Coulomb-Fourier transformation III.
Three charged particles in the continuum.

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Content

Introduction

We suggest a new adiabatic approach for description of three charged particles in the continuum. This approach is based on the Coulomb-Fourier transformation (CFT) of three body Hamiltonian, which allows to develop a scheme, alternative to Born-Oppenheimer one. The approach appears as an expansion of the kernels of corresponding integral transformations in terms of small mass-ratio parameter. To be specific, the results are presented for the system ppe in the continuum. The wave function of a such system is compared with that one which is used for estimation of the rate for triple reaction $p + p + e \to d + \nu$, which take place as a step of pp-cycle in the center of the Sun. The problem of microscopic screening for this particular reaction is discussed.

Application to the case of two heavy and one light particles $A^+\ B^+\ e^-$

Hamiltonian of the system in configuration space:

$$H = -\Delta_{\mathbf{x}_1} - \Delta_{\mathbf{y}_1} + v_s(\mathbf{x}_1) + n_1/x_1 + n_2/x_2 + n_3/x_3.$$

Coulomb Fourier Transformed Hamiltonian in momentum space:

$$\hat{H}(\mathbf{P},\mathbf{P}') = \langle \psi^0_{\mathbf{p}_1}\psi^c_{\mathbf{k}_1}|H|\psi^0_{\mathbf{p}'_1}\psi^c_{\mathbf{k}'_1}\rangle = \tag{1}$$

$$= (\mathbf{k}_1^2+\mathbf{p}_1^2)\delta(\mathbf{k}_1-\mathbf{k}'_1)\delta(\mathbf{p}_1-\mathbf{p}'_1)+\hat{v}_s(\mathbf{k}_1,\mathbf{k}'_1)\delta(\mathbf{p}_1-\mathbf{p}'_1)+$$

$$+W_2(\mathbf{P},\mathbf{P}')+W_3(\mathbf{P},\mathbf{P}')$$

operating on CF-transformed wave function $\hat{\Psi}(\mathbf{P})$.

$$\hat{v}_s(\mathbf{k}_1,\mathbf{k}'_1) = \langle \psi^c_{\mathbf{k}_1}|v_s|\psi^c_{\mathbf{k}'_1}\rangle = \int d\mathbf{x}_1\, \psi^{c*}_{\mathbf{k}_1}(\mathbf{x}_1)v_s(\mathbf{x}_1)\psi^c_{\mathbf{k}'_1}(\mathbf{x}_1)$$

and W_j are Coulomb potentials $n_j/x_j,\quad j=2,3$ in CF representation.

Let us notice, that the contribution from n_1/x_1 potential has been eliminated by CF transform.

$$W_j(\mathbf{P},\mathbf{P}') = |s_{j1}|^{-3}\hat{v}_j^c\left(\frac{\mathbf{p}_1-\mathbf{p}_1'}{s_{j1}}\right)\mathscr{L}_j(\mathbf{P},\mathbf{P}'),$$

where

$$\hat{v}_j^c(\mathbf{q}) = \frac{1}{2\pi^2}\frac{n_j}{|\mathbf{q}|^2}$$

is the familiar Fourier transform of Coulomb potential n_j/x_j and the functions $\mathscr{L}_j(\mathbf{P},\mathbf{P}')$, $j=2,3$ are given by the integrals

$$\mathscr{L}_j(\mathbf{P},\mathbf{P}') = \lim_{\lambda\to+0}\int d\mathbf{x}_1\, e^{i\tau_j\langle\mathbf{x}_1,\mathbf{p}-\mathbf{p}'\rangle-\lambda|\mathbf{x}_1|}\psi_{\mathbf{k}_1}^{c*}(\mathbf{x}_1)\psi_{\mathbf{k}_1'}^{c}(\mathbf{x}_1). \qquad (2)$$

The parameters τ_j, $j=2,3$, have the kinematical origin and are represented in terms of kinematic rotation matrix elements as

$$\tau_j = c_{j1}/s_{j1}.$$

$$\tau_2 = \sqrt{m_e/2m_p}(1+O(m_e/m_p)), \quad \tau_3 = -\tau_2,$$

what shows that τ_j are small.

The expansion of the kernel $\mathscr{L}_j(\mathbf{P},\mathbf{P}')$:

$$\mathscr{L}_j(\mathbf{P},\mathbf{P}') = \delta(\mathbf{k}_1 - \mathbf{k}_1') + \tag{3}$$

$$\frac{i\tau_j}{1!}L^{(1)}(\mathbf{P},\mathbf{P}') + \frac{(i\tau_j)^2}{2!}L^{(2)}(\mathbf{P},\mathbf{P}') + \frac{(i\tau_j)^3}{3!}L^{(3)}(\mathbf{P},\mathbf{P}') + \ldots$$

Here $L^{(l)}(\mathbf{P},\mathbf{P}')$ are integrals

$$L^{(l)}(\mathbf{P},\mathbf{P}') =$$

$$= \lim_{\lambda\to+0}\int d\mathbf{x}_1\, e^{-\lambda|\mathbf{x}_1|}\psi^{c*}_{\mathbf{k}_1}(\mathbf{x}_1)\langle\mathbf{x}_1,\mathbf{p}_1-\mathbf{p}_1'\rangle^l\psi^{c}_{\mathbf{k}_1'}(\mathbf{x}_1)$$

which are independent on j.

Since $\tau_2 = -\tau_3$

$$W_2(\mathbf{P},\mathbf{P}') + W_3(\mathbf{P},\mathbf{P}') = \hat{v}^c_{eff}(\mathbf{p}_1,\mathbf{p}'_1)\times$$

$$\left\{\delta(\mathbf{k}_1 - \mathbf{k}'_1) - \frac{\tau^2}{2!}L^{(2)}(\mathbf{P},\mathbf{P}') + \frac{\tau^4}{4!}L^{(4)}(\mathbf{P},\mathbf{P}') - \ldots\right\},$$

where we have introduced parameter $\tau = |\tau_2|$.

$$\hat{v}^c_{eff}(\mathbf{p}_1,\mathbf{p}'_1) = \frac{1}{2\pi^2}\frac{n_{eff}}{|\mathbf{p}-\mathbf{p}'|^2}$$

with $n_{eff} = -2e^2\sqrt{2\frac{m_e 2m_p}{m_e+2m_p}} \sim -2e^2\sqrt{2m_e}$.

We arrive at the representation of the CF-transformed Hamiltonian $\hat{H}$ which plays a central role in the solution of the problem

$$\hat{H} = \hat{H}_0 + \hat{V}_s + \hat{V}^c_{eff} + \tau^2 \hat{W}. \tag{4}$$

The kernels of operators involved in (4) read

$$\hat{H}_0(\mathbf{P}, \mathbf{P}') = \left(\mathbf{k}_1^2 + \mathbf{p}_1^2\right) \delta(\mathbf{P} - \mathbf{P}'),$$

$$\hat{V}_s(\mathbf{P}, \mathbf{P}') = \hat{v}_s(\mathbf{k}_1, \mathbf{k}_1')\delta(\mathbf{p}_1 - \mathbf{p}_1'),$$

$$\hat{V}^c_{eff}(\mathbf{P}, \mathbf{P}') = \hat{v}^c_{eff}(\mathbf{p}_1, \mathbf{p}_1')\delta(\mathbf{k}_1 - \mathbf{k}_1')$$

and

$$\hat{W}(\mathbf{P}, \mathbf{P}') = -\hat{W}^{(2)}(\mathbf{P}, \mathbf{P}') + \tau^2 \hat{W}^{(4)}(\mathbf{P}, \mathbf{P}') - \tau^4 \hat{W}^{(6)}(\mathbf{P}, \mathbf{P}') + ...,$$

$$\hat{W}^{(l)}(\mathbf{P}, \mathbf{P}') = \hat{v}^c_{eff}(\mathbf{p}_1, \mathbf{p}_1')\frac{1}{l!} L^{(l)}(\mathbf{P}, \mathbf{P}') \tag{5}$$

The structure of the Hamiltonian (4) suggests now the natural perturbative scheme for solution of Schrödinger equation. Let us represent the wave-function $\hat{\Psi}$ as power series in τ^2

$$\hat{\Psi} = \hat{\Psi}_0 + \tau^2\hat{\Psi}_2 + \tau^4\hat{\Psi}_4 + \ldots \tag{6}$$

We obtain the recursive set of equations for $\hat{\Psi}_k$, i.e.

$$\left(\hat{H}_0 + \hat{V}_s + \hat{V}^c_{eff}\right)\hat{\Psi}_0 = E\hat{\Psi}_0, \tag{7}$$

$$\left(\hat{H}_0 + \hat{V}_s + \hat{V}^c_{eff}\right)\hat{\Psi}_{2l} = E\hat{\Psi}_{2l} - \sum_{s=0}^{l-1}(-1)^{l-s}\hat{W}^{(2l-2s)}\hat{\Psi}_{2s}, \tag{8}$$

$$l = 1, 2, 3, \ldots$$

The scheme (7, 8) has a remarkable property, namely, the solution of the three-body problem in framework of this scheme can be obtained in terms of solutions of two-body problem.

The solution of the first equation (7) reads

$$\hat{\Psi}_0(\mathbf{P},\mathbf{P}^{in}) = \hat{\psi}^{+}_{\mathbf{k}_1^{in}}(\mathbf{k}_1)\,\hat{\psi}^{ce}_{\mathbf{p}_1^{in}}(\mathbf{p}_1), \tag{9}$$

$$\mathbf{P}^{in2} = \mathbf{k}_1^{in2} + \mathbf{p}_1^{in2} = E\,.$$

$$\hat{\Psi}_{2l} = -\hat{G}_{s,eff}(E+i0)\sum_{s=0}^{l-1}(-1)^{l-s}\hat{W}^{(2l-2s)}\hat{\Psi}_{2s}.$$

Here the kernel of the operator

$$\hat{G}_{s,eff}(z) = \left(\hat{H}_0 + \hat{V}_s + \hat{V}^c_{eff} - z\right)^{-1}$$

The kernel of the operator $\hat{G}_{s,eff}(z) = \left(\hat{H}_0 + \hat{V}_s + \hat{V}^c_{eff} - z\right)^{-1}$ is represented through two-body Green's functions $\hat{g}_s$ for potential $\hat{v}_s$ and $\hat{g}^c_{eff}$ for potential $\hat{v}^c_{eff}$ by the convolution integral

$$\hat{G}_{s,eff}(\mathbf{P},\mathbf{P}',z) = \frac{1}{2i\pi}\oint_C d\zeta\, \hat{g}_s(\mathbf{k}_1,\mathbf{k}'_1,\zeta)\hat{g}^c_{eff}(\mathbf{p}_1,\mathbf{p}'_1,z-\zeta)$$

with counter C encircling the cut of $\hat{g}_s$ in anticlockwise direction.
So that, we have constructed the formal solution to the CF transformed Schrödinger equation for the system ppe in the continuum. The configuration space wave function can be obtained from $\hat{\Psi}$

$$\Psi(\mathbf{X},\mathbf{P}^{in}) = \int d\mathbf{P}\Psi_{c0}(\mathbf{X},\mathbf{P})\hat{\Psi}(\mathbf{P},\mathbf{P}^{in}). \tag{10}$$

Now one can see, that the structure of our solution (6) and respective series in τ^2 for $\Psi(\mathbf{X},\mathbf{P}^{in})$ generated from (10) by (6) and the structure of the representation for the Hamiltonian (4) outline the framework of our approach as an alternative to Born-Oppenheimer one.

It is worth mentioning here, that the formalism given above is rather general and with minor evident modifications is applicable for the three charged particle systems for the case of different masses when the mass of one particle is significantly smaller than the masses of others.
Let us give some explicit formulas for approximation to the wave function $\Psi(\mathbf{X},\mathbf{P}^{in})$ generated by our complete formal solution. Introducing (9) into (10) we get

$$\Psi(\mathbf{X},\mathbf{P}^{in}) = \psi^{+}_{\mathbf{k}_1^{in}}(\mathbf{x}_1)\psi^{ce}_{\mathbf{p}_1^{in}}(\mathbf{y}_1) + \tau^2\Psi_2(\mathbf{X},\mathbf{P}^{in}) + O(\tau^4). \qquad (11)$$

Here

$$\psi^{+}_{\mathbf{k}_1^{in}}(\mathbf{x}_1) = \int d\mathbf{k}_1 \psi^{c}_{\mathbf{k}_1}(\mathbf{x}_1)\hat{\psi}^{+}_{\mathbf{k}_1^{in}}(\mathbf{k}_1)$$

is the two body scattering wave function corresponding to the potential $v^s(\mathbf{x}) + n_1/x$ and $\Psi_2(\mathbf{X},\mathbf{P}^{in})$ is given by transform (10) of $\hat{\Psi}_2$ calculated through $\hat{\Psi}_0$ from (9) by formula

$$\hat{\Psi}_2 = -\hat{G}_{s,eff}(E+i0)\hat{W}^{(2)}\hat{\Psi}_0. \qquad (12)$$

The complete investigation of properties of the solutions to (7,8) is out of the scope of this paper and will be made elsewhere. Below in this section, we consider two points which plays the key role for the formalism, namely the singular structure of operator $\hat{W}$ and the structure of correction term $\hat{\Psi}_2$ (and consequently Ψ_2) which possesses the most important properties specific for all correction terms $\hat{\Psi}_{2l}$.

The most singular part of the integral $L^l(\mathbf{P},\mathbf{P}')$ has the form

$$L^{(l)}(\mathbf{P},\mathbf{P}') \simeq \delta(\hat{\mathbf{k}},\hat{\mathbf{k}}')\delta^{(l)}(k-k')\langle\hat{\mathbf{k}},\mathbf{p}-\mathbf{p}'\rangle^l \mathscr{L}^{(l)}(k,k'). \qquad (13)$$

Here and in what follows we use two notational options, first the $\simeq$ sign manifests the fact that the less singular terms are not presented in the equation and second subscript 1 is omitted from definition of momentums and coordinates . Delta-function $\delta(\hat{\mathbf{k}},\hat{\mathbf{k}}')$ on unit sphere $S^2 = \{\mathbf{k} : k = 1\}$ and l-th derivative of delta-function $\delta^{(l)}(k-k')$ are defined by

$$\int_{S^2} d\hat{\mathbf{k}}\,\delta(\hat{\mathbf{k}},\hat{\mathbf{k}}')g(\hat{\mathbf{k}}') = f(\hat{\mathbf{k}}),$$

$$\int_{-\infty}^{\infty} dk'\,\delta^{(l)}(k-k')g(k) = (-1)^l g^{(l)}(k).$$

The function $\mathscr{L}^{(l)}(k,k')$ being a smooth function of k and k' for l-even has the form

$$\mathscr{L}^{(l)}(k,k') = \frac{1}{kk'} e^{-i(\sigma(k)-\sigma(k'))} \frac{\sinh \frac{\pi(\gamma-\gamma')}{2}}{\frac{\pi(\gamma-\gamma')}{2}} \times$$

$$\Re\left[e^{i(\sigma_0(k)-\sigma_0(k'))+i\frac{\pi l}{2}} (2k)^{-i\gamma} (2k')^{i\gamma'} \Gamma(1-i(\gamma-\gamma')) \right].$$

Above formulas for $L^{(l)}(\mathbf{P},\mathbf{P}')$ can be used to compute the action of $\hat{W}^{(2)}$ operator on $\hat{\Psi}_0$

$$\hat{W}^{(2)}\hat{\Psi}_0(\mathbf{P},\mathbf{P}^{in}) = \frac{1}{2} I_1(\mathbf{k},\mathbf{k}^{in}) I_2(\mathbf{p},\mathbf{p}^{in},\hat{\mathbf{k}}) \tag{14}$$

where

$$I_1(\mathbf{k},\mathbf{k}^{in}) = \int d\mathbf{k}'\, \delta(\hat{\mathbf{k}},\hat{\mathbf{k}}') \delta^{(2)}(k-k') \mathscr{L}^{(2)}(k,k') \hat{\psi}^{+}_{\mathbf{k}^{in}}(\mathbf{k}')$$

and

$$I_2(\mathbf{p},\mathbf{p}^{in},\hat{\mathbf{k}}) = \int d\mathbf{p}'\, \hat{v}^{c}_{eff}(\mathbf{p},\mathbf{p}') \langle \hat{\mathbf{k}}, \mathbf{p}-\mathbf{p}' \rangle^2 \hat{\psi}^{ce}_{\mathbf{p}^{in}}(\mathbf{p}').$$

Both integrals I_j are singular distributions. For the second integral, it is useful to make a linear change of variables to get

$$I_2(\mathbf{p},\mathbf{p}^{in},\hat{\mathbf{k}}) = \frac{n_{eff}}{2\pi^2}\int d\mathbf{q}\,\langle\hat{\mathbf{k}},\hat{\mathbf{q}}\rangle^2\hat{\psi}^{ce}_{\mathbf{p}^{in}}(\mathbf{q}+\mathbf{p})$$

and then using Fourier transform for $\hat{\psi}^{ce}_{\mathbf{p}^{in}}(\mathbf{p})$ rewrite this integral as

$$I_2(\mathbf{p},\mathbf{p}^{in},\hat{\mathbf{k}}) = \frac{n_{eff}}{2\pi^2}\int d\mathbf{y}\,D(\mathbf{y},\hat{\mathbf{k}})e^{-i\langle\mathbf{p},\mathbf{y}\rangle}\psi^{ce}_{\mathbf{p}^{in}}(\mathbf{y}). \qquad (15)$$

Here the function $D(\mathbf{y},\hat{\mathbf{k}})$ is given by

$$D(\mathbf{y},\hat{\mathbf{k}}) = \lim_{\lambda\to+0}\frac{1}{(2\pi)^{3/2}}\int d\mathbf{q}\,e^{-i\langle\mathbf{q},\mathbf{y}\rangle-\lambda q}\langle\hat{\mathbf{k}},\hat{\mathbf{q}}\rangle^2.$$

It is shown that the main singular part of $D(\mathbf{y},\hat{\mathbf{k}})$ is proportional to delta-function, i.e.

$$D(\mathbf{y},\hat{\mathbf{k}}) \simeq \frac{1}{3(2\pi)^{3/2}}\delta(\mathbf{y}).$$

The latter gives for the most singular part of the integral $I_2(\mathbf{p},\mathbf{p}^{in},\hat{\mathbf{k}})$

$$I_2(\mathbf{p},\mathbf{p}^{in},\hat{\mathbf{k}}) \simeq N_{eff}\,\psi^{ce}_{\mathbf{p}^{in}}(0), \tag{16}$$

$$N_{eff} = \frac{n_{eff}}{2\pi^2}\frac{1}{3(2\pi)^{3/2}}.$$

Now, inserting formulas given above in (12) we can represent the correction term $\hat{\Psi}_2$ as the integral

$$\hat{\Psi}_2(\mathbf{P},\mathbf{P}^{in}) = -\frac{1}{4\pi i}\int d\mathbf{P}' \oint_C d\zeta\, \hat{g}_s(\mathbf{k},\mathbf{k}',\zeta)\hat{g}^c_{eff}(\mathbf{p},\mathbf{p}',E-\zeta+i0)\times \tag{17}$$

$$I_1(\mathbf{k}',\mathbf{k}^{in})I_2(\mathbf{p}',\mathbf{p}^{in},\hat{\mathbf{k}}').$$

This general formula can be simplified if we take instead of full Green's function $\hat{g}_s(\mathbf{k},\mathbf{k}',\zeta)$ its main singular part which is Green's function of the two-body kinetic energy operator $\delta(\mathbf{k}-\mathbf{k}')(k^2-\zeta)^{-1}$. This case in fact has the particular physical sense, since taking into account that we left only delta-function for $\hat{\psi}^+_s$ the resulting approximation is exactly equivalent to the neglect of the short-range potential V_s from the very beginning.

Formula (17) is transformed in this case to

$$\hat{\Psi}_2(\mathbf{P},\mathbf{P}^{in}) \simeq \tag{18}$$

$$-\frac{1}{2}N_{eff}I_1(\mathbf{k},\mathbf{k}^{in})\int d\mathbf{p}'\hat{g}^c_{eff}(\mathbf{p},\mathbf{p}',E-k^2+i0)\psi^{ce}_{\mathbf{p}^{in}}(0).$$

The configuration space representation for Ψ_2 which can be obtained from formula (10) is reduced now to the integral

$$\Psi_2(\mathbf{X},\mathbf{P}^{in}) \simeq$$

$$-\frac{1}{2}N_{eff}\psi^{ce}_{\mathbf{p}^{in}}(0)\int d\mathbf{k}\,\psi^c_{\mathbf{k}}(\mathbf{x})I^s_1(\mathbf{k},\mathbf{k}^{in})g^c_{eff}(\mathbf{y},0,E-k^2+i0).$$

Final form for this integral follows immediately from delta-functional factors of I_1 and reads

$$\Psi_2(\mathbf{X},\mathbf{P}^{in}) \simeq -\frac{1}{2}N_{eff}\psi^{ce}_{\mathbf{p}^{in}}(0)\times \tag{19}$$

$$\frac{\partial^2}{\partial t^2}\left[t^2\mathscr{L}^{(2)}(t,k^{in})\psi^c_{t\hat{\mathbf{k}}^{in}}(\mathbf{x})g^c_{eff}(\mathbf{y},0,E-t^2+i0)\right]\Big|_{t=k^{in}}.$$

This formula describes the correction term Ψ_2 for the ppe system when strong pp interaction is neglected and at the same time is approximation to the term Ψ_2 in the general case.

The formula (19) is useful for constructing the coordinate asymptotics of $\Psi_2(\mathbf{X}, \mathbf{P}^{in})$ as $y \to \infty$. One needs to use well known coordinate asymptotics of Coulomb Green's function as $y \to \infty$ and y' is bound

$$g^c_{eff}(\mathbf{y}, \mathbf{y}', s^2 + i0) \sim \frac{\exp\{isy - i\frac{n_{eff}}{2s}\log 2sy\}}{4\pi y}\psi^{ce*}_{-s\hat{\mathbf{y}}}(\mathbf{y}')$$

This asymptotic formula gives the following asymptotics of $\Psi_2(\mathbf{X}, \mathbf{P}^{in})$

$$\Psi_2(\mathbf{X}, \mathbf{P}^{in}) \sim \tag{20}$$

$$\mathscr{A}(\mathbf{x}, \mathbf{k}^{in}, \mathbf{p}^{in}, \hat{\mathbf{y}}) \frac{\exp\{ip^{in}y - i\frac{n_{eff}}{2p^{in}} \log 2p^{in}y\}}{4\pi y} \left(1 + O\left(y\frac{k^{in}}{p^{in}}\right)\right)$$

where the amplitude $\mathscr{A}$ has the explicit form

$$\mathscr{A}(\mathbf{x}, \mathbf{k}^{in}, \mathbf{p}^{in}, \hat{\mathbf{y}}) = -\frac{1}{2} N_{eff} \psi^{ce}_{\mathbf{p}^{in}}(0) \times$$

$$\frac{\partial^2}{\partial t^2} \left[t^2 \mathscr{L}^{(2)}(t, k^{in}) \psi^{c}_{t\hat{\mathbf{k}}^{in}}(\mathbf{x}) \psi^{ce*}_{-\sqrt{E-t^2}\hat{\mathbf{y}}}(0) \right] \Big|_{t=k^{in}}$$

Here by $O\left(y\frac{k^{in}}{p^{in}}\right)$ we have denoted terms corresponding to derivatives of exponential factor in (19). The order of this terms shows the range of validity of the asymptotics (20), i.e. $y\frac{k^{in}}{p^{in}}$ has to be small, what in terms of masses must be equivalent to the fact that $y\tau$ has to be small.

Astrophysical Examples

Now let us discuss ways of describing some reactions of the pp-cycle on the Sun, which can be done on the ground of 3-body wave function given by (11). In other words, we will consider situations when in the initial state the system consists of three charged particles in the continuum and the mass of one of them is much smaller than other masses.
The first example gives the reaction

$$p + p + e \to d + \nu \tag{21}$$

considered. As it follows from the form of the main term in the right hand side of (11), with very good accuracy we have separation of the Jacobi coordinates in the wave function of the initial state for the reaction (21). This means that the rate of three-body process (21) can be expressed in terms of a binary process

$$p + p \to d + e^{+} + \nu \tag{22}$$

This is just the main result of paper. Now it becomes clear that the physi-

cal background of the above result from the point of view of the few-body theory consists in two points:
a) the system has two heavy and one light particle such that the parameter $\frac{m}{M} \ll 1$ and therefore one can neglect the second term in the right-hand side of (11).
b) heavy particles are slow enough to neglect higher partial waves in their relative motion. One should emphasize that free "effective charge of the initial nuclear system" Z, introduced in , can now be fixed to value $Z = 2$ which is supported by the structure of (11).
Let us consider another example of 3-body initial state

$$p + {}^7Be + e, \tag{23}$$

which can produce ^{8}Be or ^{7}Li nuclei via the following reactions

$$p + {}^7Be + e \begin{cases} \nearrow\ {}^8B + e \ \ (\text{or } {}^8B + e + \gamma) \\ \searrow\ {}^7Li + p + \nu \end{cases}$$

First, from the previous discussion one can see that due to different masses of heavy particles in this case the contribution from the linear term τ is

nonzero in contrast to the $p+p+e$ system, and this contribution should be estimated.

If the electron in state (23) is in the continuum, then again due to the separation of Jacobi coordinate in the first term of (11) the rate of the proton capture from the initial three-body state (23) can be expressed via the rate of the binary reaction $p+{}^7Be \rightarrow {}^8B+\gamma$.

However, the rate of the electron capture from the initial three-body state (23), as it follows from (11), (modified for the state (23)), will be defined by the Coulomb wave function of the electron moving in the Coulomb field with the charge $Z=5$ instead of $Z=4$ for the capture from the two-body state ^{7}Be+e. This means that the production rate of ^{7}Li from the three-body state (23) cannot be expressed via the binary (e+^{7}Be$\rightarrow{}^7$Li+ν) reaction rate. Roughly speaking, the ratio of these rates will be proportional to the ratio of the corresponding electron Coulomb functions at energy in the center of the Sun E_s. In other words,

$$\frac{w_3}{w_2} \sim \left|\frac{\psi_c(0,E_s,Z=5)}{\psi_c(0,E_s,Z=4)}\right|^2 \sim \frac{5}{4}.$$

Now let us discuss the problem of screening of the Coulomb interaction between two protons for the system $p+p+e$. We restrict ourselves by lowest order in the ratio m_e/m_p for the three-body wave function, i.e. consider only first term in the (11). It is evident, that the screening effect in this approximation appears due to the electronic wave function $\psi^{ce}_{\mathbf{p}^{in}}(\mathbf{y})$ where $\mathbf{y} = \sqrt{\frac{4m_e m_p}{m_e+2m_p}}\left(\frac{\mathbf{R}}{2}+\mathbf{r}\right)$, $\mathbf{R}$ being the distance between protons and $\mathbf{r}$ being the distance between electron and one of the protons. Taking the asymptotics of this function in the region where $R \gg r$, one can see that the Coulomb phases of pp wave function and electronic wave function can cancel each other for the specific configurations of initial momentums $\mathbf{k}^{in}$ and $\mathbf{p}^{in}$ of three-body system under consideration. Hence the resulting motion of two protons in this configuration would be described by plane wave, which means the total screening effect.

Conclusion

In conclusion we emphasize, that the CF-transformed three-body Hamiltonian (4) for the system of two heavy and one light particles can be used for realization of adiabatic expansion which is alternative to the Born-Oppenheimer one. This approach makes possible to treat screening effects on the microscopic level. The application of zero order in τ approximation to some astrophysical reactions allows to express rates of three-body processes in terms of rates of binary processes.