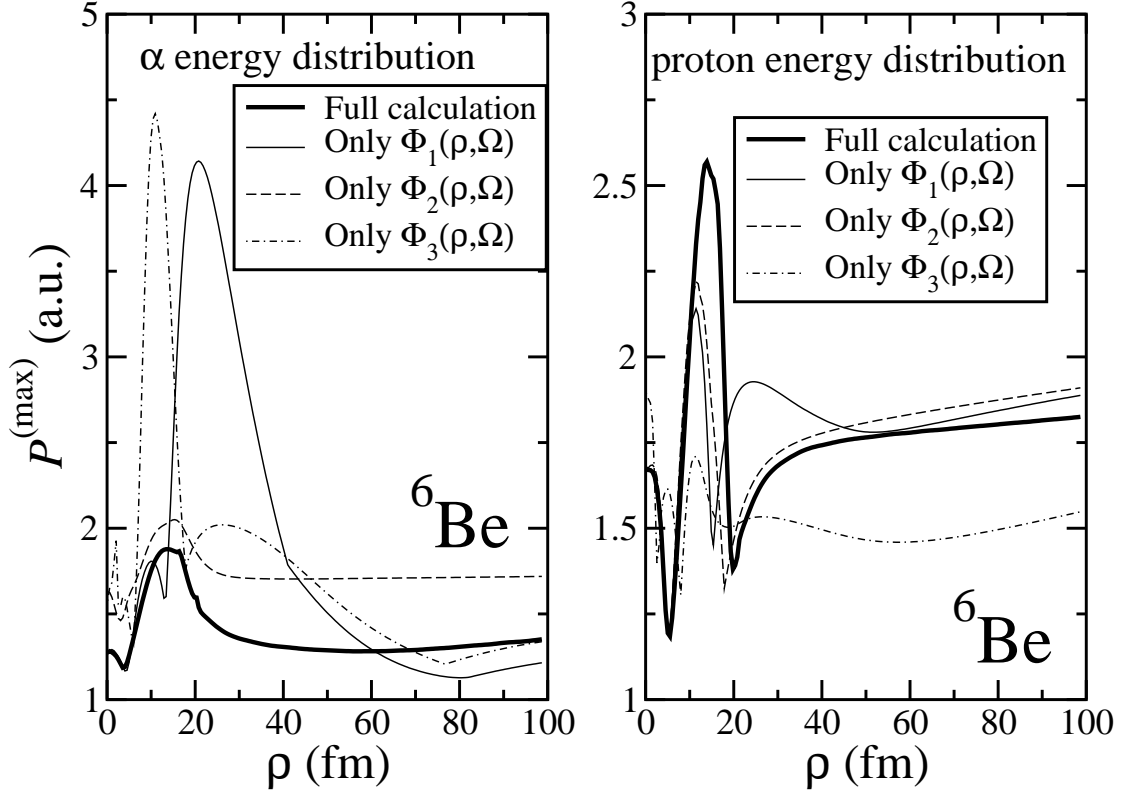


Figure 24: Projections of kinetic energy distributions for ${}^6\text{Be}$ -decay. Thick curves: Projection of the α (left panel) and proton (right panel) kinetic energy distributions Fig.(23) on the $E_{\alpha,p}/E_{\alpha,p}^{(max)} = 1$ plane. They are then the profile originating from the maximum values of the energy distribution for each value of ρ . The thin curves are the same profile but when respectively only the first adiabatic term (solid), only the second adiabatic term (dashed), or only the third adiabatic term (dot-dashed) is included in the calculation.

The kinetic energy distribution is redistributed with ρ
The total distribution is much more stable (thick curves)

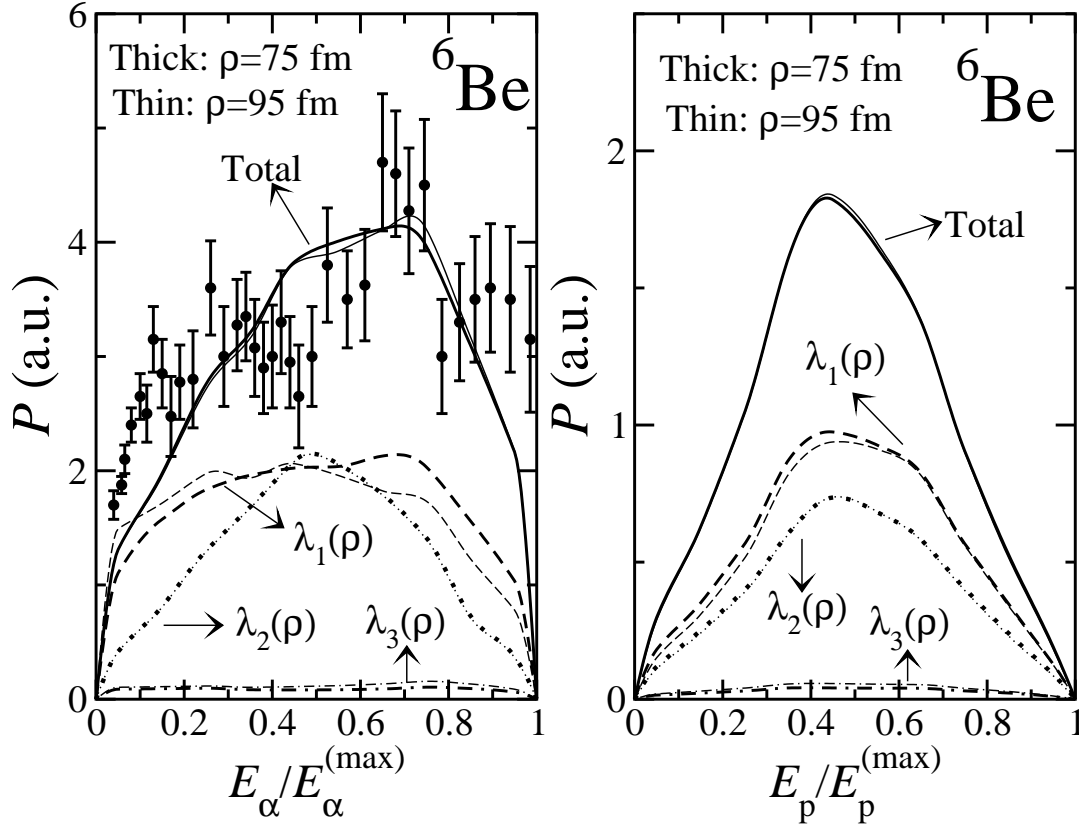


Figure 25: The kinetic energy distribution of the α -particle (upper part) and the proton (lower part) after decay of the 2^+ -resonance in ${}^6\text{Be}$. The scaling angle is $\theta = 0.15$ and the two sets of curves are for $\rho = 75, 95$ fm. The points are extracted from the measurements in [8]. Contributions from the lowest adiabatic potentials are shown individually.

Old data for α -particle
 Contributions from several adiabatic potentials

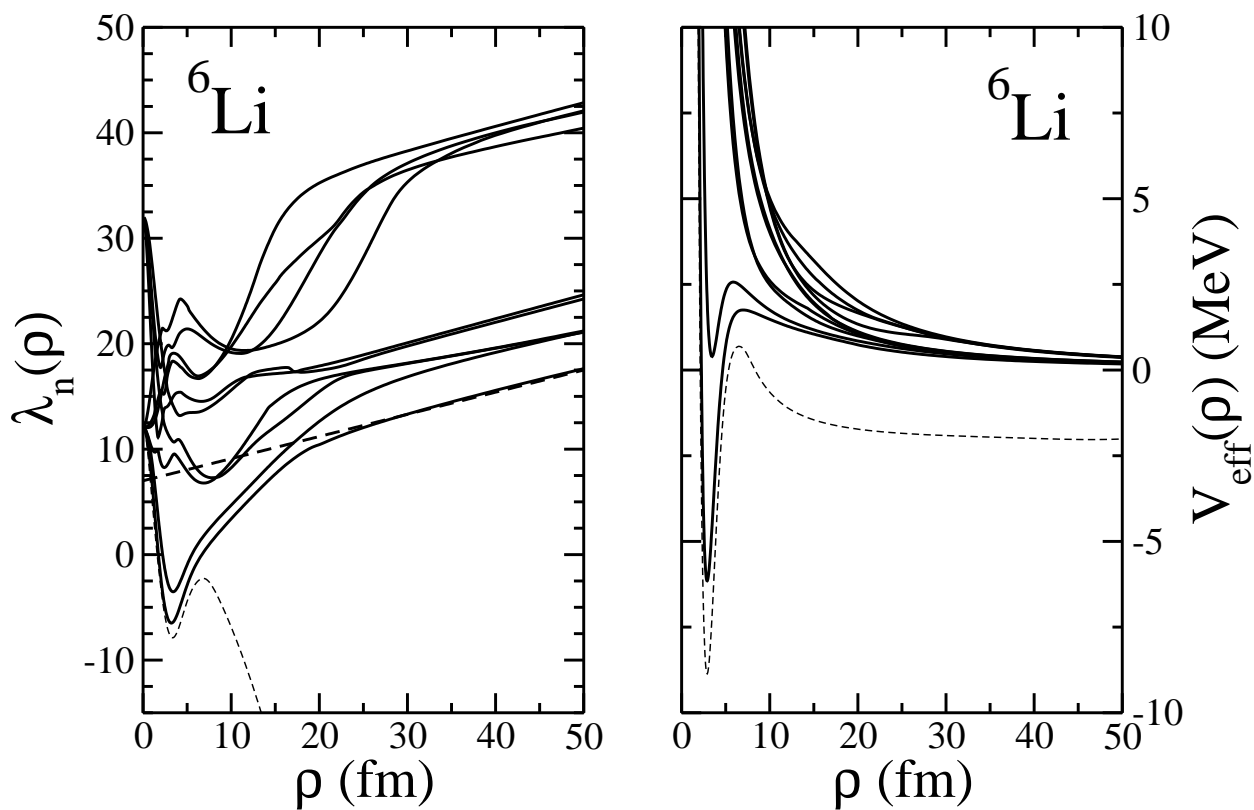


Figure 26: Real parts of the lowest 10 angular eigenvalues (left) and their corresponding adiabatic potentials (right) as functions of ρ for the 2^+ -resonance in ${}^6\text{Li}$. The scaling angle is $\theta = 0.10$. The dashed line is the estimated behaviour at large distances for the lowest angular eigenvalue.

The ${}^6\text{He}$ analog 2^+ -resonance in ${}^6\text{Li}$

Only difference is Coulomb

Small ρ : Same structure as for ${}^6\text{He}$

For large ρ :

Angular eigenvalues: linear in ρ , Potentials: $1/\rho$

One more state:

α -deuteron structure, eigenvalue as $-\rho^2$, potential to constant

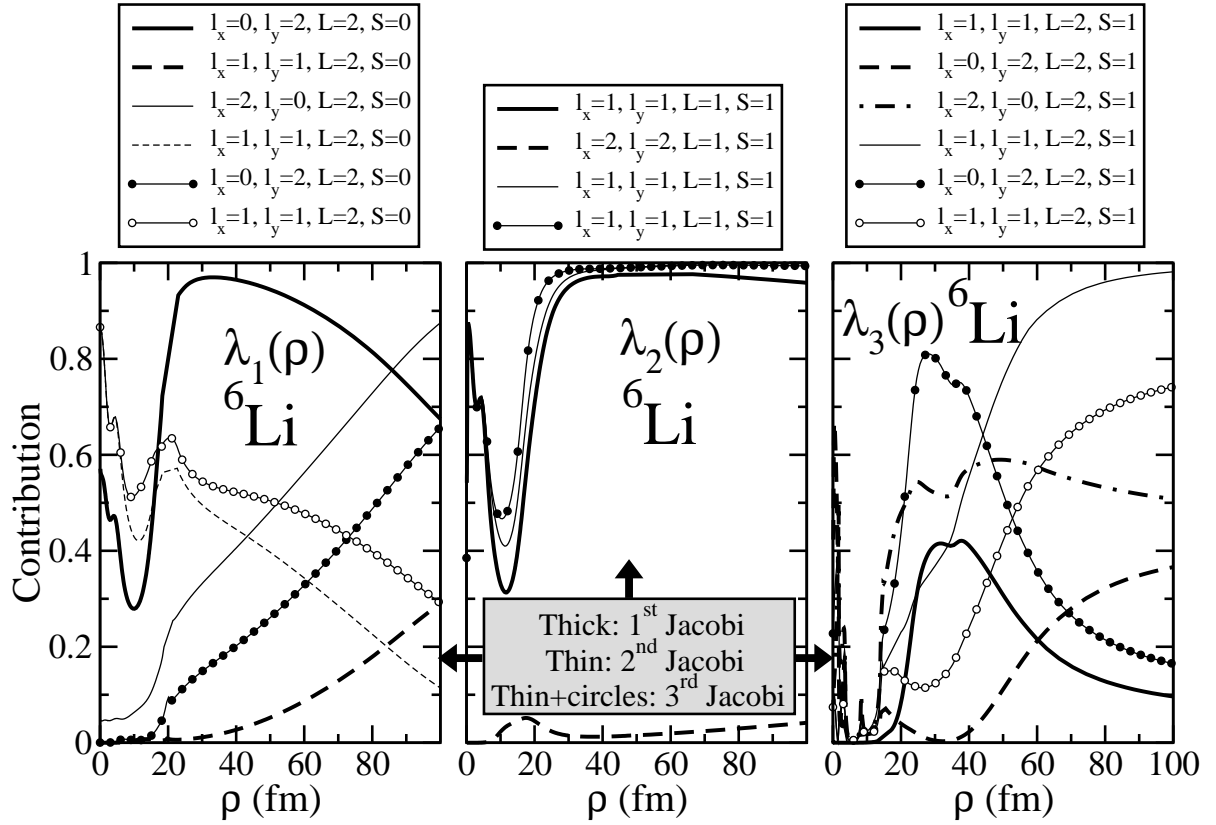


Figure 27: The fraction of the dominating components in the angular eigenfunction for the three lowest adiabatic potentials as function of ρ for ${}^6\text{Li}$ (2^+). The quantum numbers are as given in table 1. We omitted the almost decoupled lowest eigenfunction of deuteron- α character. In the second Jacobi set (thin lines) the x refers to the proton- α system and y to its center of mass motion relative to the neutron. In the third Jacobi set (thin+circle lines) the x refers to the neutron- α system and y to its center of mass motion relative to the proton.

Partial wave decomposition of angular wavefunction

Strong variation from small to large distance

First eigenvalue:

Proton-proton s-wave vary but dominates at all distances

Proton-proton p -wave increases with ρ , isospin 0 increase with ρ

α -proton d -wave dominates at large distance

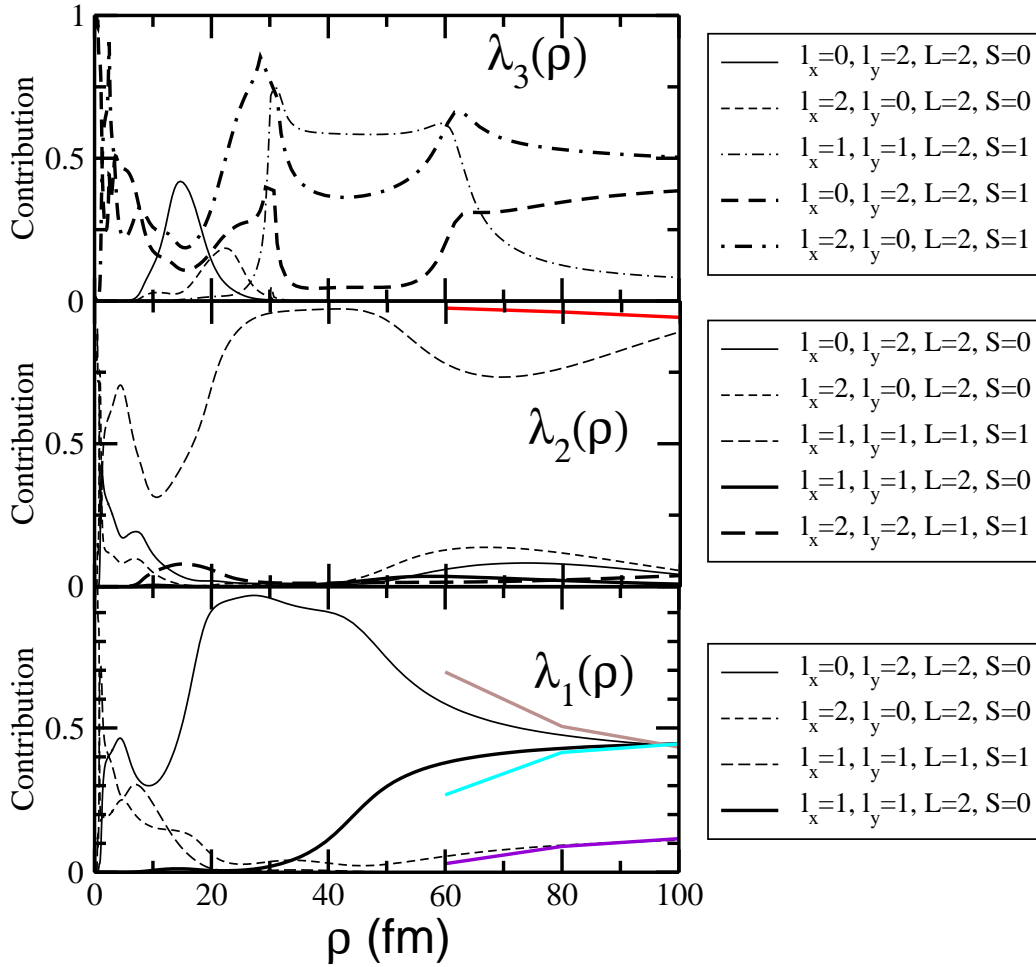


Figure 28: The fraction of the dominating components in the angular eigenfunction for the three lowest adiabatic potentials as function of ρ for ${}^6\text{Li} (2^+)$. The same as the previous figure for the Argonne potential.

First eigenvalue:

Proton-proton p -wave increases with ρ , isospin 0 increase with ρ

Much larger basis at large distance changes numerical values but maintain the picture. Second lambda remains as for $\rho \approx 50$ fm.

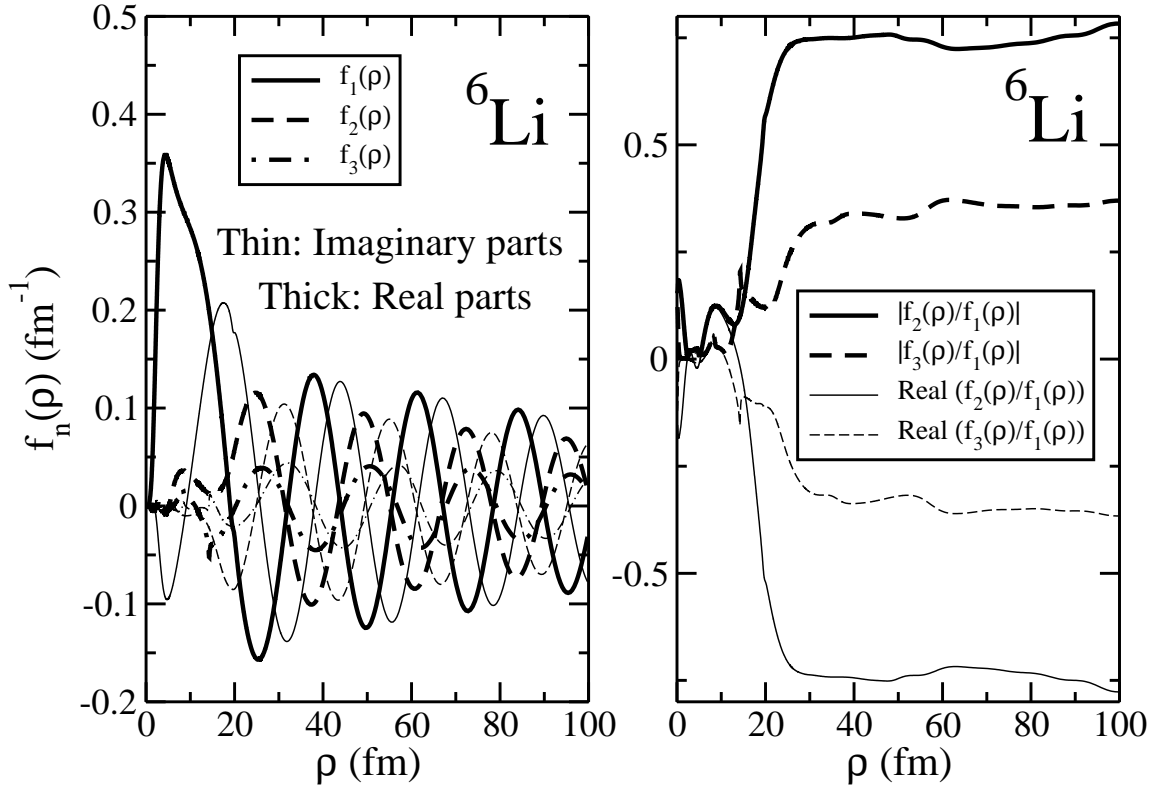


Figure 29: Left: Radial wave functions corresponding to the three first adiabatic potentials for the 2^+ -resonance in ${}^6\text{Li}$. The real and imaginary parts are shown by the thick and thin curves, respectively. Right: Absolute values (thick curves) and the real parts (thin curves) of the ratios between the radial wave functions. The probability distribution has for each ρ been normalized to 1 as function of α .

Each fall off exponentially while oscillating around zero
 Relative size at large distance determine energy distribution
 Numerical stability is established

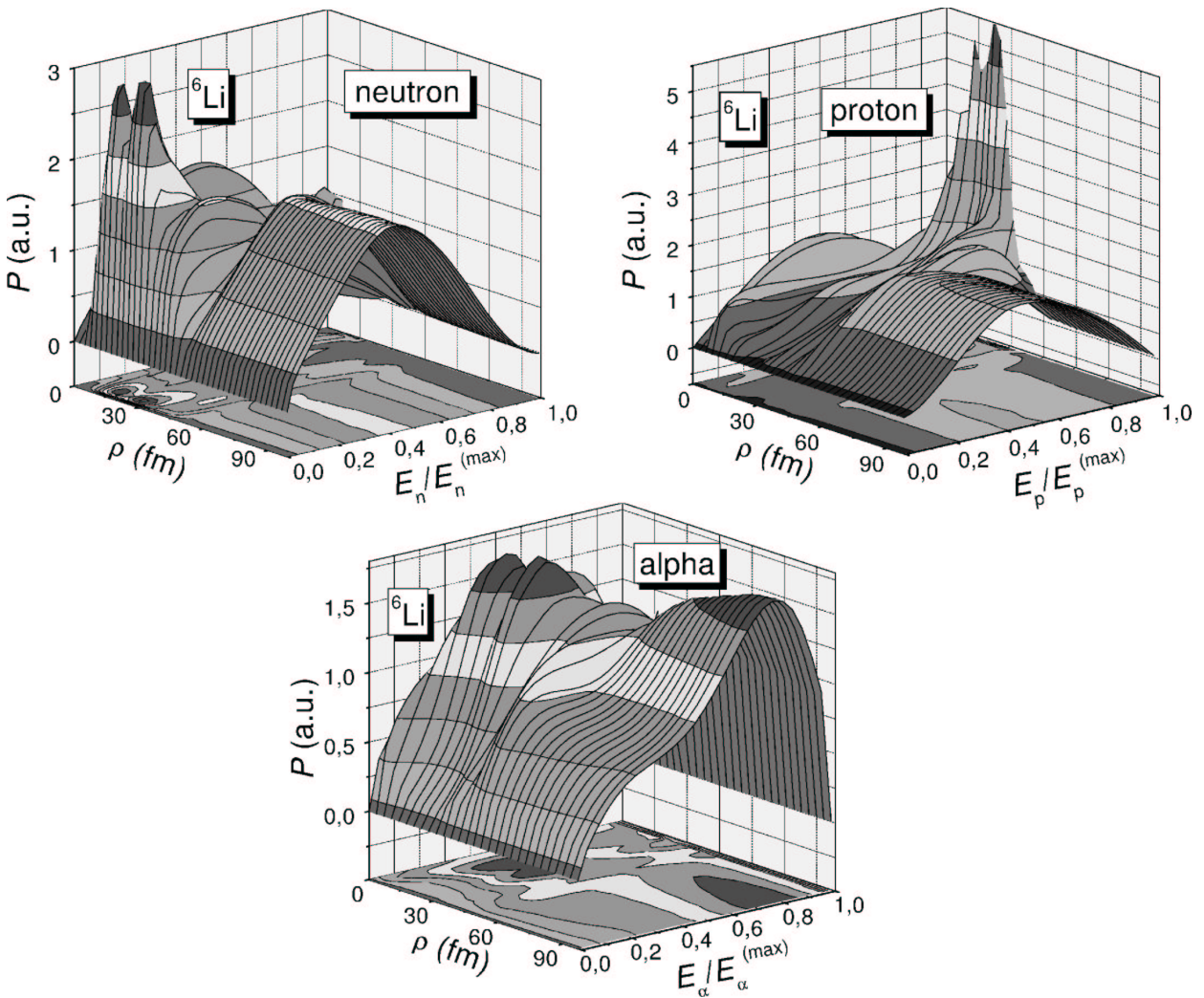


Figure 30: Kinetic energy distributions of protons (upper-right panel), neutrons (upper-left panel), and α -particles (lower panel) after decay of the 2^+ -resonance in ${}^6\text{Li}$ corresponding to that of Fig. 10. The three-dimensional plot show the dependence on ρ with inclusion of 10 adiabatic wave functions. The maximum energies $E_{n,p,\alpha}^{(max)}$ are $(m_\alpha + m_p)/(m_\alpha + m_n + m_p)E_R$, $(m_\alpha + m_n)/(m_\alpha + m_n + m_p)E_R$ and $(m_n + m_p)/(m_\alpha + m_n + m_p)E_R$ for the neutron, the proton, and the α -particle, respectively, where E_R is the energy of the decaying resonance.

Kinetic energy distribution of third particle:

$$P(k_y^2) \propto P(\cos^2 \alpha) \propto \sin(2\alpha) \int d\Omega_x d\Omega_y |\Psi(\rho, \alpha, \Omega_x, \Omega_y)|^2$$

Protons peak at intermediate (higher) energy

Neutrons peak at intermediate (lower) energy

α -particles with broad peak towards large energy

Neutron and proton tend to go together

Virtual neutron-proton state is active

Coulomb is broadening distributions

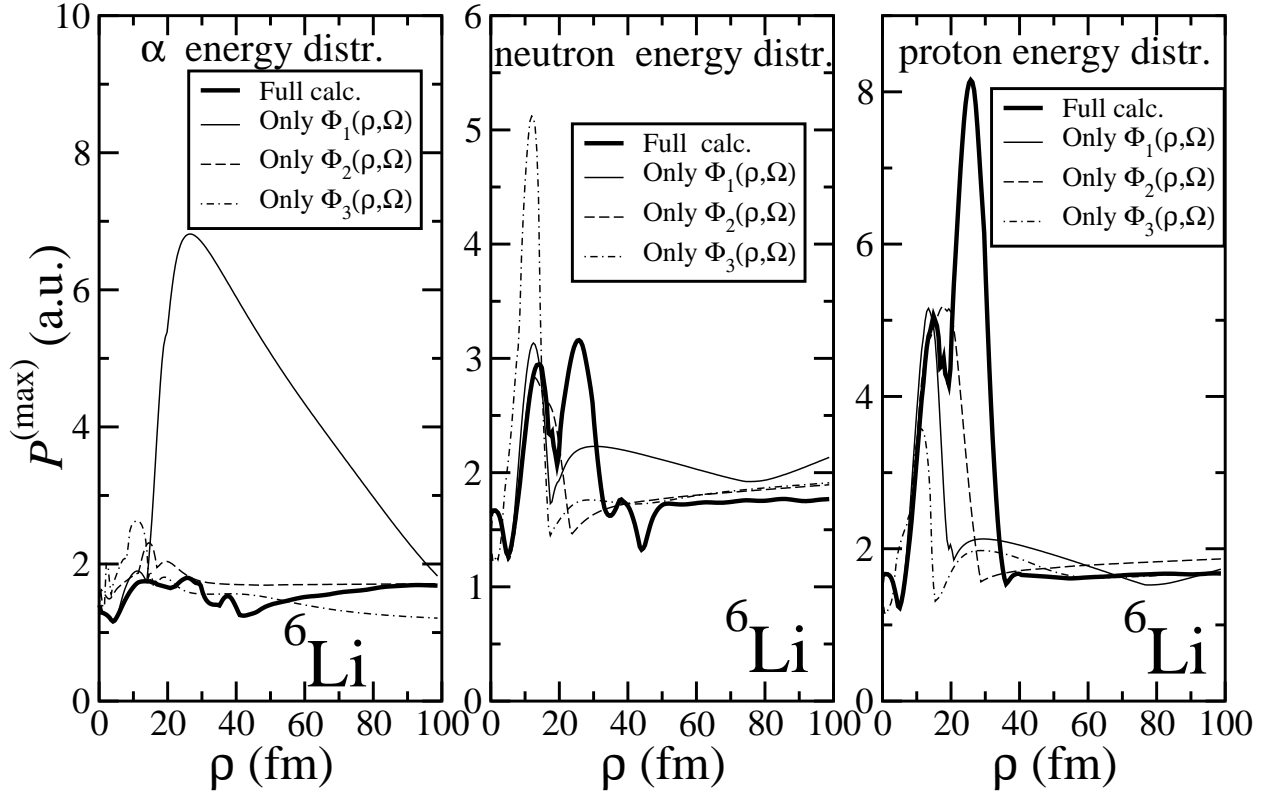


Figure 31: Projections of kinetic energy distributions as functions of ρ for ${}^6\text{Li}(2^+)$. Thick curves: Projection of the α (left panel) and proton (right panel) kinetic energy distributions (Fig. 10) on the $E_{\alpha,p}/E_{\alpha,p}^{(max)} = 1$ plane. They are then the profile originated by the maximum values of the energy distribution for each value of ρ . The thin curves are the same profile but when only the first adiabatic term (solid), only the second adiabatic term (dashed), or only the third adiabatic term (dot-dashed) is included in the calculation.

The kinetic energy distribution is redistributed with ρ
The total distribution is much more stable (thick curves)

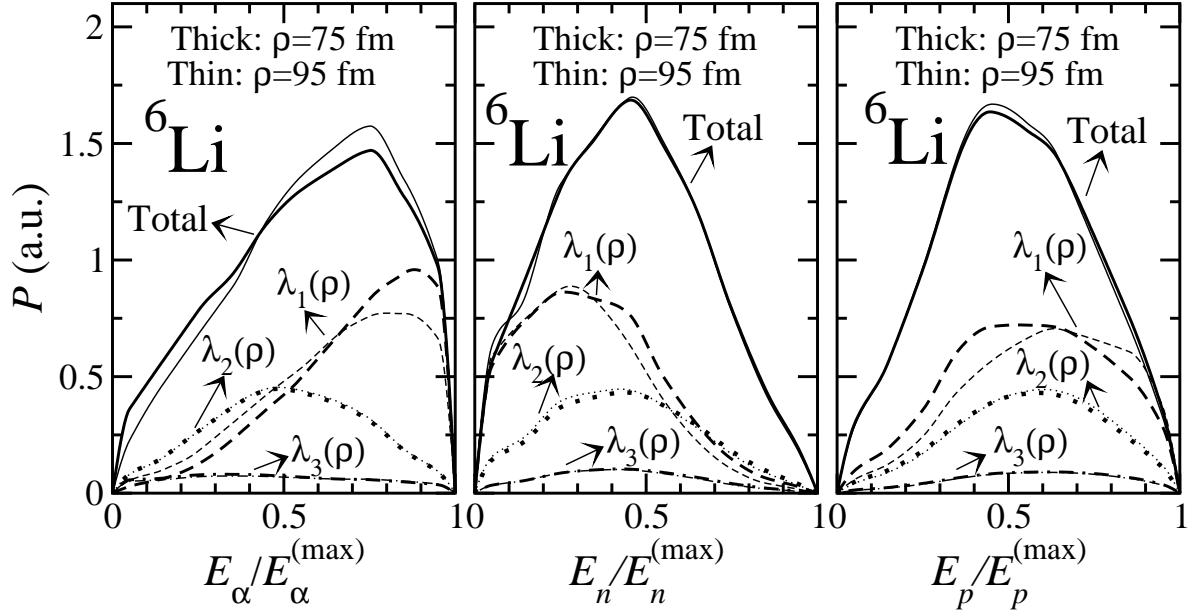


Figure 32: The kinetic energy distribution of the α -particle (upper part), the neutron (middle part) and the proton (lower part) after decay of the isobaric analog 2^+ -resonance in ${}^6\text{Li}$. The scaling angle is $\theta = 0.10$ and the two sets of curves are for $\rho = 75, 95$ fm. The points are extracted from the measurements in [8]. Contributions from the lowest adiabatic potentials are shown individually.

Old data for α -particle exist in another frame
 Contributions from several adiabatic components
 Protons peak at intermediate (higher) energy
 Neutrons peak at intermediate (lower) energy
 α -particles with broad peak towards large energy
 Neutron and proton tend to go together
 Virtual neutron-proton state is active
 Coulomb is broadening the distributions

CONCLUSIONS

1. Three-body decay of many-body resonances
2. A model working in practice
3. Energies must be artificially adjusted as in α -decay
4. Width is estimated as hyperspherical barrier penetrability
5. Asymptotic wavefunction behavior determines the final state energy distributions
6. Asymptotics are established at intermediate distances where basis size is manageable
7. Large scattering lengths can be handled, Efimov effect
8. Coulomb can be handled in the cases investigated
9. Isospin mixing is a dynamic effect occurring outside the range of short-range interactions

DIFFERENCES TO GRIGORENKO ET AL

1. The spin dependence of the proton-core interaction in ^{17}Ne computations is in conflict with the mean-field spin-orbit interaction, i.e. the valence $d_{3/2}$ and $d_{5/2}$ states can not be independently populated, only specific combinations. Same problem in ^{19}Mg and ^{45}Fe . Maybe corrected later on.

2. The hyperspherical method with only one Faddeev component is used, i.e. it is not possible to describe (i) Efimov effect, (ii) close to Efimov structure, (iii) two simultaneous 2-body substructures, (iv) one 2-body resonance unless, as he sometimes does, allow this structure as a variational degree of freedom.

3. The hyperspherical method has rather small $Kmax \simeq 20-25$. With three Faddeev components we need at least $Kmax \simeq 100-150$ with only the short range interaction. The Coulomb interaction and the unavoidable couplings at large distances does not reduce the required basis size.

4. The three-body interaction is ρ^{-3} at large distance, i.e. exactly of the same form as the effective hyperradial potentials. This should not be necessary if the proper components are accurately included. Instead this interaction should be of much shorter range in a three-body coordinate, e.g. exponential or Gaussian.

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