Charmonium polarization in high energy collisions

V.A. Saleev and D. V. Vasin

Samara State University, Samara, Russia

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High energy factorization

In the phenomenology of the strong interactions at high energy it is needed to describe the QCD evolution of the gluon distribution functions in colliding particles starting from the scale μ_0, which controls a nonperturbative regime, to the typical scale of the hard scattering processes

$$\mu \sim M_T = \sqrt{M^2 + |\mathbf{p}_T|^2},$$

where M_T is the transverse mass of the produced in a hard process particle.

In the region of very high energies

$$x = \frac{\mu}{\sqrt{S}} \ll 1.$$

This fact leads to the big logarithmic contributions $\sim (\alpha_s \log(1/x))^n$ in the resummation procedure, which is described by the BFKL [Balitskiy, Fadin, Kuraev, Lipatov] evolution equation for an unintegrated gluon distribution function

$$\Phi(x, |\mathbf{k}_T|^2, \mu^2).$$

In the k_T-factorization approach:

$$p + p \to \mathcal{H} + X \tag{1}$$

and

$$R + R \to \mathcal{H} + X \tag{2}$$

are connected as follows:

$$\begin{aligned}
&d\sigma^{\mathrm{KT}}(p + p \to \mathcal{H} + X, S) = \\
&\int \frac{dx_1}{x_1} \int d|\mathbf{k}_{1T}|^2 \int \frac{d\varphi_1}{2\pi} \Phi(x_1, |\mathbf{k}_{1T}|^2, \mu^2) \times \\
&\times \int \frac{dx_2}{x_2} \int d|\mathbf{k}_{2T}|^2 \int \frac{d\varphi_2}{2\pi} \Phi(x_2, |\mathbf{k}_{2T}|^2, \mu^2) \times \\
&\times d\hat{\sigma}(R + R \to \mathcal{H} + X, \mathbf{k}_{1T}, \mathbf{k}_{2T}, \hat{s}),
\end{aligned} \tag{3}$$

where $\hat{s} = x_1 x_2 S - (\mathbf{k}_{1T} + \mathbf{k}_{2T})^2$.

$\Phi(x, |\mathbf{k}_T|^2, \mu^2)$:

[JB] by Bluemlein,

[JS] by Jung and Salam,

[KMR] by Kimber, Martin and Ryskin.

$$\sigma^{PM}(p + p \to \mathcal{H} + X, S) = \int dx_1 G(x_1, \mu^2) \int dx_2 G(x_2, \mu^2) \times \hat{\sigma}(g + g \to \mathcal{H} + X, \hat{s}), \quad (4)$$

where $\hat{s} = x_1 x_2 S$.

$$xG(x, \mu^2) \simeq \int d|\mathbf{k}_T|^2 \Phi(x, |\mathbf{k}_T|^2, \mu^2). \quad (5)$$

This implies that the cross section (3) is normalized approximately for the parton model cross section, so that when $\mathbf{k}_{1T} = \mathbf{k}_{2T} = 0$ we recover the usual gluon-gluon result for the on-shell gluons.

NRQCD formalism

In the framework of the (NRQCD) approach the heavy quarkonium $\mathcal{H}$ production cross section in a partonic process $\hat{\sigma}(a + b \to \mathcal{H} + X)$ may be presented as a sum of terms in which the effects of the long and short distances are factorized:

$$d\hat{\sigma}(\mathcal{H}) = \sum_n d\hat{\sigma}(Q\bar{Q}[n])\langle\mathcal{O}^{\mathcal{H}}[n]\rangle. \qquad (6)$$

Here n denotes the set of the color, spin and orbital quantum numbers of the $Q\bar{Q}$-pair, $\hat{\sigma}(Q\bar{Q}[n])$ is the cross section of the $Q\bar{Q}$-pair production with quantum numbers n and with the equal 4-momenta. The last one can be calculated using the perturbative approach of the QCD by the small constant of the strong interaction α_s and using the nonrelativistic approximation for the relative motion of the heavy quarks into a $Q\bar{Q}$-pair. The nonperturbative transition of a $Q\bar{Q}$-pair into a final quarkonium $\mathcal{H}$ is described by the long distance matrix element $\langle\mathcal{O}^{\mathcal{H}}[n]\rangle$.

$[n] = [^3S_1^{(1)}, {}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(8)}]$ for $\mathcal{H} = J/\psi, \psi'$.

$[n] = [^3P_J^{(1)}, {}^3S_1^{(8)}]$ for $\mathcal{H} = \chi_{cJ}$, where $J = 0, 1$ and 2.

$$\begin{aligned} \langle \mathcal{O}^{\psi(nS)}[^3P_J^{(8)}] \rangle &= (2J+1)\langle \mathcal{O}^{\psi(nS)}[^3P_0^{(8)}] \rangle, \\ \langle \mathcal{O}^{\chi_{cJ}}[^3P_J^{(1)}] \rangle &= (2J+1)\langle \mathcal{O}^{\chi_{c0}}[^3P_0^{(1)}] \rangle, \\ \langle \mathcal{O}^{\chi_{cJ}}[^3S_1^{(8)}] \rangle &= (2J+1)\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}] \rangle, \end{aligned} \quad (7)$$

$$\begin{aligned} |J/\psi\rangle &= \mathcal{O}(v^0)|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)|c\bar{c}[^3P_J^{(8)}]g\rangle + \\ &+ \mathcal{O}(v^2)|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \mathcal{O}(v^2)|c\bar{c}[^1S_0^{(8)}]g\rangle + ..., \end{aligned} \quad (8)$$

$$\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}] \rangle = 2N_c(2J+1)|\Psi(0)|^2, \quad (9)$$

where $N_c = 3$ and $J = 1$.

$$\langle \mathcal{O}^{\chi_{cJ}}[^3P_J^{(1)}] \rangle = 2N_c(2J+1)|\Psi'(0)|^2. \quad (10)$$

Charmonium production by reggeized gluons

In this part we obtain squared amplitudes for the charmonium production via the fusion of two reggeized gluons in the framework of the NRQCD. We consider the LO in α_s and v contributions of the following partonic subprocessses:

$$R + R \to \mathcal{H}[^3S_1^{(8)}, ^1S_0^{(8)}, ^3P_J^{(1)}, ^3P_J^{(8)}], \tag{11}$$

$$R + R \to \mathcal{H}[^3S_1^{(1)}] + g, \tag{12}$$

The analysis of the NLO contribution in the processes of the reggeized gluon-gluon production of the quarkonia in the k_T-factorization approach is outside the presented paper and it needs a special investigation.

The problem of the NLO corrections in the k_T-factorization approach is a discussed question now. It was solved consistently only in part. For example, in the Ref. [Ostrovsky, Fadin,...], where the NLO corrections to the processes $R + R \to g$ were studied.

Taking into account the discussed above scheme of the calculation, we have obtained squared amplitudes for the another color-octet S-wave and the P-wave intermediate states in the fusion of two reggeized gluons.

$$\overline{|\mathcal{A}(g+g\to\mathcal{H}[^{2S+1}L_J^{(1,8)}]|^2} = \lim_{t_1,t_2\to 0}\int_0^{2\pi}\frac{d\varphi_1}{2\pi}\int_0^{2\pi}\frac{d\varphi_2}{2\pi}\times\overline{|\mathcal{A}(R+R\to\mathcal{H}[^{2S+1}L_J^{(1,8)}]|^2}.$$

Polarization formalism

Accordingly [Kniehl and Lee, Beneke and Kramer], in hadronic CM frame we have for longitudinal polarization 4-vector of spin-one boson ($^3S_1$, $^3P_1$):

$$\varepsilon^\mu(0) = Z^\mu = \frac{(PQ)P^\mu/M - MQ^\mu}{\sqrt{(PQ)^2 - M^2 S}}, \tag{13}$$

where $Q = P_1 + P_2$, $S = Q^2$.

$$\mathcal{P}^{\mu\nu} = \sum_{|\lambda|=0,1} \varepsilon^{\star\mu}(\lambda)\varepsilon^\nu(\lambda) = -g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2}, \tag{14}$$

$$\mathcal{P}_0^{\mu\nu} = \varepsilon^{\star\mu}(0)\varepsilon^\nu(0) = Z^\mu Z^\nu, \tag{15}$$

$$\mathcal{P}_1^{\mu\nu} = \sum_{|\lambda|=1}^{1} \varepsilon^{\star\mu}(\lambda)\varepsilon^\nu(\lambda) = \mathcal{P}^{\mu\nu} - \mathcal{P}_0^{\mu\nu} \tag{16}$$

In the spin-two case ($^3P_2$) we have for the polarization tensor $\mathcal{P}^{\mu\nu\rho\sigma}_{|\lambda|}$ [Cho, Wise, Trivedi]:

$$\mathcal{P}^{\mu\nu\rho\sigma}_0 = \frac{1}{6}(2\mathcal{P}^{\mu\nu} - \mathcal{P}^{\mu\nu}_1)(2\mathcal{P}^{\rho\sigma}_0 - \mathcal{P}^{\rho\sigma}_1), \tag{17}$$

$$\mathcal{P}^{\mu\nu\rho\sigma}_1 = \frac{1}{2}(\mathcal{P}^{\mu\rho}_0\mathcal{P}^{\nu\sigma}_1 + \mathcal{P}^{\mu\sigma}_0\mathcal{P}^{\nu\rho}_1 + \mathcal{P}^{\nu\rho}_0\mathcal{P}^{\mu\sigma}_1 + \mathcal{P}^{\nu\sigma}_0\mathcal{P}^{\mu\rho}_1), \tag{18}$$

$$\mathcal{P}^{\mu\nu\rho\sigma}_2 = \frac{1}{2}(\mathcal{P}^{\mu\rho}_0\mathcal{P}^{\nu\sigma}_1 + \mathcal{P}^{\mu\sigma}_0\mathcal{P}^{\nu\rho}_1 - \mathcal{P}^{\mu\nu}_0\mathcal{P}^{\rho\sigma}_1). \tag{19}$$

$$\alpha(p_T) = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}, \tag{20}$$

where $\sigma_{L,T} = \sigma_{0,1} = \dfrac{d\sigma}{dp_T}(pp \to J/\psi_{T,L} X)$

For the direct polarized J/ψ production [Cho, Leibovich]:

$$\begin{aligned} \sigma_L^{direct} \quad = \quad & \sigma_1(^3S_1^{(1)}) + \sigma_0(^3S_1^{(8)}) + \frac{1}{3}\sigma_1(^3S_0^{(8)}) + \frac{1}{3}\sigma_1(^3P_0^{(8)}) + \\ & \frac{1}{2}\sigma_1(^3P_1^{(8)}) + \frac{2}{3}\sigma_0(^3P_2^{(8)}) + \frac{1}{2}\sigma_1(^3P_2^{(8)}) \end{aligned} \tag{21}$$

For the prompt polarized J/ψ production [Kniehl, Lee]:

$$\sigma_L^{prompt} = \sigma_L^{direct} + \sigma_L^{\chi_c \to J/\psi} + \sigma_L^{\psi' \to J/\psi} + \sigma_L^{\psi' \to \chi_c \to J/\psi} \tag{22}$$

$$\sigma_L^{\chi_c\to J/\psi} = [\frac{1}{3}\sigma_0(^3P_0^{(1)}) + \frac{1}{3}\sigma_0(^3S_1^{(8)})]Br(\chi_{c0}\to J/\psi+\gamma) +$$
$$+ \quad [\frac{1}{2}\sigma_1(^3P_1^{(1)}) + \frac{1}{2}\sigma_0(^3S_1^{(8)}) + \frac{1}{4}\sigma_1(^3S_1^{(8)})]Br(\chi_{c1}\to J/\psi+\gamma) +$$
$$+ \quad [\frac{2}{3}\sigma_0(^3P_2^{(1)}) + \frac{1}{2}\sigma(^3P_2^{(1)}) + \frac{17}{30}\sigma_0(^3S_1^{(8)}) + \frac{13}{60}\sigma_1(^3S_1^{(8)})]Br(\chi_{c2}\to J/\psi+\gamma) \quad (23)$$

$$\sigma_L^{\psi'\to J/\psi} = \sigma_L^{direct,\psi'\to J/\psi} Br(\psi'\to J/\psi + X) \quad (24)$$

$$\sigma_L^{\psi'\to\chi_c\to J/\psi} = \frac{1}{3}\sigma_L^{direct,\psi'\to\chi_{c0}} Br(\psi'\to\chi_{c0})Br(\chi_{c0}\to J/\psi+\gamma) +$$
$$+(\frac{1}{2}\sigma_L^{direct,\psi'} + \frac{1}{4}\sigma_T^{direct,\psi'})Br(\psi'\to\chi_{c1})Br(\chi_{c1}\to J/\psi+\gamma) +$$
$$+(\frac{17}{30}\sigma_L^{direct,\psi'} + \frac{13}{60}\sigma_T^{direct,\psi'})Br(\psi'\to\chi_{c2})Br(\chi_{c2}\to J/\psi+\gamma) \quad (25)$$

Charmonium production at the Tevatron CDF

$\sqrt{S} = 1.8$ TeV (RUN-I) and $\sqrt{S} = 1.96$ TeV (RUN-II) $|\eta| < 0.6$.

$5 < |\mathbf{p}_T| < 20$ GeV (RUN-I) and $0 < |\mathbf{p}_T| < 20$ GeV (RUN-II).

Nowadays, the successful description of the data was archived in the framework of the NRQCD approach in the collinear parton model, including the fusion and the fragmentation mechanisms of charmonium production.

The charmonium hadroproduction was studied early within the framework of the NRQCD and the k_T-factorization approach in the fusion model in Refs. [Teryaev,..., Yuan,...., Baranov] and in the fragmentation model in Ref. [Saleev, Vasin].

Furthermore, the polarization of prompt J/ψ mesons was measured at the Fermilab Tevatron [CDF], which also provides a sensitive probe of the NRQCD mechanism. Last one was investigated in the collinear parton mode and in the k_T-factorization approach. None of these studies were able to prove or disprove the NRQCD factorization hypothesis.

The color-singlet NMEs are not fitted: $\psi(nS) \to l^+ + l^-$ and $\chi_{c2} \to \gamma + \gamma$.

The numerical values of the color-singlet NRQCD matrix elements are taken as in the Ref. [Braaten, Kniehl, Lee]:

$\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}]\rangle = 1.3$ GeV^3, $\langle \mathcal{O}^{\psi'}[^3S_1^{(1)}]\rangle = 0.65$ GeV^3,
$\langle \mathcal{O}^{\chi_{cJ}}[^3P_J^{(1)}]\rangle = (2J+1) \times 8.9 \cdot 10^{-3}$ GeV^5.

[PDG2004]: $B(J/\psi \to \mu^+ + \mu^-) = 5.88 \cdot 10^{-2}$, $B(\psi' \to J/\psi + X) = 0.576$, $B(\chi_{c0} \to J/\psi + \gamma) = 0.012$, $B(\chi_{c1} \to J/\psi + \gamma) = 0.318$ and $B(\chi_{c2} \to J/\psi + \gamma) = 0.203$.

Note. The branchings, which are used here, sufficiently differ from ones, which were used in the previous theoretical calculations before year 2004 [PDG2002].

In contrary of the previous investigations, as in the collinear parton model as in the k_T-factorization approach, our fitting procedure was performed separately for the $\langle \mathcal{O}^{\mathcal{H}}[^1S_0^{(8)}]\rangle$ and $\langle \mathcal{O}^{\mathcal{H}}[^3P_0^{(8)}]\rangle$ NMEs, where $\mathcal{H} = J/\psi, \psi'$.

$$M_r^{\mathcal{H}} = \langle \mathcal{O}^{\mathcal{H}}[^1S_0^{(8)}]\rangle + \frac{r}{m_c^2}\langle \mathcal{O}^{\mathcal{H}}[^3P_0^{(8)}]\rangle, \tag{26}$$

have been fixed, because it was impossible to separate contributions of the $\langle \mathcal{O}^{\mathcal{H}}[^1S_0^{(8)}]\rangle$ and $\langle \mathcal{O}^{\mathcal{H}}[^3P_0^{(8)}]\rangle$ NMEs using RUN-I CDF data for the $|\mathbf{p}_T| > 5$ GeV.

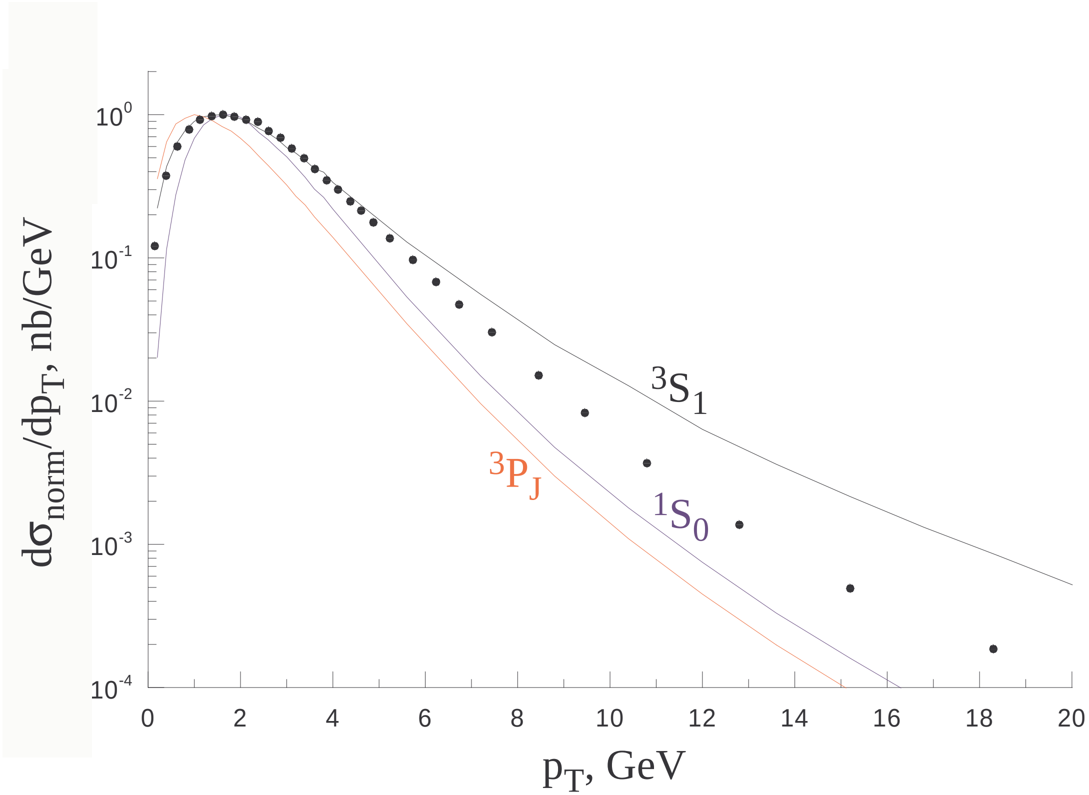

Figure 1: The contributions of different intermediate color-octet states in the direct J/ψ p_T-spectrum at $\sqrt{s} = 1.96$ TeV and $|y| < 0.6$. All curves are normalized to the unity in the points of maximums.

Table 1: NMEs for J/ψ-, ψ'- and χ_{cJ}-mesons.

n / n	PM	Fit JB	Fit JS	Fit KMR
$\langle\mathcal{O}^{J/\psi}[^1S_0^{(8)}]\rangle$, GeV^3	$4.3\cdot10^{-2}$	$4.5\cdot10^{-3}$	$3.9\cdot10^{-3}$	$4.0\cdot10^{-3}$
$\langle\mathcal{O}^{J/\psi}[^3S_1^{(1)}]\rangle$, GeV^3	1.3	1.3	1.3	1.3
$\langle\mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle$, GeV^3	$4.4\cdot10^{-3}$	$1.6\cdot10^{-3}$	$6.6\cdot10^{-3}$	$2.8\cdot10^{-3}$
$\langle\mathcal{O}^{J/\psi}[^3P_0^{(8)}]\rangle$, GeV^5	$2.8\cdot10^{-2}$	0.0	0.0	0.0
$\langle\mathcal{O}^{\psi'}[^1S_0^{(8)}]\rangle$, GeV^3	$6.9\cdot10^{-3}$	0.0	0.0	0.0
$\langle\mathcal{O}^{\psi'}[^3S_1^{(1)}]\rangle$, GeV^3	$6.5\cdot10^{-1}$	$6.5\cdot10^{-1}$	$6.5\cdot10^{-1}$	$6.5\cdot10^{-1}$
$\langle\mathcal{O}^{\psi'}[^3S_1^{(8)}]\rangle$, GeV^3	$4.2\cdot10^{-3}$	$3.0\cdot10^{-4}$	$1.4\cdot10^{-3}$	$8.4\cdot10^{-4}$
$\langle\mathcal{O}^{\psi'}[^3P_0^{(8)}]\rangle$, GeV^5	$3.9\cdot10^{-3}$	0.0	0.0	0.0
$\langle\mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}]\rangle$, GeV^3	$4.4\cdot10^{-3}$	0.0	$2.5\cdot10^{-4}$	$2.6\cdot10^{-4}$
$\langle\mathcal{O}^{\chi_{c0}}[^3P_0^{(1)}]\rangle$, GeV^5	$8.9\cdot10^{-2}$	$8.9\cdot10^{-2}$	$8.9\cdot10^{-2}$	$8.9\cdot10^{-2}$
χ^2	—	2.1	4.6	4.6

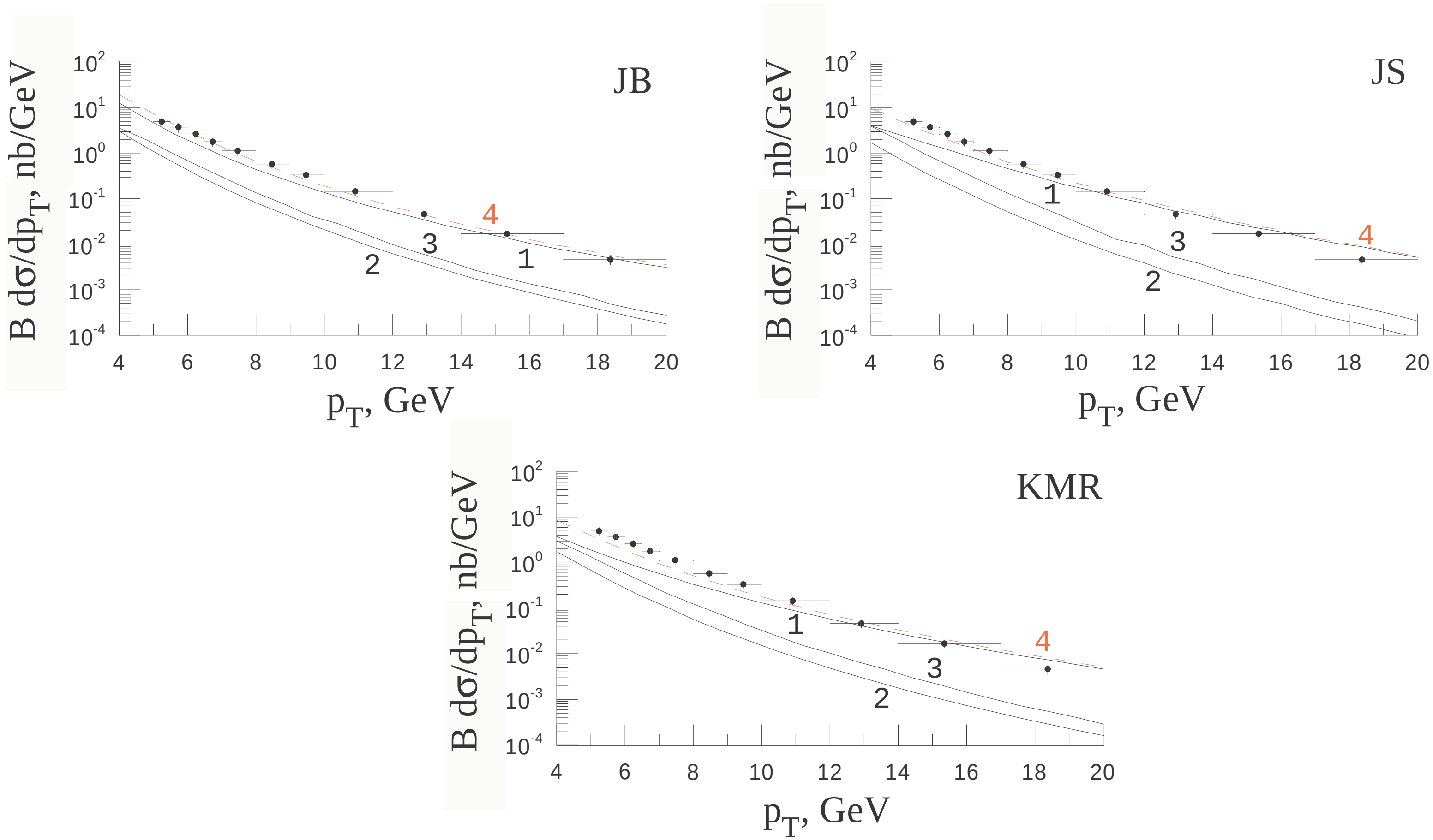

Figure 2: The p_T-spectrum of the direct J/ψ-meson at $\sqrt{S} = 1.8$ TeV and $|\eta| < 0.6$. The curve 1 is the contribution from $R + R \to J/\psi[^3S_1^{(8)}]$, curve 2 is the sum of $R + R \to J/\psi[^1S_0^{(8)}]$ and $R + R \to J/\psi[^3P_J^{(8)}]$, curve 3 is from $R + R \to g + J/\psi[^3S_1^{(1)}]$, curve 4 is the sum of curves 1, 2 and 3. The B is the J/ψ lepton branching.

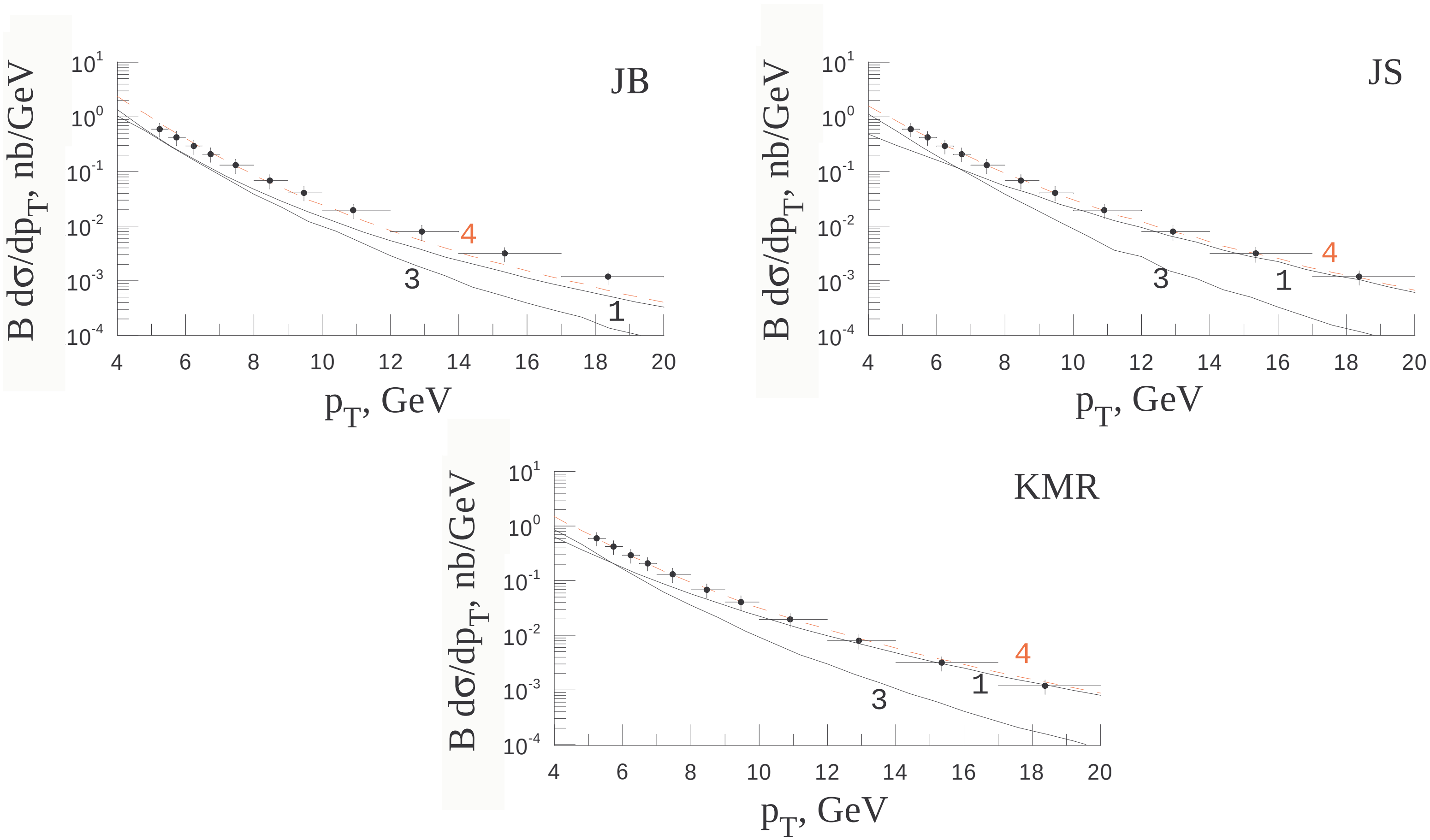

Figure 3: The p_T-spectrum of the J/ψ-meson from ψ' decays at $\sqrt{S} = 1.8$ TeV and $|\eta| < 0.6$. The curve 1 is the contribution from $R+R \to \psi'[{}^3S_1^{(8)}]$, curve 3 is from $R+R \to g+\psi'[{}^3S_1^{(1)}]$, curve 4 is the sum of curves 1 and 3. The B is the J/ψ lepton branching.

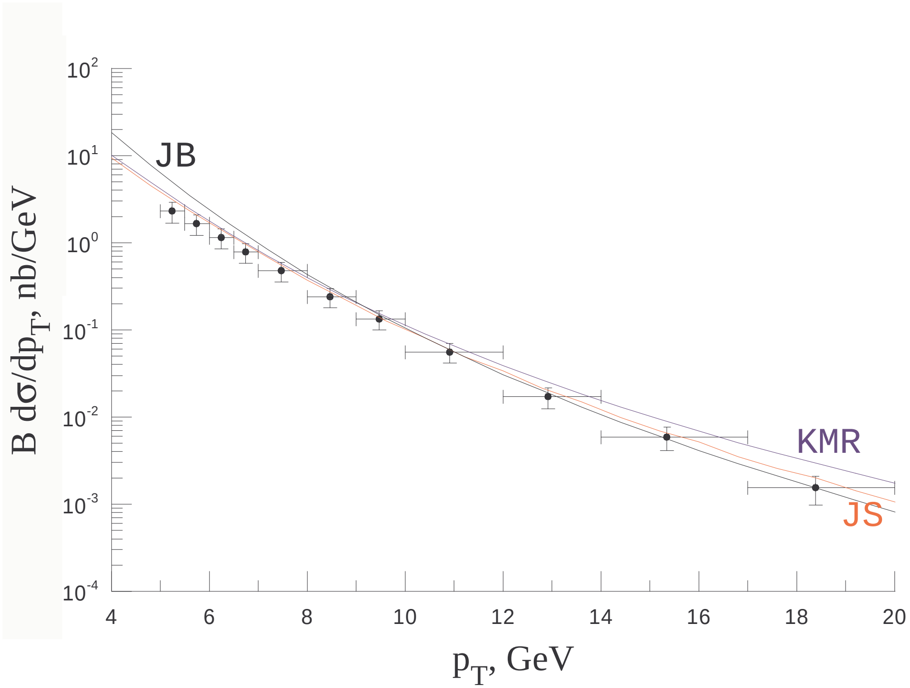

Figure 4: The p_T-spectrum of the J/ψ-meson from χ_{cJ}decays at $\sqrt{S} = 1.8$ TeV and $|\eta| < 0.6$. The curves are the sums of contributions from $R + R \to \chi_{cJ}[^3P_J^{(1)}]$ and $R + R \to \chi_{cJ}[^3S_1^{(8)}]$ (the second term is very small). The B is the J/ψ lepton branching.

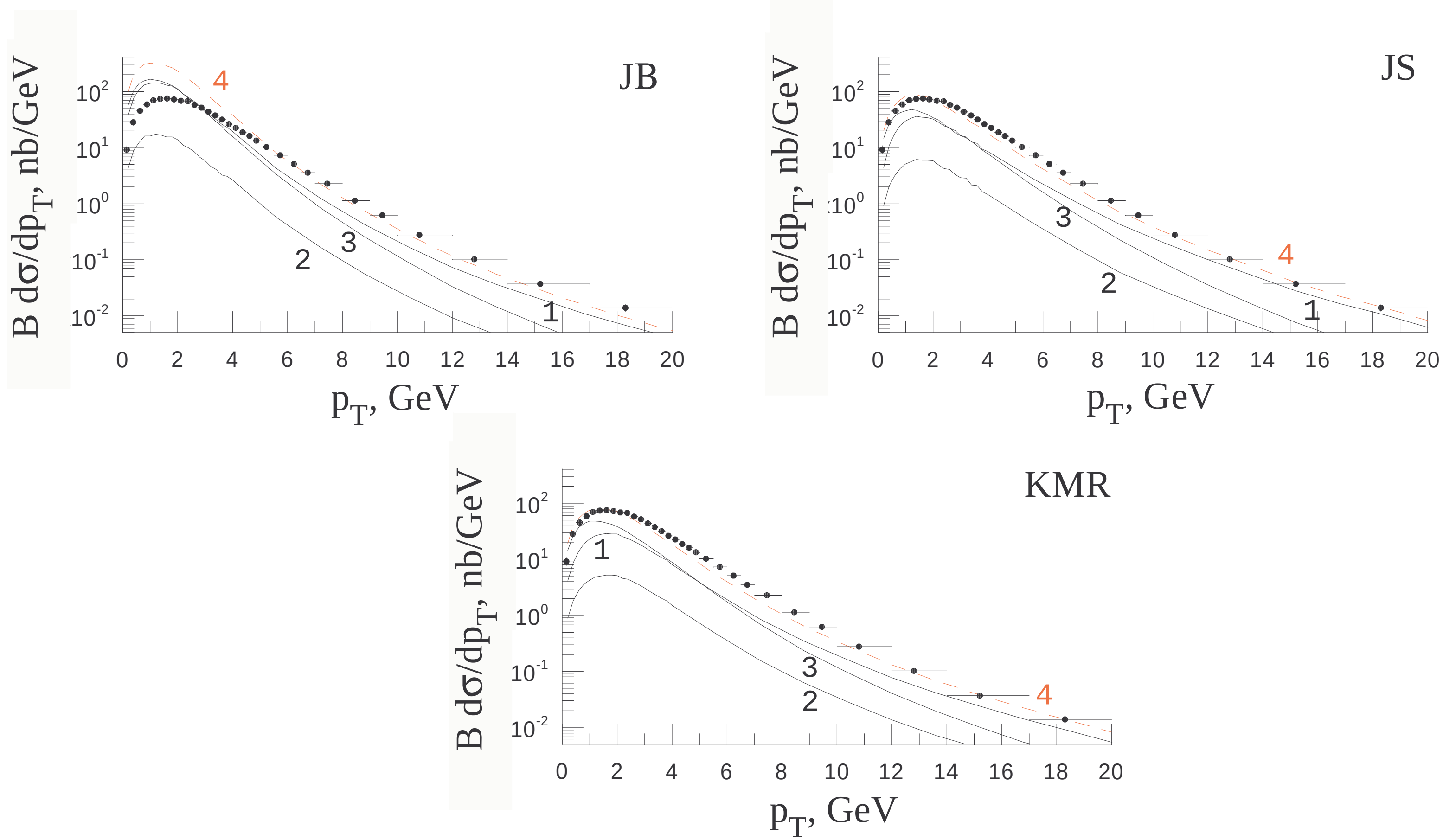

Figure 5: The prompt J/ψ-meson p_T-spectrum at $\sqrt{S} = 1.96$ TeV and $|y| < 0.6$. The curve 1 is the contribution from direct J/ψ-mesons, the curve 2 is J/ψ-mesons from ψ' decays, the curve 3 is the J/ψ-meson from χ_{cJ} decays, the curve 4 is the sum of curves 1, 2 and 3. The B is the J/ψ lepton branching.

The RUN-II Tevatron data at the small $\mathbf{p}_T|^2$ discriminate the JB parameterization at the small values of argument $|\mathbf{k}_T|^2$ and we do fitting procedure in case of the JB parameterization using RUN-I data at the $|\mathbf{p}_T| > 5$ GeV only.

After comparing the J/ψ, ψ' and χ_{cJ} production mechanisms and the relevant color-octet NMEs, we can formulate following rule for nonzero transitions from the color-octet to the color-singlet states: $\triangle S \simeq 0$ and $\triangle L \simeq 0$, i.e. these transitions are double chromoelectric and their preserve as the spin as the orbital momentum of the heavy quarks.

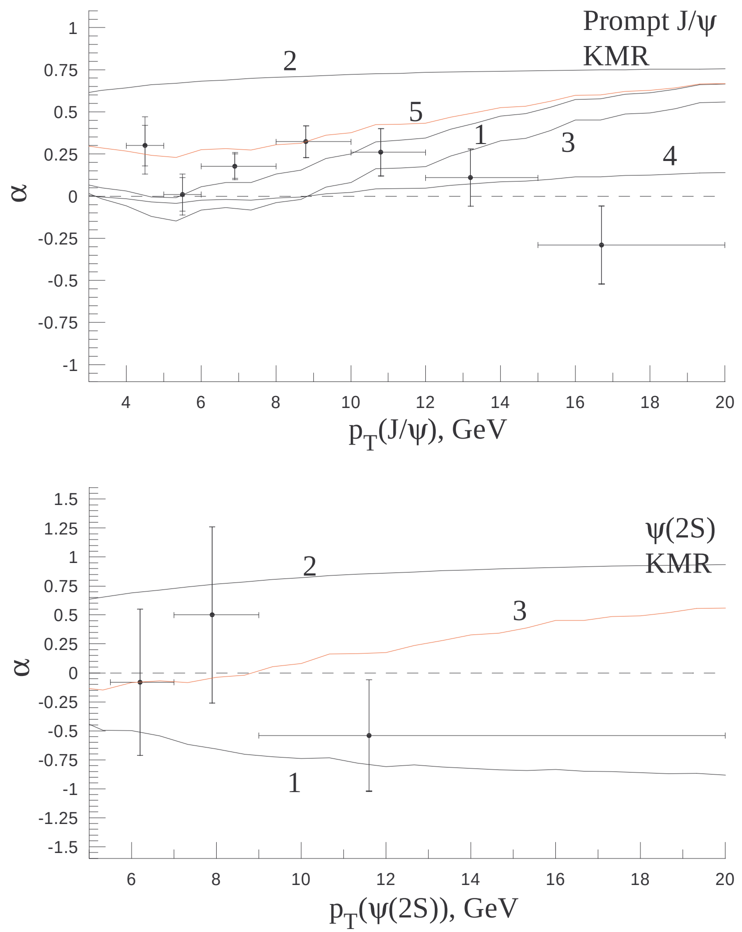

Figure 6: Polarization parameter $\alpha(p_T)$. A: Curve 1 - direct, 2 - $\chi_c \to J/\Psi$, 3 - $\psi' \to J/\psi$, 4 - $\psi' \to \chi_c \to J/\psi$, 5 - total. B: 1 - singlet, 2 - octet, 3 - direct.

Conclusions

- We have obtained analytical formulas for the processes $RR \to \mathcal{H}[1,8]$ and $RR \to \mathcal{H}[1] + g$, where $\mathcal{H}$ may be in the polarized state.
- We have obtained new set of the color-octet NMEs ($\triangle S \approx \triangle L \approx 0$).
- $\langle \mathcal{O}^H[^3S_1,^{(8)}] \rangle$ ($H = J/\psi, \psi'$) are approximately equal in the k_T-factorization approach and the parton model.
- We have predicted $\alpha(p_T)$ for direct ψ' and prompt J/ψ. Our predictions are coincide with the collinear parton model calculations rather than with previous k_T-factorization results.