Generalized parton distributions: analysis and applications

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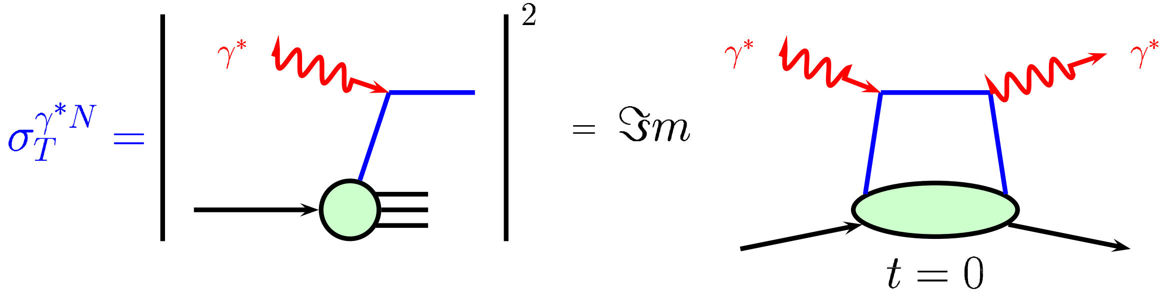
Outline:

- **Introduction**
- **Analysis of form factors**
- **Moments of GPDs**
- **Physical interpretation**
- **Applications: WACS, meson photoproduction**
- **Summary**

Parton Distributions



$$F_2(x) = x \sum_q e_q^2 q(x)$$

determination of $q(x)$, $g(x)$ with the help of DGLAP (evolve with Q^2)

Polarized DIS: Δq, Δg

describe longitudinal momentum distributions of partons within proton (IMF)

30 years of experimental (SLAC, CERN, HERA, FNAL, JLab)

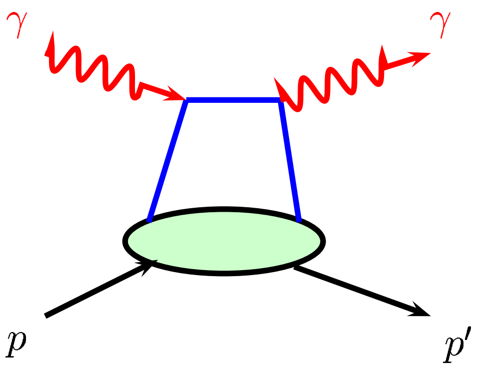
and theoretical (Barger-Phillips (74), GRV, CTEQ, MRST, ...)

effort has lead to a fair knowledge of the PDFs

(although not perfect, e.g. $x \to 0, 1$, g, Δg)

Generalized Parton Distributions

D. Müller et al (94), Ji(97), Radyushkin (97)



occurs in

DVES ($\gamma^* p \to \gamma p\,, Mp$) Q^2 large, t small

WAES ($\gamma p \to \gamma p\,, Mp$) Q^2 small, t large

soft physics: GPDs $H^q(x,\xi,t)$, $\widetilde{H}^q$, E^q, $\widetilde{E}^q$ (skewness: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$)

- reduction formulas: $H^q(x,0;0) = q(x)$; $\widetilde{H}^q(x,0;0) = \Delta q(x)$
- sum rules: $h^q_{10}(t) = \int_{-1}^{1} dx H^q(x,\xi,t)$; $F_1 = \sum_q e_q h^q_{10}$;
  $E^q \to F^q_2$; $\widetilde{H}^q \to F^q_A$; $\widetilde{E}^q \to F^q_P$
- polynomiality: e.g. $\int_{-1}^{1} dx\; x^{n-1} H^q(x,\xi,t) = \sum_{i=0}^{[n/2]} h^q_{n,i}(t)\xi^i$

universality, evolution, positivity constraints, Ji's sum rule

Interpretation: (Burkhardt) FT $\mathbf{\Delta} \Rightarrow \mathbf{b}$ $H^q(x,0,t=-\Delta^2) \Rightarrow q(x,0,\mathbf{b})$

long. momentum and trans. position distribution of partons within the proton

GPD analysis - what can be done?

DFJK hep-ph/0408173 (similar Guidal et al hep-ph/0410251)
analogue to PDF analyses

use all available data on $G_M^p\,, G_M^n\,, G_E^p\,, G_E^n\,(\Rightarrow F_1^p\,, F_1^n\,, F_2^p\,, F_2^n)\,, F_A$

exploit sum rules at $\xi = 0$

$$F_1^{p(n)}(t) = \int_0^1 dx \Big[e_{u(d)}\, H_v^u(x,t) + e_{d(u)}\, H_v^d(x,t)\Big] \qquad F_2 \Rightarrow E_v$$

$$F_A(t) = \int_0^1 dx \Big[\widetilde{H}_v^u(x,t) - \widetilde{H}_v^d(x,t)\Big] + 2\int_0^1 dx \Big[\widetilde{H}^{\bar{u}}(x,t) - \widetilde{H}^{\bar{d}}(x,t)\Big]$$

($H_v = H^q - H^{\bar{q}}$; $s - \bar{s}$, $c - \bar{c}$ and sea quark contribution to F_A neglected, probably very small)

in order to determine $H_v^{u,d}\,, \widetilde{H}_v^{u,d}\,, E_v^{u,d}$

in a strict mathematical sense an ill-posed problem BUT

Parameterisation of the GPDs

ANSATZ: $$H_v^q(x,t) \;=\; q_v(x)\,\exp\left[f_q(x)t\right]$$
$$f_q = \left[\alpha' \log(1/x) + B_q\right](1-x)^{n+1} + A_q\, x(1-x)^n$$
$\alpha' = 0.9\,\mathrm{GeV}^{-2}$ (fixed); $n = 1, 2$; $q_v(x)$ from CTEQ (INPUT)

Motivation:

for large $-t$ and x: overlaps of Gaussian LC wavefunctions

$$H_v^q(x,t) \;\longrightarrow\; \exp\left[a^2 t \frac{1-x}{2x}\right] q_v(x)$$

low $-t$, very small x: Regge behaviour expected

$$H_v^q(x,t) \;\longrightarrow\; x^{-\alpha(0)} \exp\left[\alpha' t\,\log(1/x)\right]$$

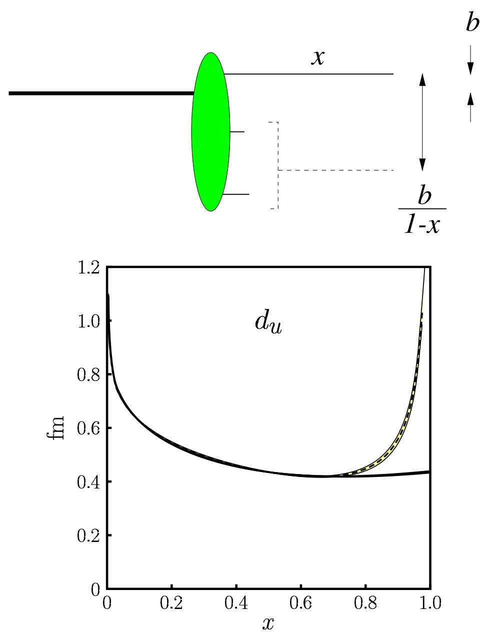
criteria for good parameterization (met by ansatz):

- simplicity
- consistency with theor. and phenom. constraints
- plausible interpretation of parameters (if possible)
- stability with respect to variation of PDFs
- stability under evolution (scale dependence of GPDs can be absorbed into parameters)

$n = 1$ or 2?

Fourier transform to impact parameter plane (Burkardt)

$$q_v(x, \mathbf{b}) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2}\, e^{-i\mathbf{b}\mathbf{\Delta}} H_v^q(x, t = -\Delta^2)$$



$\mathbf{b}$ transverse distance between struck quark and hadron's center of momentum:
$\sum_i x_i \mathbf{b_i} = 0$ (chosen, $\quad \sum_i x_i = 1$)
$\mathbf{b}/(1-x)$ relative distance between struck parton and cluster of spectators

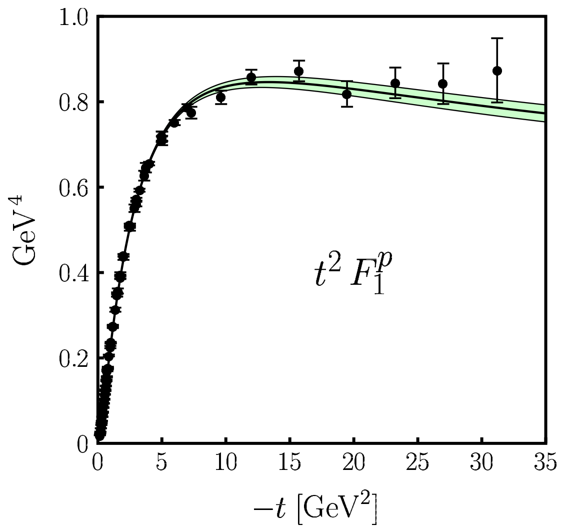
average distance: $d_q = \frac{\sqrt{\langle b^2 \rangle_x^q}}{1-x}$ $\qquad d_q$ provides estimate of size of hadron

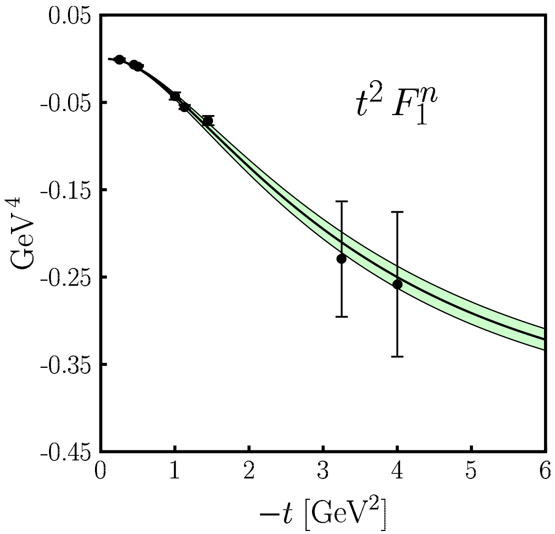
For $x \to 1 \qquad d_q \to \infty$ for $n = 1$ while d_q remains finite for $n = 2$

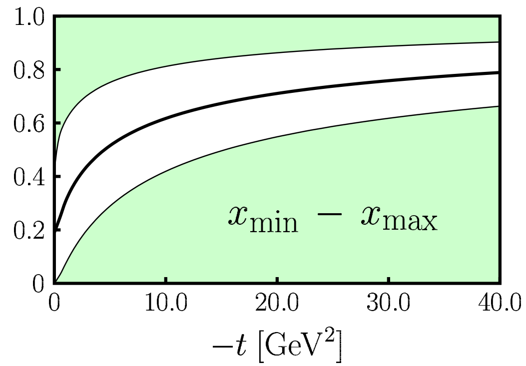
expected for system subject to confinement

Dirac form factors

$$F_1^{p(n)} = \int_0^1 dx\,[e_{u(d)} H_v^u + e_{d(u)} H_v^d]$$





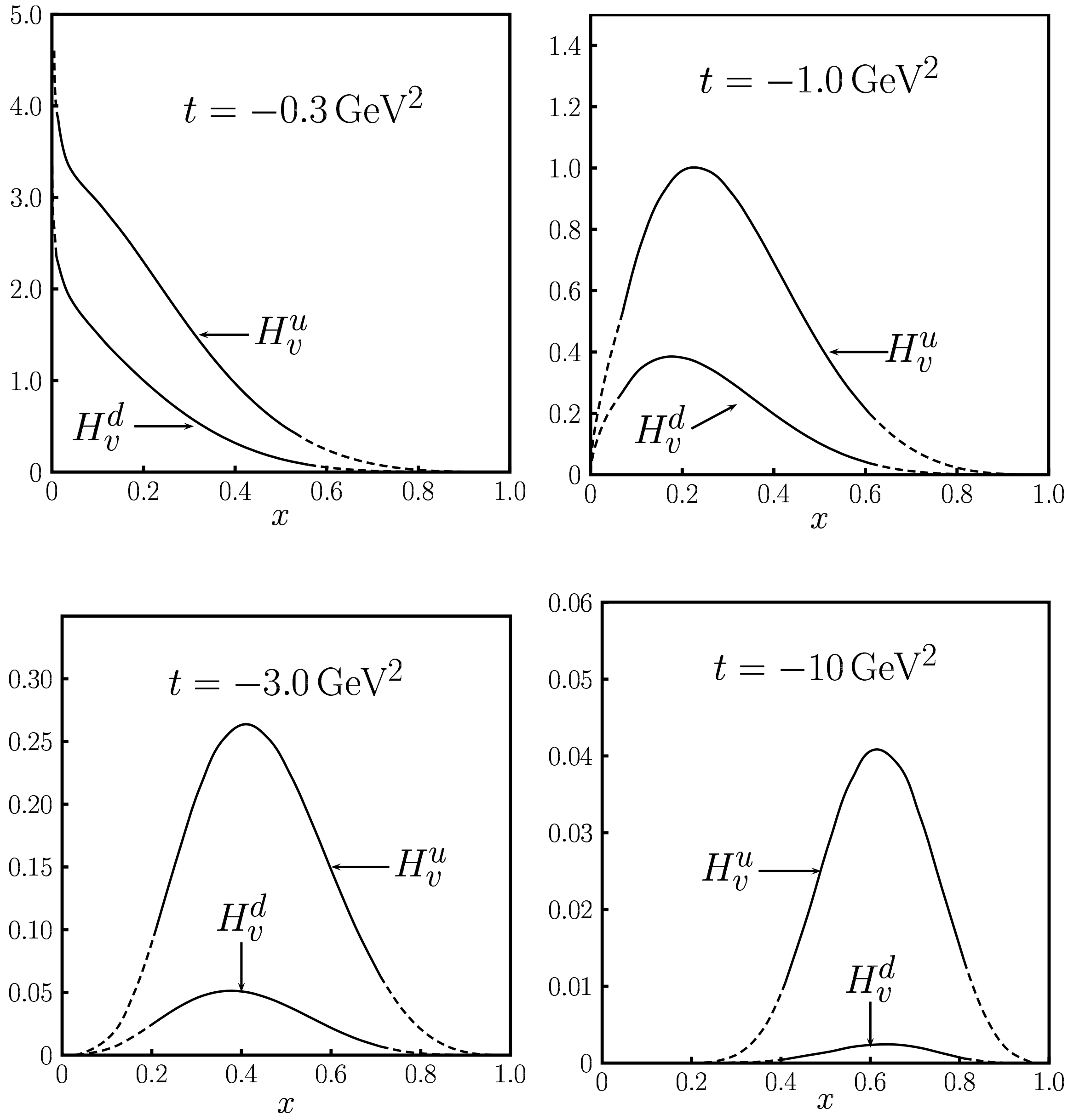


$$\int_{x_{max}(0)}^{1(x_{min})} dx \sum e_q H_v^q(x,t) = 5\% F_1^p(t)$$

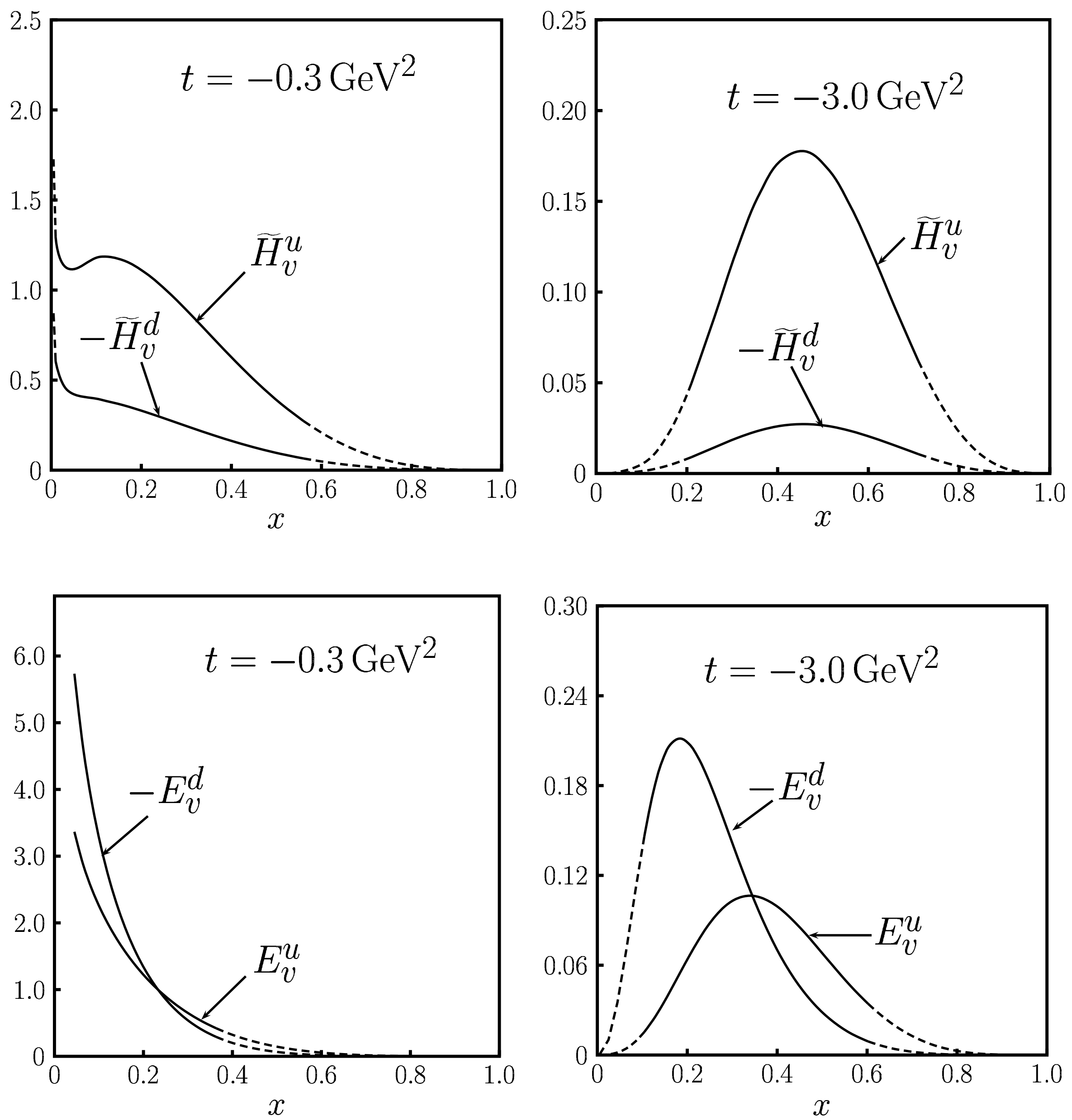
parameters for $n = 2$ fit ($\mu = 2\,\mathrm{GeV}$):

$$B_u = B_d = (0.59 \pm 0.03)\,\mathrm{GeV}^{-2}\,,\; A_u = (1.22 \pm 0.02)\,\mathrm{GeV}^{-2}\,,\; A_d = (2.59 \pm 0.29)\,\mathrm{GeV}^{-2}$$

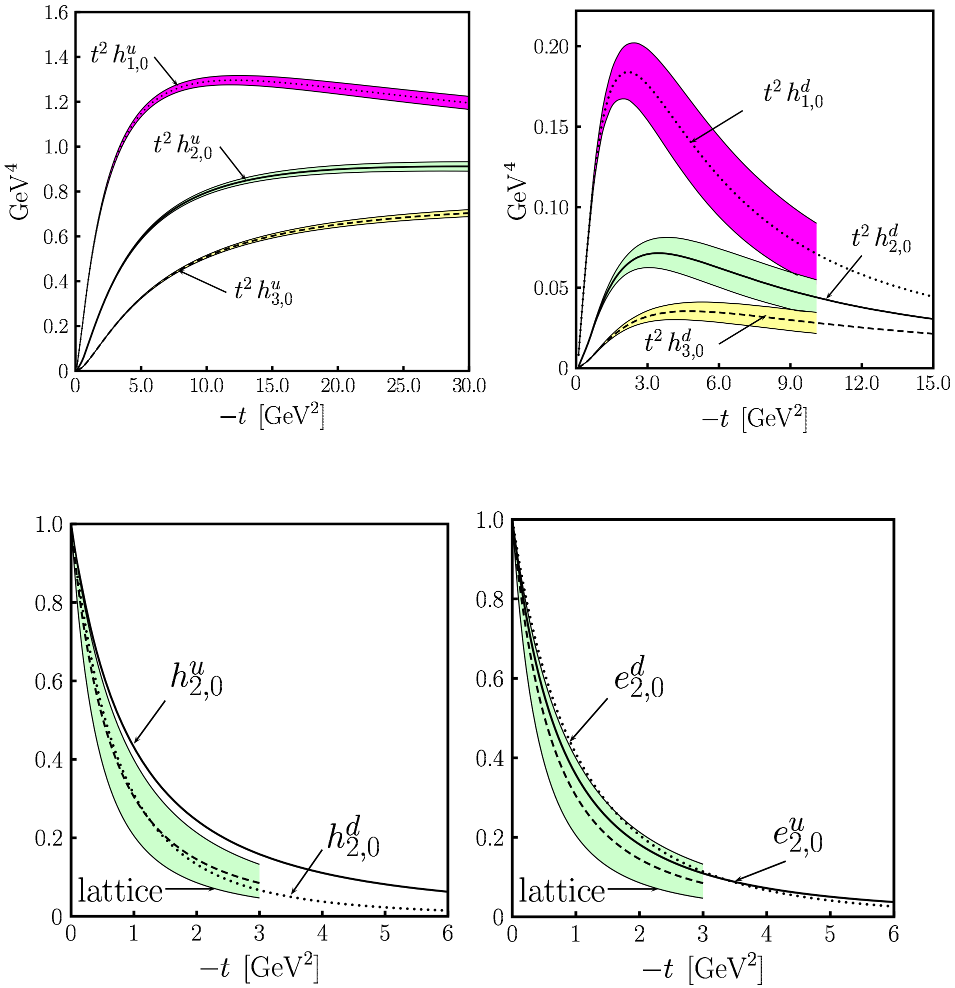
The GPD H $(\mu = 2\,\mathrm{GeV})$



The GPDs $\widetilde{H}$, E ($\mu = 2\,\mathrm{GeV}$)



Moments



u-moments behave similar to F_1^p

d-moments small, die out rapidly

almost negligible for $-t > 1\,\mathrm{GeV}^2$

explains why F_1^n negative

Lattice: Göckeler et al,

hep-ph/0304249

heavy pion, common dipole fit

$M = (1.11 \pm 0.20)\mathrm{GeV}$

Ji's sum rule

$$\langle J_q \rangle \;=\; \int_{-1}^{1} dx\, x \left[H^q(x,\xi,t=0) + E^q(x,\xi,t=0) \right]$$

valence quark contribution to sum rule respective to orbital angular momentum for our parameterization

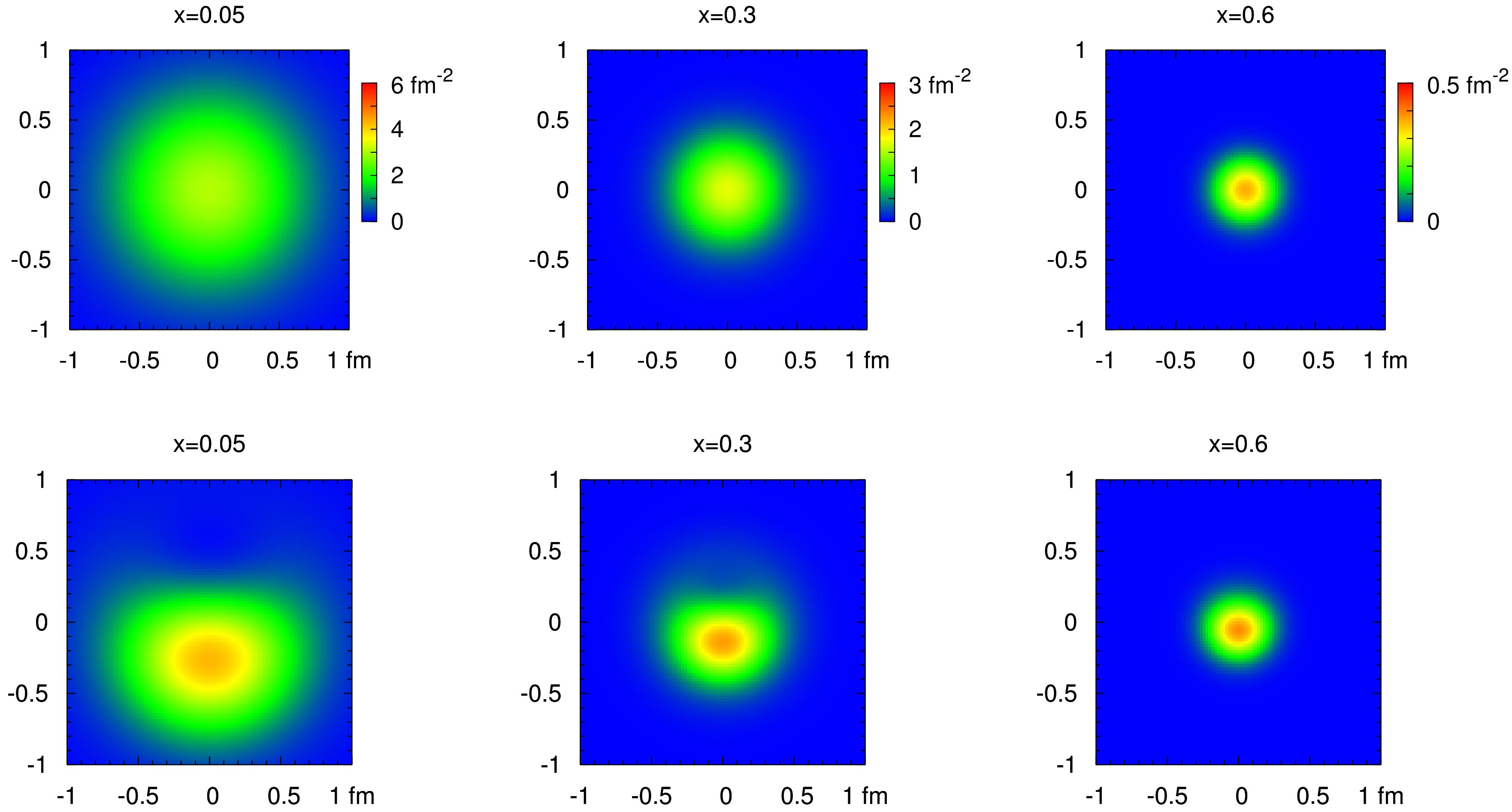
$$\langle L_v^q \rangle \;=\; \frac{1}{2} \int_0^1 dx \left[x e_v^q(x) + x q_v(x) - \Delta q_v(x) \right]$$

e_v^q from our analysis, q_v known from PDFs
Δq_v known from axial-vector couplings of nucleon and hyperons

$e_{2,0}^u(t=0)$ and $e_{2,0}^d(t=0)$ almost equal in magnitude but opposite in sign
contributions from E cancel to a large extent in sum

$$\begin{aligned} -2\langle L_v^{u+d} \rangle &= 0.09 - 0.24 \\ -2\langle L_v^{u-d} \rangle &= 0.47 - 0.54 \qquad \text{at } \mu = 2\,\mathrm{GeV} \end{aligned}$$

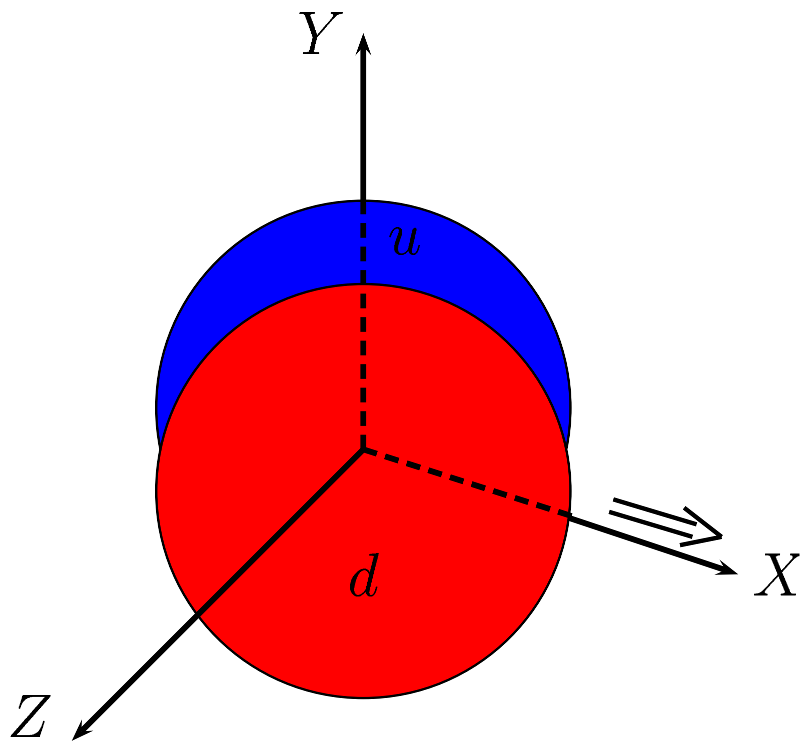
Tomography of d_v quarks



$$q_v^X(x, \mathbf{b}) = q_v(x, \mathbf{b}) - \frac{b^y}{m} \frac{\partial}{\partial \mathbf{b}^2} e_v^q(x, \mathbf{b})$$

flavor segregation

in transversely polarized proton



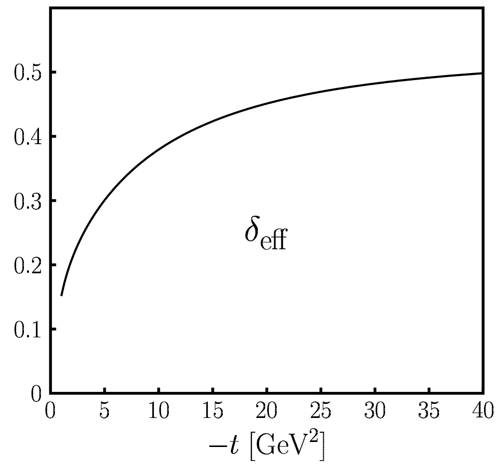
responsible for asymmetries in e.g. $p \uparrow p \longrightarrow \pi^{\pm}$?

Feynman mechanism

k, k' momenta of active parton (before and after it is struck)
l momentum of spectator system; Λ typical hadronic scale
soft region: $1 - x \sim \Lambda/\sqrt{|t|}$, $|k^2|, |k'^2| \sim \Lambda\sqrt{|t|}$ Feynman mechanism applies
ultrasoft: $1 - x \sim \Lambda/|t|$, $|k^2|, |k'^2| \sim \Lambda^2$

at large t: dominance of narrow region of large x, approx.: $q_v \sim (1-x)^{\beta_q}$, $f_q \sim A_q(1-x)^n$
Saddle point method provides $1 - x_s = \left(\frac{n}{\beta_q} A_q |t|\right)^{-1/n}$, $F_1^q \sim |t|^{-(1+\beta_q)/n}$
(similar to Drell-Yan)
$n = 2$: x_s in soft region and in sensitive x-region $\Rightarrow$ early onset of power beh. ($t \simeq 5\,\mathrm{GeV}^2$)

$n = 1$: x_s in ultrasoft region (suspect - confinement!)
requires large t in order to have x_s in sensitive region
power behaviour of F_1 does not set in before
$-t \simeq 30\,\mathrm{GeV}^2$

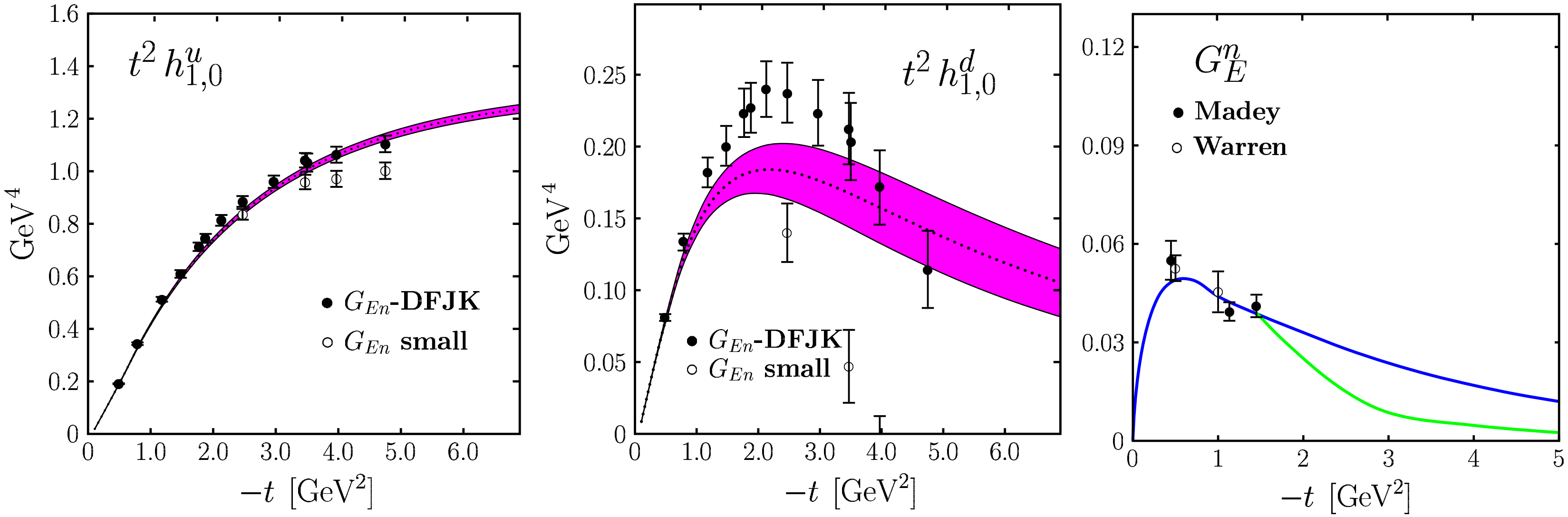


Testing the power laws:
$\delta_{\rm eff} = t\frac{d}{dt}\log[1 - \langle x\rangle_t]$

soft region: $\delta_{\rm eff} = 1/2$ ultrasoft: $\delta_{\rm eff} = 1$ $n = 1, 2$ practically same $\delta_{\rm eff}$ in fit

fit implies dominance of Feynman mechanism

u and d quark contributions to Dirac form factor



preliminary CLAS data on G^n_M used

$F^q_1 \sim |t|^{-(1+\beta_q)/n}$ CTEQ PDFs: $\beta_u \simeq 3.4$, $\beta_d \simeq 5$ (similar to DY)

dominance of u over d quarks in FF at large t corresponds to that in PDFs at large x

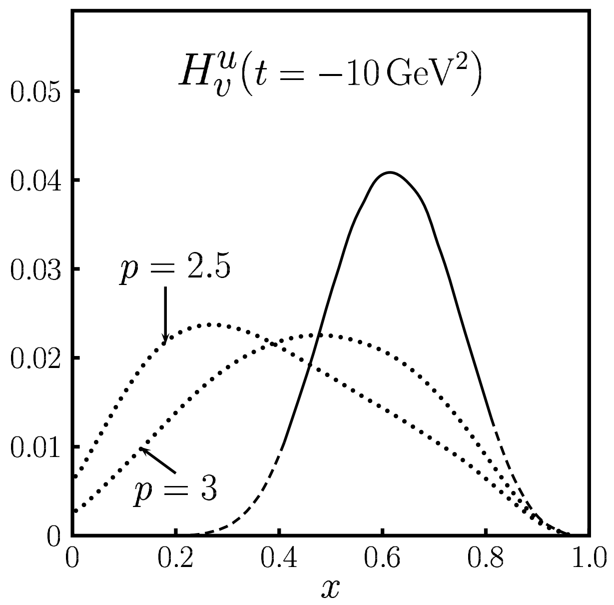
except G^n_E is much larger than expected (wait for JLab)

Power law behaviour

to cover a large range of possibilities: $H_v^q = q_v(x)\left[1 - \frac{t f_q(x)}{p}\right]^{-p}$

finite p: • not stable under evolution $\qquad p \to \infty$: exponential behaviour as before

- connection to Regge limit lost
- reasonable fits to data obtained for $p \gtrsim 2.5$



broader shape of H

$H(x = 0, t)$ remains finite

i.e. small x contribute to F at large t

significant ambiguity: correlated $x - t$ dependence of GPD not fixed by present data

theoretical input (bias) needed: combination of Regge behaviour at small x and t with dynamics of soft Feynman mechanism at large t is a physical attractive feature

The Compton amplitudes

$s,\ -t,\ -u \ll \Lambda^2$

$$\mathcal{M}_{\mu'+,\mu+} = 2\pi\alpha_{elm}\Big\{\mathcal{H}_{\mu'+,\mu+}\big[R_V + R_A\big] + \mathcal{H}_{\mu'-,\mu-}\big[R_V - R_A\big]\Big\}$$

$$\mathcal{M}_{\mu'-,\mu+} = -\pi\alpha_{elm}\frac{\sqrt{-t}}{m}\Big\{\mathcal{H}_{\mu'+,\mu+} + \mathcal{H}_{\mu'-,\mu-}\Big\}R_T$$

$$R_V(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x} H_v^q(x,t)$$

$$R_A(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x} \widetilde{H}_v^q(x,t)$$

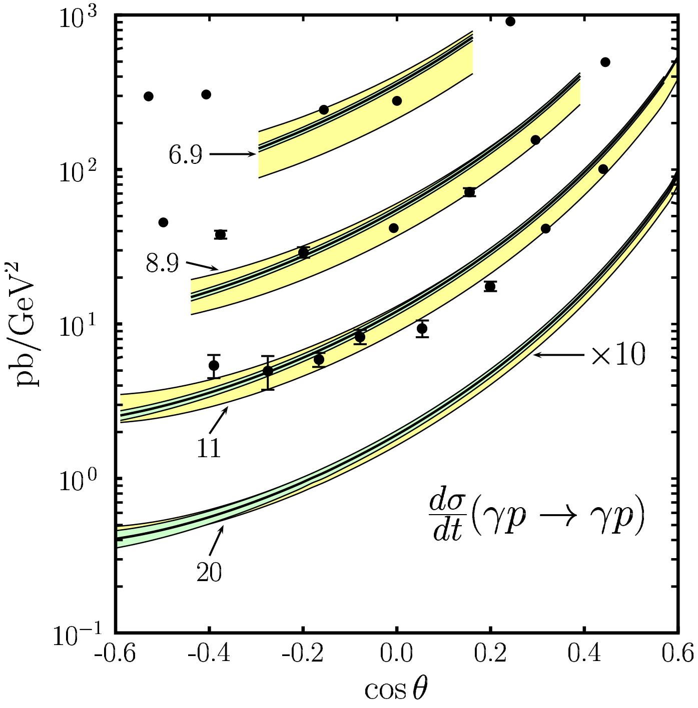
$$R_T(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x} E_v^q(x,t)$$

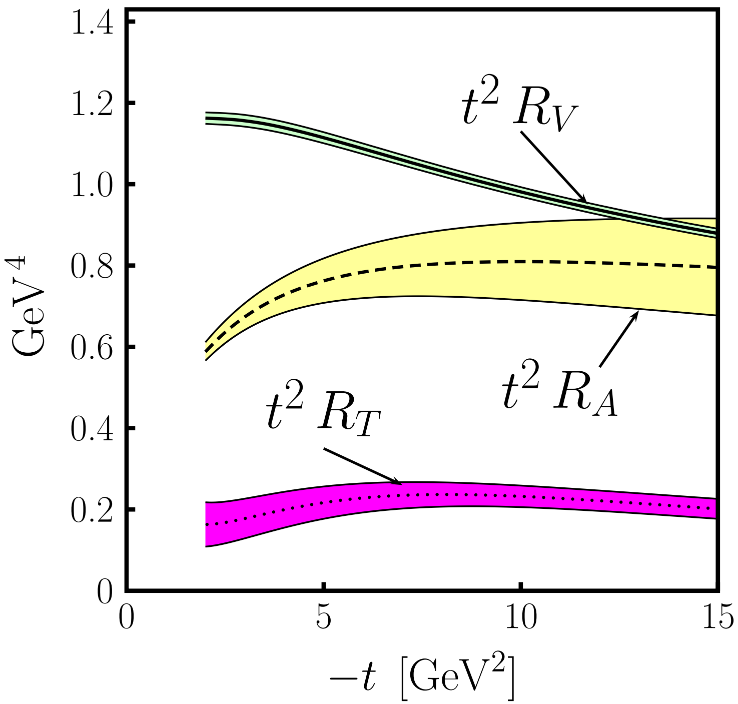
$\widetilde{E}$ decouples in symmetric frame ($\xi = 0$); $\quad H_v^q = H^q - H^{\bar{q}}$

$\mathcal{H}(s,t)$: NLO $\gamma q \to \gamma q$ amplitudes ($+\ \gamma g \to \gamma g$, R_i^g)

Radyushkin hep-ph/9803316; DFJK hep-ph/9811253; Huang-K-Morii hep-ph/0110208

The Compton cross section

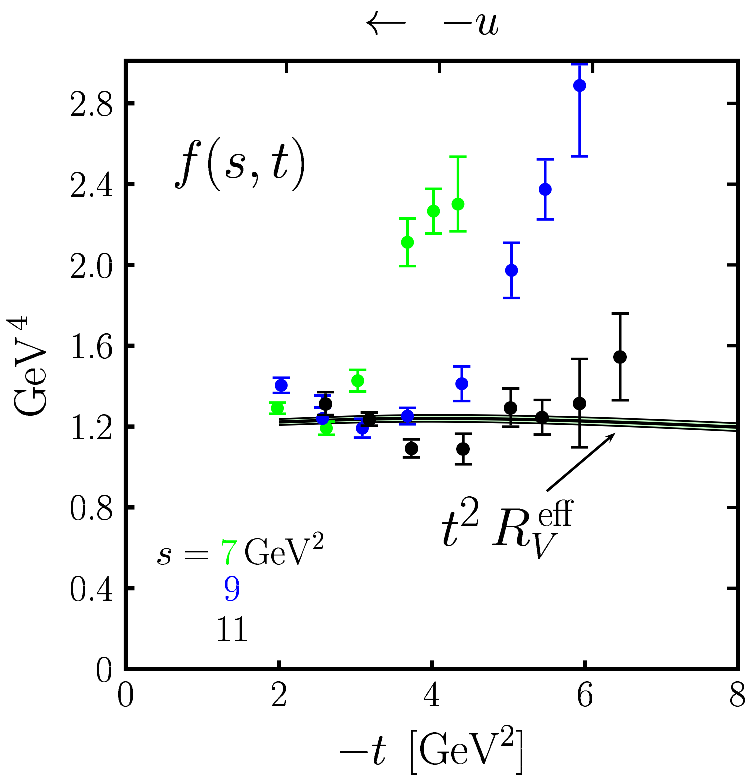




$$\frac{d\sigma}{dt} = \frac{d\hat{\sigma}}{dt}\Big\{\frac{1}{2}\,[R_V^2 + \frac{-t}{4m^2}\,R_T^2 + R_A^2] - \frac{us}{s^2+u^2}[R_V^2 + \frac{-t}{4m^2}\,R_T^2 - R_A^2]\Big\} + \mathcal{O}(\alpha_s)$$

$\frac{d\hat{\sigma}}{dt}$ Klein-Nishina cross section data: JLab E99-114

The onset of factorization



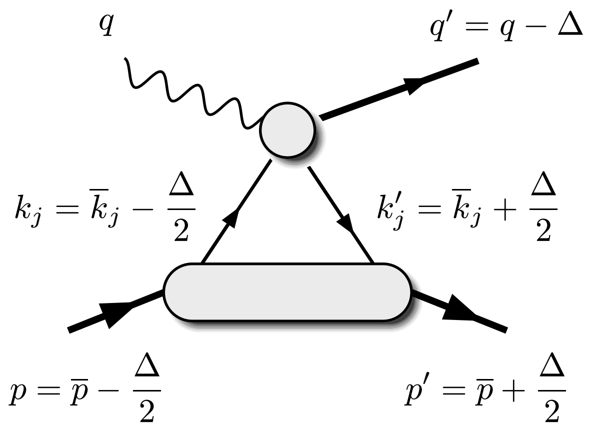
$$f(s,t) = t^2 \sqrt{\frac{d\sigma_{exp}}{dt}} \left[\frac{d\hat{\sigma}}{dt}\, \frac{1}{2}\, \frac{(s-u)^2}{s^2+u^2} \right]^{-1/2}$$

$$\Longrightarrow t^2 R_V(t) \sqrt{1+\kappa_T^2} + \mathcal{O}(R_A, \alpha_s)$$

data: JLab E99-114

Photo- and electroproduction of pions

handbag contribution $(s,\,-t,\,-u \ll \Lambda^2)$



$$\mathcal{M}^{\pi}_{0+\mu+} = \frac{e}{2}\Big\{\mathcal{H}^{\pi}_{0+\mu+}\left[R^{\pi}_V + R^{\pi}_A\right] + \mathcal{H}^{\pi}_{0-\mu-}\left[R^{\pi}_V - R^{\pi}_A\right]\Big\}$$

$$\mathcal{M}^{\pi}_{0+\mu-} = \frac{e}{2}\Big\{\mathcal{H}^{\pi}_{0+\mu+} + \mathcal{H}^{\pi}_{0-\mu-}\Big\}R^{\pi}_T$$

For each flavor: $R^{\pi q}_i \simeq R^q_i$ known, universality (sea quarks neglected)

$$R^{\pi^0}_i = 1/\sqrt{2}\left[e_u R^u_i - e_d R^d_i\right] \qquad R^{\pi^+}_i = R^{\pi^-}_i = R^u_i - R^d_i$$

Huang-K hep-ph/0005318; Huang et al hep-ph/0309071

How to improve the analysis?

More data on form factors

JLab: G_M^n up to $\simeq 5.5\,\mathrm{GeV}^2$ preliminary data from CLAS

G_E^p up to $\simeq 9\quad\mathrm{GeV}^2$ (run in 2007)

G_E^n up to $\simeq 4.2\,\mathrm{GeV}^2$ (run in 2006)

JLab upgrade $\lesssim 13\mathrm{GeV}^2$

axial vector form factor?

More moments (to become independent of parameterization)

from lattice claculations (provided reliable chiral extrapolation is made)

$1/x$ moments from Compton scattering

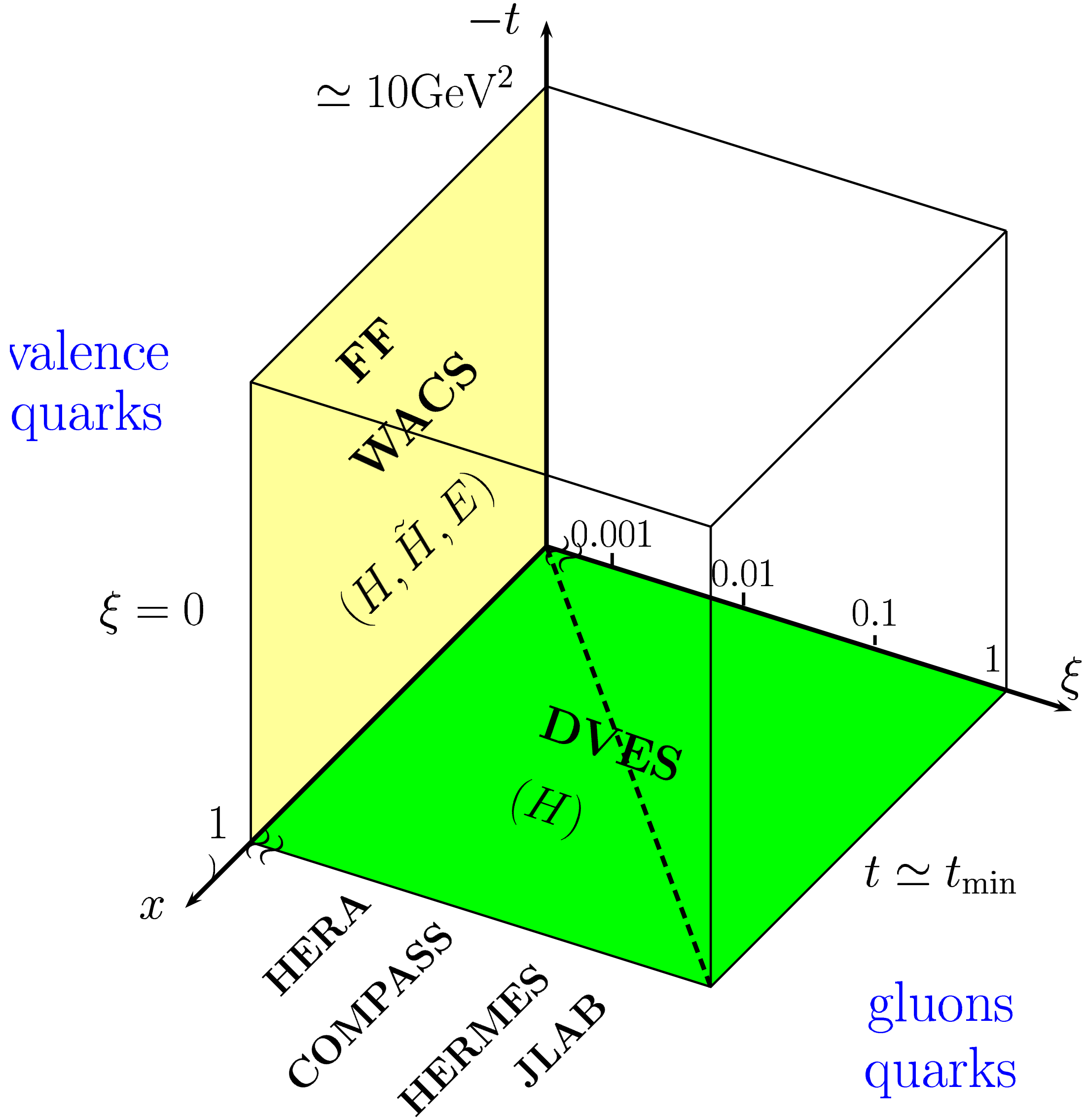
(provided power corrections have been died out, wait for Jlab upgrade)

Summary

- A first attempt to extract the GPDs from form factor data in analogy to the analyses of the PDFs
- On the basis of physically motivated parameterizations information on $H, \tilde{H}, E$ for valence quarks and at $\xi = 0$ has been obtained
- Parameterization is not unique but results are theoretically consistent and imply the physics of the Feynman mechanism at large t
- Some surprising results have been found:

  e.g. elm. FF are dominated by u-quarks at larger t

  e.g. orbital angular momentum carried by valence quarks small but non-zero
- Polarized and unpolarized WACS can be predicted now and found to be in fair agreement with experiment



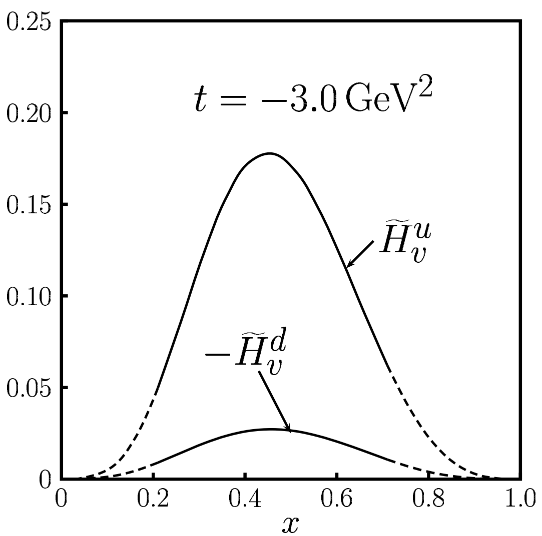
$-t$
$\simeq 10\text{GeV}^2$
valence
quarks
FF
WACS
$(H, \tilde{H}, E)$
$\xi = 0$
0.001
0.01
0.1
1
ξ
DVES
(H)
1
x
$t \simeq t_{\min}$
HERA
COMPASS
HERMES
JLAB
gluons
quarks

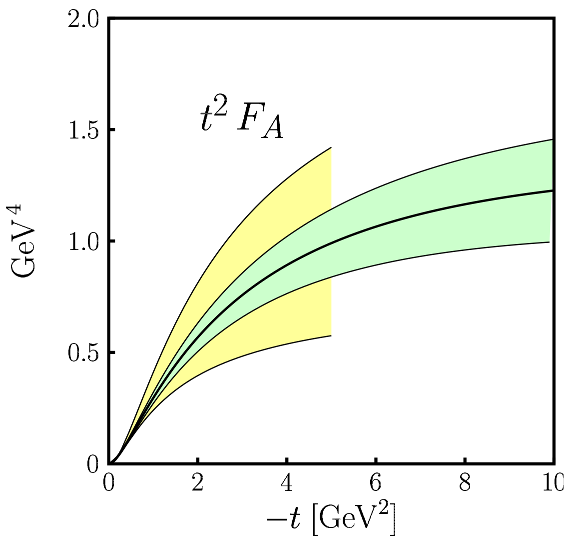
The GPD $\widetilde{H}$

$$F_A = \int_{-1}^{1} dx \, [\widetilde{H}^u(x,t) - \widetilde{H}^d(x,t)] = \int_0^1 dx \, [\widetilde{H}_v^u - \widetilde{H}_v^d] + 2 \int_0^1 dx [\widetilde{H}^{\bar{u}} - \widetilde{H}^{\bar{d}}]$$

ANSATZ: $\widetilde{H}^q = \Delta q_v \exp[t \tilde{f}_q]; \quad \Delta\bar{u} - \Delta\bar{d} \simeq 0 \quad \rightarrow \quad \widetilde{H}_v^q \simeq \widetilde{H}^q; \qquad \tilde{f}_q = f_q$

Δq_v from Blümlein-Böttcher (INPUT)





Data: Kitagaki et al (83)

$\nu_\mu n \rightarrow \mu^- p$

$(0.1 \leq -t \leq 3\,\mathrm{GeV}^2)$

presented as (no justification):

$F_A = \frac{F_A(0)}{(1-t/M_A^2)^2}$

$F_A(0) = 1.23 \pm 0.01$

$M_A = 1.05^{+0.12}_{-0.16}\,\mathrm{GeV}$

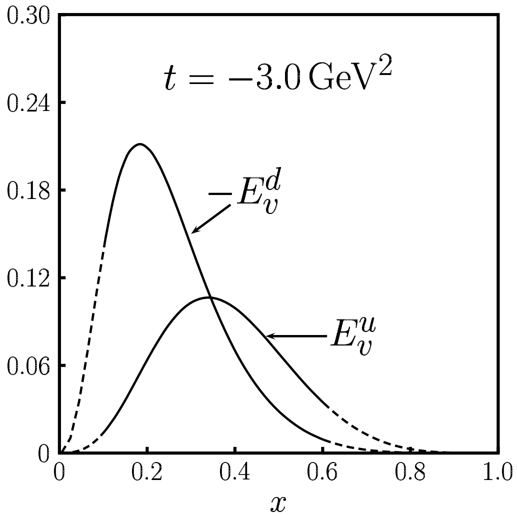
more low t data needed

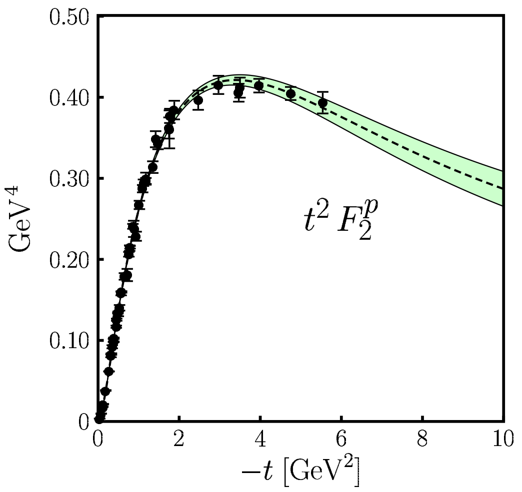
The GPD E

$$F_2^{p(n)} = \int_0^1 dx\,[e_{u(d)}E_v^u + e_{d(u)}E_v^d]$$

ANSATZ: $E_v^q = e_v^q \exp[tg_q]; \quad e_v^q(x) = N_q \kappa_q x^{-\alpha}(1-x)^{\beta_q}$

$$g_q = \alpha'(1-x)^3 \log\frac{1}{x} + D_q(1-x)^3 + C_q x(1-x)^2$$





positivity constraints:

$0 < g_q(x) < f_q(x)$

for our ansatz:

$[e_v^q(x)]^2 \leq$

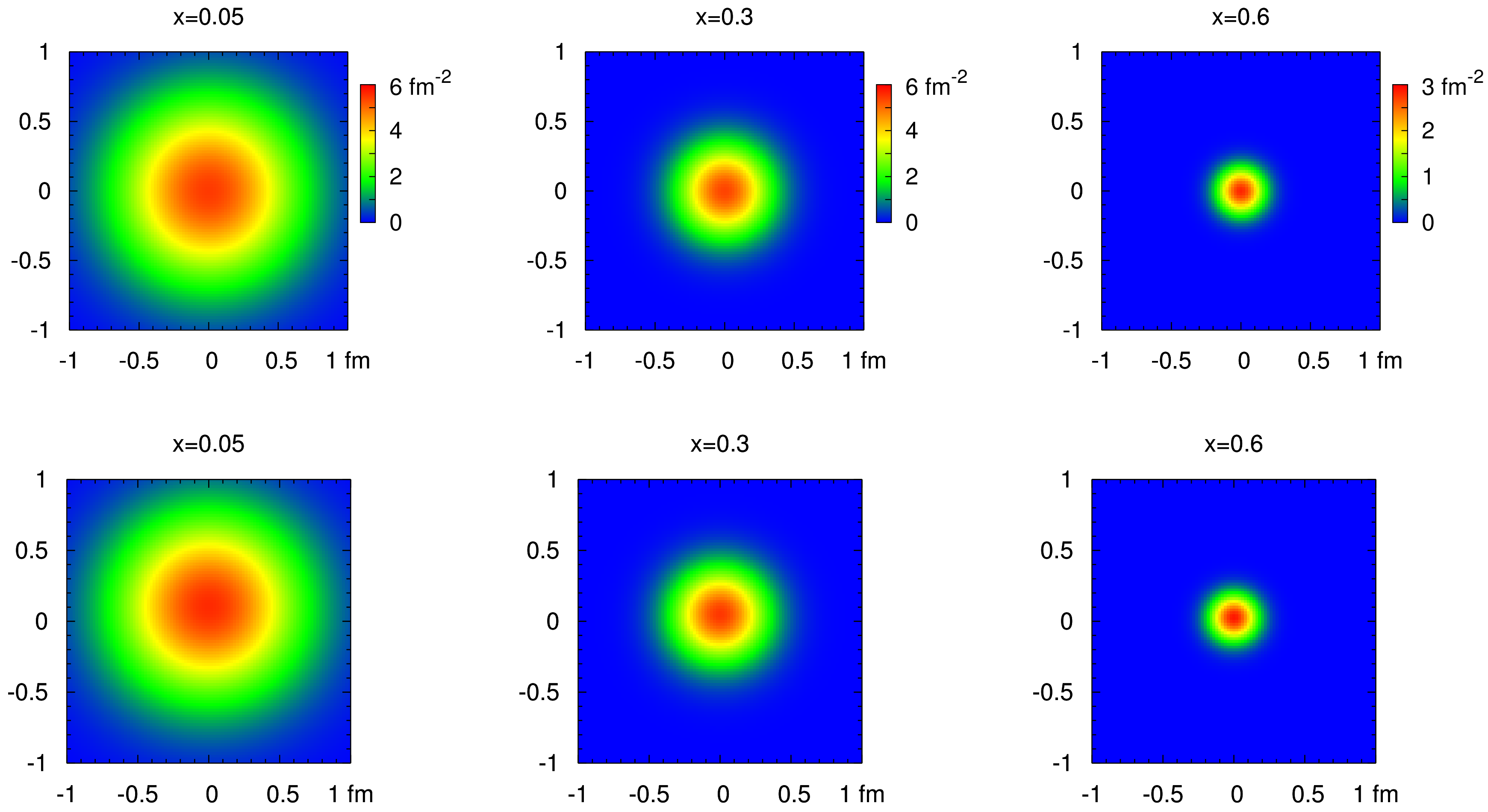
$4m^2 f_q(x)[[q_v(x)]^2 - [\Delta q_v(x)]^2]$

parameters (at $\mu = 2\,\mathrm{GeV}$): $\alpha = 0.55; \quad \beta_u = \beta_d - 1.6 = 3.99 \pm 0.22$

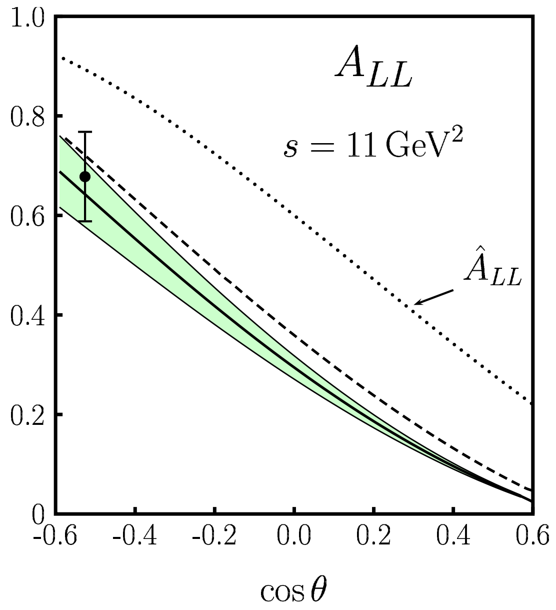
$C_u = 1.22\,\mathrm{GeV}^{-2}, \quad C_d = 2.59\,\mathrm{GeV}^{-2}$

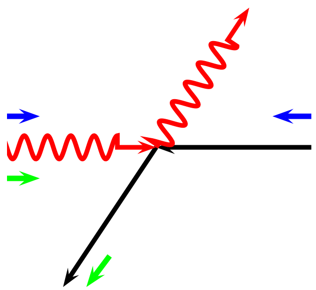
$D_u = (0.38 \pm 0.11)\,\mathrm{GeV}^{-2}, \quad D_d = -(0.75 \pm 0.05)\,\mathrm{GeV}^{-2}$

Tomography of u_v quarks



Spin correlation $A_{LL}, K_{LL}, A_{LS}, K_{LS}$





$$\hat{A}_{LL} = \frac{s^2 - u^2}{s^2 + u^2}$$

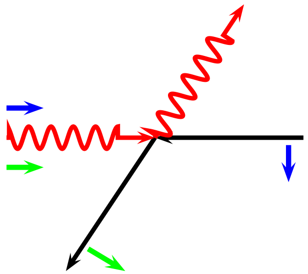
$$A_{LL} = K_{LL} \simeq \hat{A}_{LL} \frac{R_A}{R_V}$$

JLab E99-114 data at $E_\gamma = 3.23\mathrm{GeV}$: $\cos\theta = -0.5,\ t = -4\,\mathrm{GeV}^2,\ u = -1.14\,\mathrm{GeV}^2$

$$\hat{A}_{LS} = 0$$

$$A_{LS} = -K_{LS} \simeq \frac{R_A}{R_V} \hat{A}_{LL} \left(\frac{\sqrt{-t}}{2m} \frac{R_T}{R_V} - \beta\right)$$

$$\beta = \frac{2m}{\sqrt{s}} \frac{\sqrt{-t}}{\sqrt{s} + \sqrt{-u}}$$



$K_{LS} = 0.111 \pm 0.078 \pm 0.04$ (exp)

$= 0.10 \pm 0.02$ (theory)