COMPLETE POLARIZATION EXPERIMENTS

for $NN \to N\Delta$ and p+d→(pp)($^1S_0$)+Δ^0

IN COLLINEAR KINEMATICS

Yu.N. Uzikov

V.P. Dzhelepov Laboratory of Nuclear Problems, JINR, Dubna, RF

In collaboration with *J. Haidenbauer (IKP FZJ, Jülich)*

Dubna, JINR, SPIN05, 1 October, 2005

Content:

- **Triple spin correlations for binary reaction** $\vec{1} + \vec{2} \to \vec{3} + 4$
- $\vec{N}\vec{N} \to N\vec{\Delta}$ **:**
  four spin amplitudes $\to$ **observables**
- $\vec{p} + \vec{d} \to$ **pp**($^1S_0$)+$\vec{\Delta}$ **:**
  - **three spin amplitudes & observables** $\to$ **C.P.E.**
    - ⋆ **transversal,**
    - ⋆ **longitudinal**
- **Relation between** $\vec{p} + \vec{d} \to$ **pp**($^1S_0$)+$\vec{\Delta}$ **and** $NN \to N\Delta$ **within the IA**

- Δ-isobar is very important for:

  $NN \to NN$ due to coupling to $NN \to N\Delta$ & $NN \to \Delta\Delta$

  $NN \to NN\pi$ systems (even near the threshold and above)

  3NF forces ($pd \to pd$ Sagara puzzle; $\frac{d\sigma}{d\Omega}$, 60 - 200 MeV, A_y)

  3NF forces in medium

  Δ-mechanism in pd-backward elastic scattering, $T > 400$MeV T_{20} problem

  $\Delta\sigma_L$ & $\Delta\sigma_T$ in pn-, pp- interaction below 1 GeV

Amplitude analysis of the exp.data on $NN \to N\Delta$ at 500-800 MeV ($d\sigma/d\Omega$, A_{N0}, A_{L0}, A_{S0}, A_{NN}, A_{LL}, $P_y\rho_{33}$):

A.B. Wicklund et al., Phys. Rev. D **34** (1986) 19.
R.L. Shypit et. al., Phys. Rev. Lett. **60** (1988) 901; Phys. Rev. C **40** (1989) 2203.
D.V. Bugg, Proceedings of the 3rd International Symposium on πN and NN Physics, LNPI, Gatchina (1989) p.57.
V.V. Anisovich, A.V. Sarantsev, and D.V. Bugg, Nucl. Phys. **A537** (1992) 501.

- Complete polarization experiment for $NN \to NN$ (5 amplitudes) $\Longrightarrow$ SAID
- 16 amplitudes for $NN \to N\Delta$ in general case

- Collinear kinematics (only one direction $\mathbf{k}$) $\Longrightarrow$ simplification of spin dependence

Polarizations with axial symmetry is not a limiting case of the general formalism

Examples:

- $dp \to pd$ in collinear kinematics

V.P. Ladygin, N.B. Ladygina, J.Phys.G: Part.Nucl.**23** (1997) 847
M.P. Rekalo, N.M. Piskunov, and I.M. Sitnik, Few-Bod.Syst. **23** (1998) 187.

Review of the near threshold formalism:
E. Tomasi-Gustaffson and M.P. Rekalo, Fiz. Elem. Chat. At.Yadr. **33** (2002) 436 (Phys. Part.Nucl. **33** (2002) 220).

$\vec{1} + \vec{2} \to \vec{3} + 4$

Beam direction $\mathbf{k}||OZ$

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} =< \mu_1, \mu_2|F|\mu_3\mu_4 >= \sum_{S_i\,M_i\,S_f\,M_f\,L\,m} (j_1\mu_1\, j_2\mu_2|S_iM_i)(j_3\mu_3\, j_4\,\mu_4|S_f\,M_f) \times$$

$$\times(S_i\,M_i\,L\,m|S_f\,M_f)Y_{Lm}(\hat{\mathbf{k}})a^{LS_i}_{S_f}. \quad (1)$$

$$(-1)^L = \pi_1\pi_2\pi_3\pi_4,$$

Triple spin correlation

$$K^{J_3\,M_3}_{J_1\,M_1,J_2\,M_2} = \frac{Tr\,\{T_{J_3M_3}(3)F\,T_{J_1M_1}(1)\,T_{J_2M_2}(2)\,F^+\}}{TrFF^+}, \quad (2)$$

Rank $J_i = 0, 1, \ldots, 2j_i$

$$M_k = -J_k, -J_k + 1, \cdots, J_k$$

S.M.Bilenky, L.I.Lapidus, L.D.Puzikov, R.M. Ryndin, Nucl.Phys. **7** (1958) 646.

$$K^{J_3\,M_3}_{J_1\,M_1,J_2\,M_2}\,TrFF^{+} = \frac{1}{4\pi}\sqrt{(2J_1+1)(2J_2+1)(2J_3+1)}$$

$$\sum_{S\,S'\,J\,J'\,L\,L'\,J_0\,J_0'} (2J+1)(2J'+1)\sqrt{(2L+1)\,(2L'+1)}\,\times$$

$$\sqrt{(2S+1)\,(2S'+1)\,(2J_0+1)}(-1)^{j_3+j_4+J+L+S'-S}\,\times$$

$$\times(J_0\;-M_3\,J_3\,M_3|J_0'\,0)(L'\,0\,L\,0\,|J_0'0)(J_1M_1J_2M_2|J_0-M_3)\,\times$$

$$\begin{Bmatrix} j_3 & j_4 & J \\ J' & J_3 & j_3 \end{Bmatrix}\begin{Bmatrix} S & j_1 & j_2 \\ S' & j_1 & j_2 \\ J_0 & J_1 & J_2 \end{Bmatrix}\begin{Bmatrix} S & J & L \\ S' & J' & L' \\ J_0 & J_1 & J_0' \end{Bmatrix} a^{L\,S}_{J}(a^{L'\,S'}_{J'})^{*}. \qquad (3)$$

$$L+L'+J_0' \quad is\;\; even, \qquad J_0' \;\; is\;\; even$$

$$M_1+M_2+M_3=0. \qquad (4)$$

Symmetry:

$$K^{J_3\,-M_3}_{J_1\,-M_1,J_2\,-M_2} = (-1)^{J_1+J_2+J_3}K^{J_3\,M_3}_{J_1\,M_1,J_2\,M_2} \qquad (5)$$

Yu.N. U. Phys.Lett. B595 (2004) 277; hep-ph/0411113

• The reaction $NN \to \Delta N$

Parity and angular momentum conservation $\to a^{L\,S_i}_{S_f}$:

$$B_1 = a^{2\,0}_2, \underbrace{B_2 = a^{2\,1}_2, B_3 = a^{2\,1}_1, B_4 = a^{0\,1}_1}_{\text{(spin-triplet)}}$$

$$|B_1|^2 = \{\frac{1}{5}(K^{20}_{10,10} - K^{20}_{00,00}) + \frac{2}{5}K^{00}_{11,1-1}\}\Sigma, \tag{6}$$

$$|B_2|^2 = \frac{1}{5}\{1 + K^{20}_{00,00} - \sqrt{6}K^{2-2}_{11,11} - 3K^{20}_{10,10}\}\Sigma, \tag{7}$$

$$|\sqrt{2}B_3 - B_4|^2 = \{K^{20}_{10,10} - K^{20}_{00,00} - 2K^{00}_{11,1-1}\}\Sigma, \tag{8}$$

$$|B_3 + \sqrt{2}B_4|^2 = \frac{1}{2}\left\{1 + 3K^{20}_{00,00} - K^{20}_{10,10} + \sqrt{6}K^{2-2}_{11,11}\right\}\Sigma, \tag{9}$$

where

$$\Sigma = 4\pi Tr\, FF^{+} = \frac{16\pi d\sigma_0}{\Phi}. \tag{10}$$

$$ReB_2(B_3 + \sqrt{2}B_4)^* = \frac{\Sigma}{2\sqrt{10}}\{\sqrt{6}(K^{20}_{10,10} + K^{20}_{00,00}) - 2K^{2-2}_{11,11}\}, \tag{11}$$

$$ReB_1(\sqrt{2}B_3 - B_4)^* = \Sigma\frac{1}{30}\{\sqrt{5} + 20K^{30}_{10,00} - 2\sqrt{5}K^{20}_{00,00} + 3K^{20}_{10,10})\}. \tag{12}$$

For T-odd observables($J_1+J_2+J_3$ is odd):

$$K^{3-2}_{11,11}\,\Sigma=\frac{i5\sqrt{2}}{2}ImB_2(B_3+\sqrt{2}B_4)^*, \tag{13}$$

$$K^{30}_{1-1,11}\,\Sigma=\frac{i3}{2}ImB_1(\sqrt{2}B_3-B_4)^* \tag{14}$$

Real and imaginary parts of the products $B_1(\sqrt{2}B_3-B_4)^*$ and $B_2(B_3+\sqrt{2}B_4)^*$ $\Longrightarrow$ the relative phases $\phi_{B_1}-\phi_{\sqrt{2}B_3-B_4}$ and $\phi_{B_2}-\phi_{B_3+\sqrt{2}B_4}$.
The last relative phase, for example, $\kappa=\phi_{B_2}-\phi_{B_3+\sqrt{2}B_4}$, can be found from $K^{3-1}_{11,10}$ and $K^{3-1}_{11,00}$ $\Longrightarrow$ $\sin\kappa$ and $\cos\kappa$.

Complete polarization experiment:

$$d\sigma_0,\underbrace{K^{00}_{11,1-1},\ K^{20}_{10,10},\ K^{2-2}_{11,11},\ K^{20}_{00,00},\ \ K^{30}_{10,00},\ K^{31}_{11,00}}_{\mathrm{T-even,}}$$

$$\underbrace{K^{3-2}_{11,11},\ \ K^{30}_{1-1,11}\,and\ K^{3-1}_{11,10}}_{\mathrm{T-odd}}.$$

The reaction $pd \to \Delta^0(pp)(^1S_0)$

$$A_1 = a_{\frac{3}{2}}^{2\,\frac{1}{2}},\, A_2 = a_{\frac{3}{2}}^{0\,\frac{1}{2}},\, A_3 = a_{\frac{3}{2}}^{2\,\frac{1}{2}}$$

⋆ Transversally polarized beam and target

Six observables, $d\sigma_0$, $K^{00}_{00,20}$, $K^{00}_{1-1,11}$, $K^{20}_{00,00}$, $K^{2-1}_{00,21}$, and $K^{22}_{00,2-2}$, $\Longrightarrow$
$\Longrightarrow$ $|A_1|^2$, $|A_2|^2$, $|A_3|^2$ and cosines of $\phi_1 - \phi_2$ and $\phi_1 - \phi_3$:

$$\begin{aligned} |A_1|^2 &= I_0 + \frac{2E+F}{3}, \\ |A_2|^2 &= \frac{1}{2}C - \frac{1}{3}E - \frac{1}{6}F, \\ |A_3|^2 &= -\frac{1}{2}C - \frac{1}{3}E - \frac{1}{6}F, \\ ReA_1A_2^* &= \frac{1}{2}D + \frac{E-F}{6}, \\ ReA_1A_3^* &= \frac{1}{2}D - \frac{E-F}{6}, \end{aligned} \tag{15}$$

where

$$I_0 = |A_1|^2 + |A_2|^2 + |A_3|^2 = \pi TrFF^+ = \frac{6\pi d\sigma_0}{\Phi}, \tag{16}$$

$$E = \{4K^{00}_{1-1,11} - \frac{4}{\sqrt{3}}K^{00}_{00,20} - \frac{2}{3}\}I_0, \tag{17}$$

$$F = \{\frac{12}{\sqrt{3}}K^{00}_{00,20} - 2\sqrt{6}K^{20}_{00,00} - 1\}I_0,$$

$$C = \{\frac{4}{\sqrt{3}}K^{22}_{00,2-2} - \frac{8}{\sqrt{3}}K^{2-1}_{00,21}\}I_0,$$

$$D = -\frac{4}{\sqrt{3}}\{K^{22}_{00,2-2} + K^{2-1}_{00,21}\}I_0.$$

In order to find the sines of two phases one should measure the three T-odd observables $K^{2-1}_{00,11}$, $K^{2-1}_{11,00}$, and $K^{00}_{11,2-1}$.

$$iImA_1A_2^* = -2\{K^{00}_{11,2-1} - \frac{1}{\sqrt{3}}K^{2-1}_{00,11} + \frac{1}{\sqrt{2}}K^{2-1}_{11,00}\}I_0, \tag{18}$$

$$iImA_1A_3^* = 2\{K^{00}_{11,2-1} + \frac{1}{\sqrt{3}}K^{2-1}_{00,11} - \frac{1}{\sqrt{2}}K^{2-1}_{11,00}\}I_0,$$

For $J_1 + J_2 + J_3$ being odd, the value of $K^{J_3M_2}_{J_1M_1,J_2M_2}$ is purely imaginary.

Note: only the signs of $ImA_1A_2^*$ and $ImA_1A_3^*$ are required.

Nine observables $\Longrightarrow$ complete polarization experiment: $d\sigma_0$, $K^{00}_{00,20}$, $K^{00}_{1-1,11}$, $K^{20}_{00,00}$, $K^{2-1}_{00,21}$, $K^{22}_{00,2-2}$, $\underbrace{K^{2-1}_{00,11},\ K^{2-1}_{11,00},\ K^{00}_{11,2-1}}_{\text{T}-\text{odd}}$.

$\star$ Transversally & longitudinally polarized beam and target in $pd \to \Delta^0(pp)(^1S_0)$

Using I_0 and $\underbrace{K^{00}_{10,10}}_{\text{longitudinal}}$ $\underbrace{K^{00}_{1-1,11},\ K^{21}_{00,2-1},\ K^{22}_{00,2-2}}_{\text{transversal}}$, $\Longrightarrow$ $|A_1|^2$, $|A_2|^2$, and $|A_3|^2$.

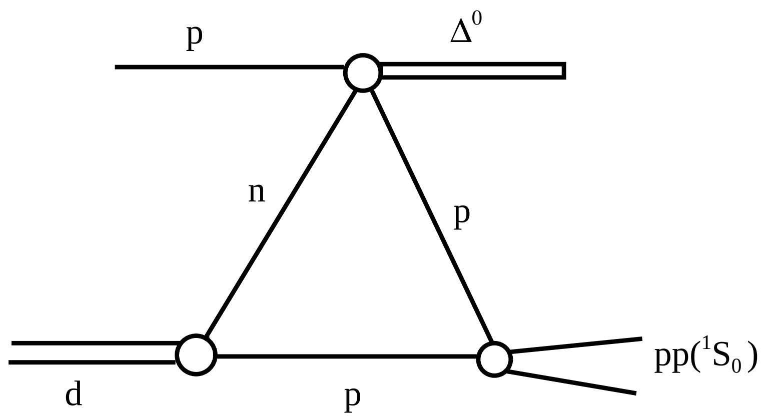
In addition, $K^{20}_{00,00} \Longrightarrow Re\, A_1 A_2^*$, $Re\, A_1 A_3^*$ and $Re\, A_2 A_3^*$.

Finally, $K^{2-1}_{00,11}$ and $K^{2-1}_{11,00}$ give

$$iImA_2A_3^* = \{\frac{4}{\sqrt{3}}K^{2-1}_{00,11} + \frac{2}{\sqrt{2}}K^{2-1}_{11,00}\}I_0 \tag{19}$$

$\phi_{23} = \phi_{13} + \phi_{21}$, where $\phi_{ij} = \phi_i - \phi_j$

The above eight observables $\Longrightarrow$ A_1, A_2 and A_3



$B_i = a_J^{LS}(pn \to \Delta^0 p) \to M^{\mu_\Delta}_{\mu_0,\lambda}(pd \to \Delta^0 (pp)_{^1S_0})$

$$A_1 = \frac{\sqrt{6\pi}}{3}(M^{+1/2}_{+,0} - \sqrt{2}M^{-1/2}_{+,-}), \quad (20)$$

$$A_2 = \frac{\sqrt{6\pi}}{3}(\sqrt{\frac{3}{2}}M^{+3/2}_{+,+} + M^{+1/2}_{+,0} + \frac{1}{\sqrt{2}}M^{-1/2}_{+,-}),$$

$$A_3 = \frac{\sqrt{6\pi}}{3}(\sqrt{\frac{3}{2}}M^{+3/2}_{+,+} - M^{+1}_{+,0} - \frac{1}{\sqrt{2}}M^{-1/2}_{+,-}),$$

$A_1,\ A_2,\ A_3$ are functions of $B_1,\ B_2,\ B_3,\ B_4$ and transition form factors

Inversion problem:

$$B_4 = -\frac{16\pi}{\sqrt{15}}\{(5S_2 + \sqrt{10}S_0)A_2 + 5S_2(A_3 - \sqrt{3}A_1)\}/Y,$$
$$\sqrt{5}B_2 - \sqrt{3}B_3 = \frac{32\pi}{\sqrt{2}}\{\sqrt{5}S_2(A_1 + A_2) + (\sqrt{2}S_0 + \sqrt{5}S_2)A_3\}/Y,$$
$$\sqrt{3}B_1 - \sqrt{2}B_2 = \frac{16\pi}{\sqrt{5}}\{(2\sqrt{2}S_0 + \sqrt{5}S_2)A_1 - 3\sqrt{5}S_2A_2 - (\sqrt{2}S_0 - \sqrt{5}S_2)A_3\}/Y, \quad (21)$$

where

$$Y = 2S_0^2 + \sqrt{10}S_0S_2 - 10S_2^2. \quad (22)$$

Transition form factor ${}^3S_1 - {}^3D_1 \to {}^1S_0$

$$S_l(Q/2) = \int_0^\infty drr^2 j_l(Qr/2)u_l(r)\psi_k^{(-)^*}(r), \quad (23)$$

where u_0 and u_2 are the S- and D-components of the deuteron
$\psi_k^{(-)}(r)$ is the NN scattering wave function in the ${}^1S_0$ state

CONCLUSION

D.V. Bugg and C. Wilkin, Nucl. Phys. A**467** (1987) 575.
Charge-exchange $dp \to (pp)_{(^1S_0)}n$ and $pn \to np$ (SATURNE, COSY, JINR)

⋆ Complete polarizations experiment for $NN \to N\Delta$ (4 amplitudes, 10 observables)
⋆ Complete polarizations experiment for $pd \to (pp)_{(^1S_0)}\Delta^0$ (3 amplitudes, 8-9 observables)
⋆ Three combinations from four amplitudes of the $NN \to N\Delta$ are related to three amplitudes of the $pd \to (pp)_{(^1S_0)}\Delta^0$

"… The whole business looks extremely complicated — but that is the nature of the problem."

$$(\frac{3}{2} \neq \frac{1}{2})$$

C. Wilkin (private communication)