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Non-singlets in

Semi-inclusive DIS and inclusive e^+e^-

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The goal

We know:

$$DIS: \qquad g_1^p - g_1^n = \frac{1}{6}\Delta q_3 \otimes (1 + \frac{\alpha_s}{2\pi}\delta C_q + \ldots)$$

$$\underbrace{\Delta q_3}_{\text{NS}} = (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d})$$

- in NLO, NNLO ... no new PD
- in Q^2 evolution – no new PD

– allows to compare meas. at diff. Q^2

We ask:

- What are the measurable quantities that single out NS in SIDIS & e^+e^--inclusive?
- What are the NSs of pol. PD determined in combined DIS & SIDIS?
- What are the NSs of FF determined in combined unpol. SIDIS & e^+e^--inclusive?
- What other info can we obtain?

SIDIS: $\overrightarrow{l} + \overrightarrow{N} \to l' + h + X$

$$A_N^h = \frac{\Delta\sigma_N^h}{\sigma_N^h} = \frac{\sum e_q^2 \,(\Delta q\, D_q^h + \Delta\bar{q}\, D_{\bar{q}}^h)}{\sum e_q^2 \,(q\, D_q^h + \bar{q}\, D_{\bar{q}}^h)}$$

SMC, HERMES, COMPASS:

Δq_V determined:

but assumps. about $\Delta\bar{q}$:

$$\Delta\bar{u} = \Delta\bar{d} = \Delta\bar{s} \quad \text{or} \quad \Delta\bar{u}/\bar{u} = \Delta\bar{d}/\bar{d} = \Delta\bar{s}/\bar{s}$$

and uncertainties in FFs:

D. de Florian, G, Navarro and R. Sassot, 2005

We suggest the difference asymmetries

$$A_{1N}^{h-\bar{h}} = \frac{\Delta\sigma_N^{h-\bar{h}}}{\sigma_N^{h-\bar{h}}}$$

- measure only NS both in PDs and FFs

notation: $\Delta\sigma_N^{h-\bar{h}} \equiv \Delta\sigma_N^h - \Delta\sigma_N^{\bar{h}}$

SIDIS: $\overrightarrow{l} + \overrightarrow{N} \to l' + h + X$

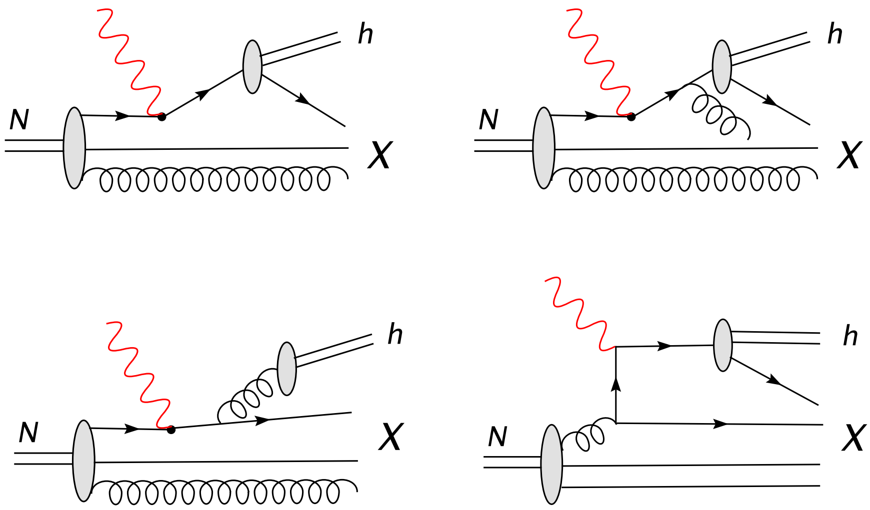
The general formula in SIDIS, $Q^2 \gg M^2$:

$$\Delta\tilde{\sigma}_N^h \propto \sum_q e_q^2 \Big\{ \Delta q \otimes \Delta\hat{\sigma}_{qq}(\gamma q \to qX) \otimes D_q^h$$
$$+ \Delta q \otimes \Delta\hat{\sigma}_{qG}(\gamma q \to GX) \otimes D_G^h$$
$$+ \Delta G \otimes \Delta\hat{\sigma}_{Gq}(\gamma G \to q\bar{q}X) \otimes (D_q^h + D_{\bar{q}}^h) \Big\}$$

$\Delta q(x,t)$ and $D_{q,G}^h(z,t) \Rightarrow$ from experiment

$\Delta\hat{\sigma}_{ff'} \Rightarrow$ theor. calculated in perturb. QCD:

$$\Delta\hat{\sigma}_{ff'} = \Delta\hat{\sigma}_{ff'}^{(0)} + \frac{\alpha_s}{2\pi}\Delta\hat{\sigma}_{ff'}^{(1)} + \dots$$



C-inv: $D_G^{h-\bar{h}} = 0,\ D_q^{h-\bar{h}} = -D_{\bar{q}}^{h-\bar{h}}$

The difference asymmetries $A_{1N}^{h-\bar{h}}$

C-inv. implies: $D_G^{h-\bar{h}} = 0$, $D_q^{h-\bar{h}} = -D_{\bar{q}}^{h-\bar{h}}$

$\Rightarrow$ In all orders in QCD all gluons cancel :

– no g, no Δg, no D_g

$$\Delta\tilde{\sigma}_N^{h-\bar{h}} \propto \left[4\Delta u_V \otimes D_u^{h-\bar{h}} + \Delta d_V \otimes D_d^{h-\bar{h}} + (\Delta s - \Delta\bar{s}) \otimes D_s^{h-\bar{h}}\right] \otimes \Delta\hat{\sigma}(\gamma q \to qX)$$

$$\Delta\hat{\sigma}_{qq} = \Delta\hat{\sigma}_{qq}^{(0)} + \frac{\alpha_s}{2\pi}\Delta\hat{\sigma}_{qq}^{(1)} + \ldots$$

- only NS $\Rightarrow$ gluons do not reappear in Q^2-evol.
- each term is a NS
- sensitive to Δu_V, Δd_V & $(\Delta s - \Delta\bar{s})$ only

Further $\Delta\tilde{\sigma}_N^{h-\bar{h}}$ depends on the final hadron h.

$SIDIS: \vec{l} + \vec{N} \to l' + \pi^{\pm} + X$

SU(2) and C: $D_u^{\pi^+-\pi^-} = -D_d^{\pi^+-\pi^-}$, $D_s^{\pi^+-\pi^-} = 0$

singles $\Delta u_V, \Delta d_V$:

$$LO: A_{1p}^{\pi^+-\pi^-}(x, z, Q^2) = \frac{4\Delta u_V - \Delta d_V}{4u_V - d_V}(x, Q^2)$$

$$A_{1n}^{\pi^+-\pi^-}(x, z, Q^2) = \frac{4\Delta d_V - \Delta u_V}{4d_V - u_V}(x, Q^2)$$

The FFs completely cancel!

Frankfurt et al, P.L. B (1989)

E. Ch. & E. Leader, P.L. B (1999)

$\Rightarrow$ 2 *algebraic* eqs. for Δu_V and Δd_V.

NLO test: $z-dep. \Rightarrow$ NLO; no $z-dep. \Rightarrow LO/NLO$

Higher orders in QCD:

$$A_{1p}^{\pi^+-\pi^-} = \frac{(4\Delta u_V - \Delta d_V)[1 + \otimes(\alpha_s)\Delta C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{(4u_V - d_V)[1 + \otimes(\alpha_s)C_{qq}\otimes]D_u^{\pi^+-\pi^-}}$$

$$A_{1n}^{\pi^+-\pi^-} = \frac{(4\Delta d_V - \Delta u_V)[1 + \otimes(\alpha_s)\Delta C_{qq}\otimes]D_u^{\pi^+-\pi^-}}{(4d_V - u_V)[1 + \otimes(\alpha_s)C_{qq}\otimes]D_u^{\pi^+-\pi^-}}$$

$$\Delta C_{qq} = \Delta C_{qq}^{(1)} + \alpha_s \Delta C_{qq}^{(2)} + \ldots$$

$\Rightarrow$ 2 eqs. for Δu_V and Δd_V

E. Ch. & E. Leader, N.P. B (2001)

JLab can measure these asymmetries!

SU(2) for the polarized sea : $(\Delta\bar{\mathrm{u}} - \Delta\bar{\mathrm{d}})$

We have in any order in QCD:

$$(\Delta\bar{u} - \Delta\bar{d}) = \Delta q_3 + \Delta d_V - \Delta u_V$$

where

$$\Delta q_3(x, Q^2) = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}).$$

Here Δq_3 is obtained directly from DIS:

LO :

$$g_1^p(x, Q^2) - g_1^n(x, Q^2) = \frac{1}{6}\Delta q_3$$

NLO :

$$g_1^p(x, Q^2) - g_1^n(x, Q^2) = \frac{1}{6}\Delta q_3 \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q\right)$$

not through $(\Delta u + \Delta\bar{u})$ & $(\Delta d + \Delta\bar{d})$ that depend on Δs & ΔG.

$\Rightarrow$ no influence from Δs and ΔG.

$(\Delta\bar{u} - \Delta\bar{d}) \simeq$ *small* $\mapsto$ NLO needed

$SIDIS \quad unpol: l + N \to l' + \pi^{\pm} + X$

From unpolarized SIDIS $\Rightarrow D_u^{\pi^+ - \pi^-}(z, Q^2)$:

$$R_p^{\pi^+-\pi^-} = \frac{[4u_V - d_V][1 + \otimes(\alpha_s) C_{qq} \otimes] D_u^{\pi^+-\pi^-}}{18 F_1^p \, [1 + 2\gamma(y)\, R^p]}$$

$$R_n^{\pi^+-\pi^-} = \frac{[4d_V - u_V][1 + \otimes(\alpha_s) C_{qq} \otimes] D_u^{\pi^+-\pi^-}}{18 F_1^n \, [1 + 2\gamma(y)\, R^n]}.$$

$\Rightarrow$ 2 eqs. for $D_u^{\pi^+-\pi^-}(z, Q^2)$

HERMES: $\sigma_p^{\pi^{\pm}}$

$\Rightarrow \Delta u_V, \Delta d_V$ and $D_u^{\pi^+-\pi^-}(z, Q^2)$ are non-singlets and don't mix with other PDs and FFs

JLab: $\sigma_{p,d}^{\pi^{\pm}}$ & $\sigma_{p,d}^{K^{\pm}}$ to be measured

SIDIS – $\pi^{\pm}$

Δu_V, Δd_V and $\Delta\bar{u} - \Delta\bar{d}$ determined in LO and NLO:

- no assumptions about FFs
- no assumptions about polarized sea densities, even $s \neq \bar{s}$ and $\Delta s \neq \Delta\bar{s} \Leftarrow D_s^{\pi^+-\pi^-} = 0$
- only SU(2) and C inv. of strong ints. assumed
- only unpolarized u_V and d_V to be known
- no knowledge of $\Delta\bar{u}$ and $\Delta\bar{d}$ required
- test for NLO of $A_N^{\pi^+-\pi^-}(x, z, Q^2)$:
  z-dep. of $A_N^{\pi^+-\pi^-} \Rightarrow$ NLO
  no z-dep. of $A_N^{\pi^+-\pi^-} \Rightarrow$ LO or NLO
  If a small dependence on $z \Rightarrow$ it can be considered as a system. th. error in LO analysis of $A_N^{\pi^+-\pi^-}$

$SIDIS: \vec{l} + \vec{N} \to l' + K^{\pm} + X,$

SIDIS $\pi^{\pm} \Rightarrow \Delta u_V, \Delta d_V$ without assumptions,

SIDIS $K^{\pm} \Rightarrow (\Delta s - \Delta \bar{s}) = 0?$ - LO & NLO

$D_d^{K^+-K^-} = 0$ assumed $\quad [K^+ = (\bar{s}u), K^- = (s\bar{u})]$

LO:

$$A_{1p}^{K^+-K^-} = \frac{4\Delta u_V D_u^{K^+-K^-} + (\Delta s - \Delta \bar{s}) D_s^{K^+-K^-}}{4 u_V D_u^{K^+-K^-}}$$

$$A_{1n}^{K^+-K^-} = \frac{4\Delta d_V D_u^{K^+-K^-} + (\Delta s - \Delta \bar{s}) D_s^{K^+-K^-}}{4 d_V D_u^{K^+-K^-}}$$

NLO: the same quantities enter

$$A_{1p}^{K^+-K^-} = \frac{[\Delta u_V D_u + (\Delta s - \Delta \bar{s}) D_s] \otimes (1 + (\alpha_s) \Delta C_{qq})}{u_V \otimes (1 + (\alpha_s)\, C_{qq} \otimes) D_u^{K^+-K^-}}$$

$$A_{1n}^{K^+-K^-} = \frac{[\Delta d_V D_u + (\Delta s - \Delta \bar{s}) D_s] \otimes (1 + (\alpha_s) \Delta C_{qq})}{d_V \otimes (1 + (\alpha_s)\, C_{qq} \otimes) D_u^{K^+-K^-}}$$

$(\Delta s - \Delta \bar{s}) D_s^{K^+-K^-} \Rightarrow$ info. about $(\Delta s - \Delta \bar{s}) \neq 0?$

$\longrightarrow$ note: $D_s^{K^+-K^-}$ is not small

- If $(\Delta s - \Delta \bar{s}) = 0$:

$$A_{1p}^{K^+-K^-}(x, z) = \frac{\Delta u_V}{u_V}(x), \quad A_{1n}^{K^+-K^-}(x, z) = \frac{\Delta d_V}{d_V}(x)$$

– cannot be tests for $(\Delta s - \Delta \bar{s}) = 0$, because $D_d^{K^+-K^-} = 0$ assumed!

SIDIS: $eN \to e + K^{\pm} + X \Rightarrow (s - \bar{s}) = 0?$

$D_d^{K^+ - K^-} = 0$ assumed

The measurable quantity is

$$R_p^{K^+ - K^-} = \frac{\sigma_p^{K^+ - K^-}}{\sigma_p^{DIS}}$$

LO:

$$R_p^{K^+ - K^-} = \frac{4\, u_V D_u^{K^+ - K^-} + (s - \bar{s}) D_s^{K^+ - K^-}}{\sigma_p^{DIS}}$$

$$R_n^{K^+ - K^-} = \frac{4\, d_V D_u^{K^+ - K^-} + (s - \bar{s}) D_s^{K^+ - K^-}}{\sigma_n^{DIS}}$$

NLO:

$$R_p^{K^+ - K^-} = \frac{[4\, u_V D_u^{K^+ - K^-} + (s - \bar{s}) D_s^{K^+ - K^-}](1 + \otimes \alpha_s C_{qq})}{\sigma_p^{DIS}}$$

$$R_n^{K^+ - K^-} = \frac{[4\, d_V D_u^{K^+ - K^-} + (s - \bar{s}) D_s^{K^+ - K^-}](1 + \otimes \alpha_s C_{qq})}{\sigma_n^{DIS}}$$

2 meas. to determine $D_u^{K^+ - K^-}$ and $(s - \bar{s}) D_s^{K^+ - K^-}$

$\Rightarrow (s - \bar{s}) D_s^{K^+ - K^-} \neq 0 \Rightarrow (s - \bar{s}) \neq 0$

$\Rightarrow (s - \bar{s}) D_s^{K^+ - K^-} = 0 \Rightarrow (s - \bar{s}) = 0$

up to now $s = \bar{s}$ assumed!

inclusive $e^+e^- \to h + X$

In general:

$$\frac{d\sigma^h}{dz\,d\cos\vartheta} \propto (1+\cos^2\vartheta)\frac{d\sigma_T^h}{dz} + \cos\vartheta\,\frac{d\sigma_A^h}{dz}$$

$$\frac{d\sigma_T^h}{dz} = \sum_q \hat{e}_q^2\, D_q^{h+\bar{h}}$$
$$\frac{d\sigma_A^h}{dz} = \sum_q \hat{a}_q\, D_q^{h-\bar{h}}$$

$$d\sigma_T^h(z) = \int_{-1}^{1} d\cos\vartheta \propto \frac{d\sigma_T^h(z)}{dz},$$
$$A_{FB}^h(z) = \left[\int_{-1}^{0} - \int_{0}^{1}\right] d\cos\vartheta \propto \frac{d\sigma_A^h(z)}{dz}$$

$d\sigma_T^h(z)$ measures $D_q^{h+\bar{h}}$, $A_{FB}^h(z)$ measures $D_q^{h-\bar{h}}$

$A_{FB}^h(z)$ measures NS $\Leftarrow$ we are interested in

$d\sigma_T^h(z)$ measures S + NS

note: neither $d\sigma_T^h(z)$ nor $A_{FB}^h(z)$ distinquishes between the down-type quarks: always $D_d + D_s$

$\Rightarrow$ sym. properties of h, or assumptions needed to distinguish D_d and D_s

$\underline{\underline{inclusive \quad e^+e^- \to \pi^\pm + X}}$

$$A_{FB}^{\pi^+-\pi^-} = (\hat{a}_u - \hat{a}_d) D_u^{\pi^+-\pi^-}(z, m_Z^2)$$

$\underline{\underline{SIDIS \quad unpol: l + N \to l' + \pi^\pm + X}}$

From unpolarized SIDIS $\Rightarrow D_u^{\pi^+-\pi^-}(z, Q^0)$:

$$R_p^{\pi^+-\pi^-} = \frac{[4u_V - d_V] D_u^{\pi^+-\pi^-} [1 + \otimes(\alpha_s) C_{qq} \otimes]}{18 F_1^p [1 + 2\gamma(y) \, R^p]}$$

$$R_n^{\pi^+-\pi^-} = \frac{[4d_V - u_V] D_u^{\pi^+-\pi^-} [1 + \otimes(\alpha_s) C_{qq} \otimes]}{18 F_1^n [1 + 2\gamma(y) \, R^n]}.$$

$\Rightarrow$ **2 eqs. for** $D_u^{\pi^+-\pi^-}(z, Q^2)$

$D_u^{\pi^+-\pi^-}(z, Q^2) =$ **NS and doesn't mix with other FFs**

- **compare** $D_u^{\pi^+-\pi^-}$ **from** $A_{FB}^{\pi^+-\pi^-}$ **&** $\sigma_p^{\pi^+-\pi^-}$

– universality of FFs with no new FFs involved?

Can we avoid $D_d^{K^+-K^-}$?

inclusive $e^+e^- \to K^\pm + X$

SLD(SLAC)

$$A_{FB}^{K^+-K^-}(z) \propto \hat{a}_u D_u^{K^+-K^-} + \hat{a}_d(D_d + D_s)^{K^+-K^-}$$

SIDIS unpol: $eN \to e + K^\pm + X$

$$\tilde{\sigma}_p^{K^+-K^-} \propto [4u_V D_u + d_V D_d + (s-\bar{s})D_s)]^{K^+-K^-}(Q^2)$$
$$\tilde{\sigma}_n^{K^+-K^-} \propto [4d_V D_u + u_V D_d + (s-\bar{s})D_s]^{K^+-K^-}(Q^2)$$

Result: SIDIS on p & n + e^+e^- = not enough to determine $D_{u,d,s}^{K^+-K^-}$ & $(s-\bar{s})$

assump. needed: $D_d^{K^+-K^-} = 0$

If $K^{\pm}$ & $K^0, \bar{K}^0$ observed

SU(2): $K^{\pm} \Leftrightarrow K^0 \Rightarrow$ distinguishes $(D_d \,\&\, D_s)^{K^+ + K^-}$:

$$D_u^{K^+ + K^-} = D_d^{K^0 + \bar{K}^0}, \qquad D_d^{K^+ + K^-} = D_u^{K^0 + \bar{K}^0},$$
$$D_s^{K^+ + K^-} = D_s^{K^0 + \bar{K}^0}.$$

no new FFs! but does not help for $D_d^{K^+ - K^-}$!

inclusive $e^+e^- \to K^{\pm}, K^0, \bar{K}^0 + X$

SLD(SLAC)...

$$\frac{d\sigma_T^{K^+ + K^- - (K^0 + \bar{K}^0)}}{dz} = (\hat{e}_u^2 - \hat{e}_d^2)_{m_Z^2} (D_u - D_d)^{K^+ + K^-}$$

SIDIS unpol: $eN \to e + K^{\pm}, K^0, \bar{K}^0 + X$

$$\tilde{\sigma}_p^{K^+ + K^- - (K^0 + \bar{K}^0)} = (4(u + \bar{u}) - (d + \bar{d}))(D_u - D_d)^{K^+ + K^-}$$
$$\tilde{\sigma}_n^{K^+ + K^- - (K^0 + \bar{K}^0)} = (4(d + \bar{d}) - (u + \bar{u}))(D_u - D_d)^{K^+ + K^-}$$

- the same NS: $(D_u - D_d)^{K^+ + K^-}$ at m_Z^2 & Q^2

  $\Rightarrow$ relations between SIDIS and e^+e^- at LO:

$$\frac{\tilde{\sigma}_p^{K^+ + K^- - (K^0 + \bar{K}^0)}(x, z, Q^2)}{d\sigma_T^{K^+ + K^- - (K^0 + \bar{K}^0)}(z, m_Z^2)|_{\downarrow Q^2}} \propto \frac{[4(u + \bar{u}) - (d + \bar{d})](x, Q^2)}{(\hat{e}_u^2 - \hat{e}_d^2)_{m_Z^2}}$$

  On the l.h.s. $d\sigma_T(m_Z^2)$ is QCD-evolved to Q^2.

  No z-dependence on the r.h.s.

CONCLUSIONS

SIDIS & e^+e^- give different pieces of model indep. inform. through the difference asymmetries
$A_N^{h-\bar{h}}$, $R_N^{h-\bar{h}}$ and $A_{FB}^{h-\bar{h}}$ in LO & NLO

advantage : only measurable quantities are used
the price : precise measurements needed

- $\pi^\pm$: no assumps.

– $A_{1N}^{\pi^+-\pi^-}$: Δu_V, Δd_V & $\Delta\bar{u} - \Delta\bar{d}$
– $R_N^{\pi^+-\pi^-}$: $D_u^{\pi^+-\pi^-}$
$\rightarrow$ no assumps. about Δq_{sea}, ΔG & FFs

- $K^\pm$: assump: $D_d^{K^+-K^-} = 0$

– $A_{1N}^{K^+-K^-}$: $(\Delta s - \Delta\bar{s}) \neq 0$?
– $R_N^{K^+-K^-}$: $(s - \bar{s}) \neq 0$?
– $R_N^{K^+-K^-}$ & $A_{FB}^{K^+-K^-}$: $(s - \bar{s}), D_u^{K^+-K^-}, D_s^{K^+-K^-}$

- $K^\pm, K^0, \bar{K}^0$: no assumps.

$R_N^{K^++K^--(K^0+\bar{K}^0)}$ & $A_{FB}^{K^++K^--(K^0+\bar{K}^0)}$ measure the same $(D_u - D_d)^{K^++K^-}$ at different Q^2.
$\Rightarrow$ different relations at LO.