Interacting Rarita–Schwinger field and its spin-parity content

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Introduction

The spin-3/2 particles in QFT are described by the spin-vector field Ψ^{μ} – so called Rarita-Schwinger field.
Main problems and paradoxes are related with "extra"components of spin-1/2 in Ψ^{μ} (Johnson,Sudarshan,1961; Velo,Zwanziger,1969).

In spite of a long history the properties of this field are discussed up to now. Main reasons for it are related with experiments on production of baryon resonances of spin-3/2, most investigated of them is the $\Delta(1232)$.

There are two different opinions concerning spin-1/2 sector.

Most straight way for investigation of non-leading contributions spin-1/2 is to construct the dressed propagator with account of all contributions. So we suggest to go in this direction.

Introduction

The standard free lagrangian (with unphysical spin-1/2 sector) is

$$\mathcal{L} = \overline{\Psi}^{\mu}\Lambda^{\mu\nu}\Psi^{\nu},$$

$$\Lambda^{\mu\nu} = (\hat{p}-M)g^{\mu\nu} + A(\gamma^{\mu}p^{\nu}+\gamma^{\nu}p^{\mu}) + \frac{1}{2}(3A^2+2A+1)\gamma^{\mu}\hat{p}\gamma^{\nu} + +M(3A^2+3A+1)\gamma^{\mu}\gamma^{\nu}.$$

Here A is an arbitrary real parameter, $p_{\mu} = i\partial_{\mu}$.
This lagrangian is invariant under the point transformation:

$$\Psi^{\mu} \to \Psi'^{\,\mu} = (g^{\mu\nu} + \alpha\gamma^{\mu}\gamma^{\nu})\Psi^{\nu}, \qquad A \to A' = \frac{A-2\alpha}{1+4\alpha},$$

with parameter $\alpha \neq -1/4$.
It's not difficult to build the corresponding free propagator $G_0^{\mu\nu}$.

As concerned for the dressed propagator, its construction is a more complicated issue and its total expression is unknown up to now.

Below we discuss the following:

- ✓ We derive an analytical expression for propagator of the interacting Rarita-Schwinger field with accounting all spin components and discuss its properties. The crucial point for it is the choosing of a suitable basis.
- ✓ We discuss also dressing of Dirac fermions in search of the nearest analogy for dressing the s=1/2 sector of Rarita-Schwinger field.

If to say about possible application for phenomenology, the considered problem is in fact a problem of exact form of resonance curve $\Delta(1232)$.

Basis for spin-tensor $S^{\mu\nu}(p)$

- The γ-matrix basis:

$$
\begin{aligned}
S^{\mu\nu}(p) =& g^{\mu\nu} \cdot s_1 + p^\mu p^\nu \cdot s_2 + \\
& + \hat{p} p^\mu p^\nu \cdot s_3 + \hat{p} g^{\mu\nu} \cdot s_4 + p^\mu \gamma^\nu \cdot s_5 + \gamma^\mu p^\nu \cdot s_6 + \\
& + \sigma^{\mu\nu} \cdot s_7 + \sigma^{\mu\lambda} p^\lambda p^\nu \cdot s_8 + \sigma^{\nu\lambda} p^\lambda p^\mu \cdot s_9 + \gamma^\lambda \gamma^5 \imath \varepsilon^{\lambda\mu\nu\rho} p^\rho \cdot s_{10}.
\end{aligned}
$$

Here $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$, $\quad \hat{p} = p_\mu \gamma^\mu$, $\quad s_i(p^2)$.

Basis

- Another known basis ($\hat{p}$ basis) is constructed through the use of:

$$(\mathcal{P}^{3/2})^{\mu\nu} = g^{\mu\nu} - \frac{2}{3}\frac{p^\mu p^\nu}{p^2} - \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3p^2}(\gamma^\mu p^\nu - \gamma^\nu p^\mu)\hat{p},$$

$$(\mathcal{P}^{1/2}_{11})^{\mu\nu} = \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{1}{3}\frac{p^\mu p^\nu}{p^2} - \frac{1}{3p^2}(\gamma^\mu p^\nu - \gamma^\nu p^\mu)\hat{p},$$

$$(\mathcal{P}^{1/2}_{22})^{\mu\nu} = \frac{p^\mu p^\nu}{p^2},$$

$$(\mathcal{P}^{1/2}_{21})^{\mu\nu} = \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3p^2}(-p^\mu + \gamma^\mu \hat{p})\hat{p}p^\nu, \qquad (\mathcal{P}^{1/2}_{12})^{\mu\nu} = \sqrt{\frac{3}{p^2}} \cdot \frac{1}{3p^2}p^\mu(-p^\nu + \gamma^\nu \hat{p})\hat{p}.$$

Decomposition of $S^{\mu\nu}(p)$:

$$S^{\mu\nu}(p) = (S_1 + S_2\hat{p})(\mathcal{P}^{3/2})^{\mu\nu} + (S_3 + S_4\hat{p})(\mathcal{P}^{1/2}_{11})^{\mu\nu} + (S_5 + S_6\hat{p})(\mathcal{P}^{1/2}_{22})^{\mu\nu} +$$

$$+ (S_7 + S_8\hat{p})(\mathcal{P}^{1/2}_{21})^{\mu\nu} + (S_9 + S_{10}\hat{p})(\mathcal{P}^{1/2}_{12})^{\mu\nu}.$$

Basis

- We suggest to use the most convenient at multiplication basis (let's call it Λ-basis). It is built from the operators $(\mathcal{P}^{3/2})^{\mu\nu}$, $(\mathcal{P}^{1/2}_{ij})^{\mu\nu}$ and off-shell projection operators $\Lambda^\pm$

$$\Lambda^\pm = \frac{1}{2}\left(1 \pm \frac{\hat{p}}{\sqrt{p^2}}\right).$$

We assume $\sqrt{p^2}$ to be the first branch of analytical function. Ten elements of this basis look as

$$\begin{aligned}
\mathcal{P}_1 &= \Lambda^+\mathcal{P}^{3/2}, & \mathcal{P}_3 &= \Lambda^+\mathcal{P}^{1/2}_{11}, & \mathcal{P}_5 &= \Lambda^+\mathcal{P}^{1/2}_{22}, & \mathcal{P}_7 &= \Lambda^+\mathcal{P}^{1/2}_{21}, & \mathcal{P}_9 &= \Lambda^+\mathcal{P}^{1/2}_{12}, \\
\mathcal{P}_2 &= \Lambda^-\mathcal{P}^{3/2}, & \mathcal{P}_4 &= \Lambda^-\mathcal{P}^{1/2}_{11}, & \mathcal{P}_6 &= \Lambda^-\mathcal{P}^{1/2}_{22}, & \mathcal{P}_8 &= \Lambda^-\mathcal{P}^{1/2}_{21}, & \mathcal{P}_{10} &= \Lambda^-\mathcal{P}^{1/2}_{12},
\end{aligned}$$

where tensor indices are omitted.
The decomposition of $S^{\mu\nu}(p)$:

$$S^{\mu\nu}(p) = \sum_{i=1}^{10} \mathcal{P}^{\mu\nu}_i \bar{S}_i(p^2).$$

Properties of Λ-basis

- The coefficients $\bar{S}_i$ are calculated in analogy with γ-matrix ones.
- The transfer matrix from γ- to Λ- basis is not singular.
- Multiplicative properties $\mathcal{P}_i(column) \times \mathcal{P}_j(row)$:

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$	$\mathcal{P}_6$	$\mathcal{P}_7$	$\mathcal{P}_8$	$\mathcal{P}_9$	$\mathcal{P}_{10}$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	0	0	0	0	0	0	0	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	0	0	0	0	0	0	0
$\mathcal{P}_3$	0	0	$\mathcal{P}_3$	0	0	0	$\mathcal{P}_7$	0	0	0
$\mathcal{P}_4$	0	0	0	$\mathcal{P}_4$	0	0	0	$\mathcal{P}_8$	0	0
$\mathcal{P}_5$	0	0	0	0	$\mathcal{P}_5$	0	0	0	$\mathcal{P}_9$	0
$\mathcal{P}_6$	0	0	0	0	0	$\mathcal{P}_6$	0	0	0	$\mathcal{P}_{10}$
$\mathcal{P}_7$	0	0	0	0	0	$\mathcal{P}_7$	0	0	0	$\mathcal{P}_3$
$\mathcal{P}_8$	0	0	0	0	$\mathcal{P}_8$	0	0	0	$\mathcal{P}_4$	0
$\mathcal{P}_9$	0	0	0	$\mathcal{P}_9$	0	0	0	$\mathcal{P}_5$	0	0
$\mathcal{P}_{10}$	0	0	$\mathcal{P}_{10}$	0	0	0	$\mathcal{P}_6$	0	0	0

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$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	0	0	0	0	0	0	0
$\mathcal{P}_3$	0	0	$\mathcal{P}_3$	0	0	0	$\mathcal{P}_7$	0	0	0
$\mathcal{P}_4$	0	0	0	$\mathcal{P}_4$	0	0	0	$\mathcal{P}_8$	0	0
$\mathcal{P}_5$	0	0	0	0	$\mathcal{P}_5$	0	0	0	$\mathcal{P}_9$	0
$\mathcal{P}_6$	0	0	0	0	0	$\mathcal{P}_6$	0	0	0	$\mathcal{P}_{10}$
$\mathcal{P}_7$	0	0	0	0	0	$\mathcal{P}_7$	0	0	0	$\mathcal{P}_3$
$\mathcal{P}_8$	0	0	0	0	$\mathcal{P}_8$	0	0	0	$\mathcal{P}_4$	0
$\mathcal{P}_9$	0	0	0	$\mathcal{P}_9$	0	0	0	$\mathcal{P}_5$	0	0
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$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	0	0	0	0	0	0	0
$\mathcal{P}_3$	0	0	$\mathcal{P}_3$	0	0	0	$\mathcal{P}_7$	0	0	0
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$\mathcal{P}_5$	0	0	0	0	$\mathcal{P}_5$	0	0	0	$\mathcal{P}_9$	0
$\mathcal{P}_6$	0	0	0	0	0	$\mathcal{P}_6$	0	0	0	$\mathcal{P}_{10}$
$\mathcal{P}_7$	0	0	0	0	0	$\mathcal{P}_7$	0	0	0	$\mathcal{P}_3$
$\mathcal{P}_8$	0	0	0	0	$\mathcal{P}_8$	0	0	0	$\mathcal{P}_4$	0
$\mathcal{P}_9$	0	0	0	$\mathcal{P}_9$	0	0	0	$\mathcal{P}_5$	0	0
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$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	0	0	0	0	0	0	0
$\mathcal{P}_3$	0	0	$\mathcal{P}_3$	0	0	0	$\mathcal{P}_7$	0	0	0
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$\mathcal{P}_6$	0	0	0	0	0	$\mathcal{P}_6$	0	0	0	$\mathcal{P}_{10}$
$\mathcal{P}_7$	0	0	0	0	0	$\mathcal{P}_7$	0	0	0	$\mathcal{P}_3$
$\mathcal{P}_8$	0	0	0	0	$\mathcal{P}_8$	0	0	0	$\mathcal{P}_4$	0
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$\mathcal{P}_{10}$	0	0	$\mathcal{P}_{10}$	0	0	0	$\mathcal{P}_6$	0	0	0

Dyson–Schwinger equation

$$G^{\mu\nu} = G_0^{\mu\nu} + G^{\mu\alpha} J^{\alpha\beta} G_0^{\beta\nu}.$$

Here $G_0^{\mu\nu}$ and $G^{\mu\nu}$ are the free and full propagators respectively, $J^{\mu\nu}$ is a self-energy contribution. The equation may be rewritten for inverse propagators as

$$(S)^{\mu\nu} = (S_0)^{\mu\nu} - J^{\mu\nu}.$$

Using the Λ-basis for $S^{\mu\nu}$, $S_0^{\mu\nu}$ and $J^{\mu\nu}$ we reduce eq.(12) to set of equations for the scalar coefficients

$$\bar{S}_i(p^2) = \bar{S}_{0i}(p^2) + \bar{J}_i(p^2), \qquad i = 1 \ldots 10$$

We look for the dressed propagator in the same form

$$G^{\mu\nu} = \sum_{i=1}^{10} \mathcal{P}_i^{\mu\nu} \cdot \bar{G}_i(p^2)$$

Dyson–Schwinger equation

The existing 6 projection operators take part in the decomposition of unity, so equation takes the form:

$$G^{\mu\nu} S^{\nu\lambda} = \sum_{i=1}^{6} \mathcal{P}_i^{\mu\lambda}.$$

In Λ-basis we obtain a set of equations for coefficients $\bar{G}_i$.

$$\begin{aligned} \bar{G}_1 \bar{S}_1 &= 1, \\ \bar{G}_2 \bar{S}_2 &= 1, \end{aligned}$$

$$\begin{aligned} \bar{G}_3 \bar{S}_3 + \bar{G}_7 \bar{S}_{10} &= 1, & \qquad \bar{G}_4 \bar{S}_4 + \bar{G}_8 \bar{S}_9 &= 1, \\ \bar{G}_3 \bar{S}_7 + \bar{G}_7 \bar{S}_6 &= 0, & \qquad \bar{G}_4 \bar{S}_8 + \bar{G}_8 \bar{S}_5 &= 0, \end{aligned}$$

$$\begin{aligned} \bar{G}_5 \bar{S}_5 + \bar{G}_9 \bar{S}_8 &= 1, & \qquad \bar{G}_6 \bar{S}_6 + \bar{G}_{10} \bar{S}_7 &= 1, \\ \bar{G}_5 \bar{S}_9 + \bar{G}_9 \bar{S}_4 &= 0, & \qquad \bar{G}_6 \bar{S}_{10} + \bar{G}_{10} \bar{S}_3 &= 0. \end{aligned}$$

Solution of the Dyson–Schwinger equation

The equations are easy to solve:

$$\bar{G}_1 = \frac{1}{\bar{S}_1}, \quad \bar{G}_2 = \frac{1}{\bar{S}_2},$$
$$\bar{G}_3 = \frac{\bar{S}_6}{\Delta_1}, \quad \bar{G}_4 = \frac{\bar{S}_5}{\Delta_2}, \quad \bar{G}_5 = \frac{\bar{S}_4}{\Delta_2}, \quad \bar{G}_6 = \frac{\bar{S}_3}{\Delta_1}, \tag{1}$$
$$\bar{G}_7 = \frac{-\bar{S}_7}{\Delta_1}, \quad \bar{G}_8 = \frac{-\bar{S}_8}{\Delta_2}, \quad \bar{G}_9 = \frac{-\bar{S}_9}{\Delta_2}, \quad \bar{G}_{10} = \frac{-\bar{S}_{10}}{\Delta_1},$$

where

$$\Delta_1 = \bar{S}_3\bar{S}_6 - \bar{S}_7\bar{S}_{10}, \qquad \Delta_2 = \bar{S}_4\bar{S}_5 - \bar{S}_8\bar{S}_9.$$

$\bar{G}_i$ are coefficients in Λ basis.

What does it mean?

- First two terms correspond to spin-3/2 contribution.
- Remaining eight terms should describe two spin-1/2 representations including the mutual transitions.
- Meaning of nilpotent operators in decomposition ?

Let's look for some analogies with Dirac fermions

Simplest example: dressing of single Dirac fermion

Dyson–Schwinger equation for the dressed fermion propagator $G(p)$:

$$G(p) = G_0 + G\Sigma G_0.$$

We will use again the off-shell projection operators $\Lambda^{\pm}$.

$$\mathcal{P}_1 \equiv \Lambda^+ = \frac{1}{2}\left(1 + \frac{\hat{p}}{\sqrt{p^2}}\right), \qquad \mathcal{P}_2 \equiv \Lambda^- = \frac{1}{2}\left(1 - \frac{\hat{p}}{\sqrt{p^2}}\right).$$

Decomposition of any 4×4 matrix depending on p:

$$S(p) = \sum_{M=1}^{2} \mathcal{P}_M \bar{S}^M.$$

D.-S. equation in this basis takes the form:

$$\bar{G}^M = \bar{G}_0^M + \bar{G}^M \bar{\Sigma}^M \bar{G}_0^M, \qquad M = 1, 2.$$

Dressing of single Dirac fermion

Dressed propagator:

$$(\bar{G}^{M})^{-1} = (\bar{G}_0^{M})^{-1} - \bar{\Sigma}^{M}.$$

In more detail:

$$(\bar{G}^{M=1})^{-1} = (\bar{G}_0^{M=1})^{-1} - \bar{\Sigma}^{M=1} = -m_0 - A(p^2) + \sqrt{p^2}\big(1 - B(p^2)\big),$$
$$(\bar{G}^{M=2})^{-1} = (\bar{G}_0^{M=2})^{-1} - \bar{\Sigma}^{M=2} = -m_0 - A(p^2) - \sqrt{p^2}\big(1 - B(p^2)\big),$$

where A, B are the conventional components of the self-energy

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) = \Lambda^+\Sigma^1 + \Lambda^-\Sigma^2,$$

$$\Sigma^1 = A + \sqrt{p^2}B, \qquad \Sigma^2 = A - \sqrt{p^2}B.$$

Dressing of single Dirac fermion (renormalization)

Standard procedure of renormalization consist in formal expansion $G^{-1}(p)$ in terms of $\hat{p} - m$ and choosing the renormalization constants to fulfill the condition

$$G^{-1}(p) = \hat{p} - m + o(\hat{p} - m).$$

With using of the projection operators basis one needs to renormalize the scalar functions G^M, depending on the argument $E = \sqrt{p^2}$.

Let us consider the $(\bar{G}^{\ 1})^{-1}$ component (recall that the bare contribution is $(G_0^1)^{-1} = -m_0 + \sqrt{p^2}$) and require its expansion in terms on $\left(\sqrt{p^2} - m\right)$ to be:

$$\left(\bar{G}^1\right)^{-1} = \sqrt{p^2} - m + o\left(\sqrt{p^2} - m\right)$$

As a result we have the dressed renormalized propagator $G(p)$, which coincides with the standard expression.

Dressing of single Dirac fermion (self-energy)

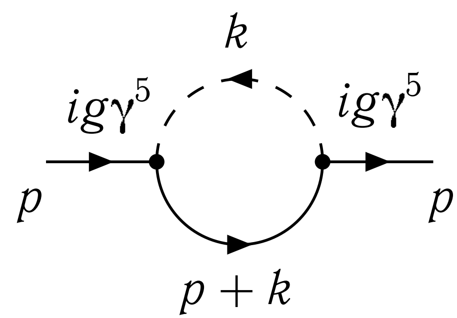
As an example we will consider the dressing of baryon resonance N' ($J^P = 1/2^{\pm}$) due to interaction with πN system. Interaction lagrangian:

$$L_{int} = g\overline{\Psi}\,'(x)\gamma^5\Psi(x)\cdot\phi(x) + h.c. \quad \text{for} \quad N' = 1/2^+$$

and

$$L_{int} = g\overline{\Psi}\,'(x)\Psi(x)\cdot\phi(x) + h.c. \quad \text{for} \quad N' = 1/2^-.$$

$J^P = 1/2^+$



$$\Sigma(p) = ig^2\int\frac{d^4k}{(2\pi)^4}\,\gamma^5\frac{1}{\hat{p}+\hat{k}-m_N}\gamma^5\frac{1}{k^2-m_\pi^2} = I\cdot A(p^2) + \hat{p}B(p^2)$$

Dressing of single Dirac fermion (self-energy)

Loop discontinuity:

$$\Delta A = -\frac{ig^2 m_N}{(2\pi)^2} I_0, \qquad \Delta B = \frac{ig^2}{(2\pi)^2} I_0 \frac{p^2 + m_N^2 - m_\pi^2}{2p^2}.$$

Here

$$I_0 = \int d^4k \delta\left(k^2 - m_\pi^2\right) \delta\left((p+k)^2 - m_N^2\right) = \frac{\pi}{2} \theta\left(p^2 - (m_N + m_\pi)^2\right) \frac{q}{E},$$

where $\lambda\left(a, b, c\right) = \left(a - b - c\right)^2 - 4bc$ and q is the CMS momentum.

Parity conservation: $P(\pi N) = (-1)^{l+1}$

l	J^P	
0	$1/2^-$	
1	$3/2^+$	$1/2^+$
2	$5/2^-$	$3/2^-$
	$\ldots$	

So in the transition $N'(1/2^+) \to N(1/2^+) + \pi(0^-)$ the πN pair should have the orbital momentum $l = 1$.

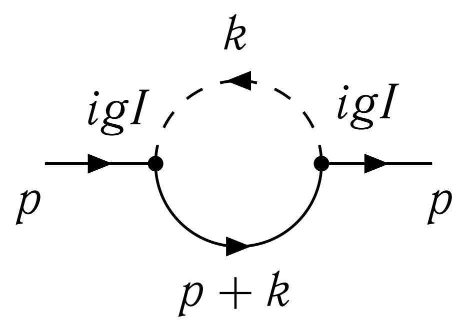
Dressing of single Dirac fermion (self-energy)

According to threshold theorems the imaginary part of a loop should behave as q^{2l+1} at $q \to 0$. But it is not seen in $\Delta A, \Delta B$.
Imaginary parts of $\bar{\Sigma}^M$ components according to (20)

$$Im\ \bar{\Sigma}^1 = Im\ \left(A + \sqrt{p^2}B\right) \sim q^3, \qquad Im\ \bar{\Sigma}^2 = Im\ \left(A - \sqrt{p^2}B\right) \sim q^1.$$

Only the first component Σ^1 demonstrates the proper threshold behavior, i.e. the proper parity.

$J^P(N') = 1/2^-$



$$\Sigma(p) = ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{\hat{k} + \hat{p} - m_N} \cdot \frac{1}{k^2 - m_\pi^2} = IA(p^2) + \hat{p}B(p^2),$$

$$\Delta A = -i\frac{g^2 m_N}{(2\pi)^2} I_0, \qquad \Delta B = \frac{-ig^2}{(2\pi)^2} I_0 \frac{p^2 + m_N^2 - m_\pi^2}{2p^2}$$

Dressing of single Dirac fermion

Imaginary parts of $\bar{\Sigma}^{1,2}$ now demonstrate $l = 0$ and $l = 1$ behavior

$$Im\ \bar{\Sigma}^1 = -\frac{g^2 I_0}{4\sqrt{p^2}(2\pi)^2}\left[\left(\sqrt{p^2} + m_N\right)^2 - m_\pi^2\right] \sim q^1,$$

$$Im\ \bar{\Sigma}^2 = \frac{g^2 I_0}{4\sqrt{p^2}(2\pi)^2}\left(\sqrt{p^2} - m_N - m_\pi\right)\left(\sqrt{p^2} - m_N + m_\pi\right) \sim q^3.$$

The considered examples show that only an „alive" component $\bar{\Sigma}^1$, which has the pole $1/\left(\sqrt{p^2} - m\right)$ demonstrates the proper threshold behavior (i.e. the proper parity). Another component $\bar{\Sigma}^2$, which has pole of the form $1/\left(-\sqrt{p^2} - m\right)$ demonstrates the opposite parity.

Dressing of Dirac fermion with pariry violation

Let us consider a dressing of the fermion state with parity violation. Such situation, arises, in particular for dressing of the t-quark propagator. Dyson–Schwinger equation has the same form but the self-energy contribution Σ contains the parity violating terms

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) + \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2).$$

Now the basis will contain four operators:

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5,$$

where $\mathcal{P}_{1,2}$ are projection operators and $\mathcal{P}_{3,4}$ are nilpotent ones. The expansion of any γ-matrix depending on p now has the form

$$S(p) = \sum_{M=1}^{4} \mathcal{P}^M \bar{S}^M.$$

Dressing of Dirac fermion with pariry violation

This set of operators has simple multiplication properties.

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	$\mathcal{P}_3$	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	$\mathcal{P}_4$
$\mathcal{P}_3$	0	$\mathcal{P}_3$	0	$\mathcal{P}_1$
$\mathcal{P}_4$	$\mathcal{P}_4$	0	$\mathcal{P}_2$	0

Let us denote the inverse dressed and bare propagators as $S(p)$ and $S_0(p)$ respectively. With using of the Λ-basis the Dyson–Schwinger equation is reduced to

$$\bar{S}^M = \left(\bar{S}_0\right)^M - \bar{\Sigma}^M, \qquad M = 1, \ldots, 4.$$

So the problem is reduced to reversing of the known $S(p)$ matrix

$$\left(\sum_{M=1}^{4} \mathcal{P}_M \bar{G}^M\right)\left(\sum_{L=1}^{4} \mathcal{P}_L \bar{S}^L\right) = \mathcal{P}_1 + \mathcal{P}_2.$$

Dressing of Dirac fermion with pariry violation

We obtain the set of equations on the unknown coefficients $\bar{G}^M$

$$\begin{aligned} \bar{G}^1\bar{S}^1+\bar{G}^3\bar{S}^4 &= 1 \qquad & \bar{G}^1\bar{S}^3+\bar{G}^3\bar{S}^2 &= 0 \\ \bar{G}^2\bar{S}^2+\bar{G}^4\bar{S}^3 &= 1 \qquad & \bar{G}^4\bar{S}^1+\bar{G}^2\bar{S}^4 &= 0, \end{aligned}$$

which are easy to solve. The answer is

$$\bar{G}_1=\frac{\bar{S}_2}{\Delta}, \quad \bar{G}_2=\frac{\bar{S}_1}{\Delta}, \quad \bar{G}_3=-\frac{\bar{S}_3}{\Delta}, \quad \bar{G}_4=-\frac{\bar{S}_4}{\Delta},$$

where $\Delta=\bar{S}_1\bar{S}_2-\bar{S}_3\bar{S}_4$.
This example resembles the dressing of the Rarita–Schwinger field by its algebraic structure but it contains only few degrees of freedom.

Joint dressing of two fermions of opposite parities

Let us consider the nearest analogy to the Rarita-Schwinger field: the joint dressing of two fermions of different parity $1/2^{\pm}$.
Now the Dyson-Schwinger equation has matrix form

$$G_{ij} = \left(G_0\right)_{ij} + G_{ik}\Sigma_{kl}\left(G_0\right)_{lj}, \qquad i,j,k,l = 1,2.$$

The basis contains four operators:

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5,$$

where $\mathcal{P}_{1,2}$ are projection operators and $\mathcal{P}_{3,4}$ are the nilpotent ones.
Decomposition of any γ-matrix, depending on p, has the form

$$S(p) = \sum_{M=1}^{4} \mathcal{P}^M \bar{S}^M,$$

but now $\bar{S}^M$ are matrices 2×2.

Joint dressing of two fermions of opposite parities

This set of operators has very simple multiplicative properties.

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$
$\mathcal{P}_1$	$\mathcal{P}_1$	0	$\mathcal{P}_3$	0
$\mathcal{P}_2$	0	$\mathcal{P}_2$	0	$\mathcal{P}_4$
$\mathcal{P}_3$	0	$\mathcal{P}_3$	0	$\mathcal{P}_1$
$\mathcal{P}_4$	$\mathcal{P}_4$	0	$\mathcal{P}_2$	0

The Dyson-Schwinger equation reduces to the matrix equations:

$$\bar{G}_1\bar{S}_1 + \bar{G}_3\bar{S}_4 = E_2,$$

$$\bar{G}_2\bar{S}_2 + \bar{G}_4\bar{S}_3 = E_2,$$

$$\bar{G}_1\bar{S}_3 + \bar{G}_3\bar{S}_2 = 0,$$

$$\bar{G}_4\bar{S}_1 + \bar{G}_2\bar{S}_4 = 0,$$

where E_2 is the unit matrix 2×2.

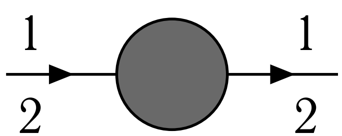
Joint dressing of two fermions of opposite parities

Solution of D.–S. equation:

$$\bar{G}_1 = \left[\bar{S}_1 - \bar{S}_3\left(\bar{S}_2\right)^{-1}\bar{S}_4\right]^{-1}, \qquad \bar{G}_2 = \left[\bar{S}_2 - \bar{S}_4\left(\bar{S}_1\right)^{-1}\bar{S}_3\right]^{-1},$$

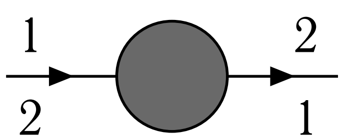
$$\bar{G}_3 = -\left[\bar{S}_1 - \bar{S}_3\left(\bar{S}_2\right)^{-1}\bar{S}_4\right]^{-1}\bar{S}_3\left(\bar{S}_2\right)^{-1}, \quad \bar{G}_4 = -\left[\bar{S}_2 - \bar{S}_4\left(\bar{S}_1\right)^{-1}\bar{S}_3\right]^{-1}\bar{S}_4\left(\bar{S}_1\right)^{-1}.$$

Now let us concretize these general formulae. Suppose that we have two fermions of opposite parities, but there is no parity violation in lagrangian. It means that the diagonal loops contain only the I and $\hat{p}$



$$\Sigma_{ii} \sim I,\ \hat{p},$$

while the non-diagonal ones should have γ^5



$\Sigma_{ij} \sim \gamma^5,\ \hat{p}\gamma^5$ for $i \neq j$

Joint dressing of two fermions of opposite parities

So the decomposition of inverse propagator in this basis has the form

$$
\begin{aligned}
S(p) &= \mathcal{P}_1 \begin{pmatrix} -m_1 + E - \bar{\Sigma}^{(1)}_{11} & 0 \\ 0 & -m_2 + E - \bar{\Sigma}^{(1)}_{22} \end{pmatrix} \\
&+ \mathcal{P}_2 \begin{pmatrix} -m_1 - E - \bar{\Sigma}^{(2)}_{11} & 0 \\ 0 & -m_2 - E - \bar{\Sigma}^{(2)}_{22} \end{pmatrix} \\
&+ \mathcal{P}_3 \begin{pmatrix} 0 & -\bar{\Sigma}^{(3)}_{12} \\ -\bar{\Sigma}^{(3)}_{21} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & -\bar{\Sigma}^{(4)}_{12} \\ -\bar{\Sigma}^{(4)}_{21} & 0 \end{pmatrix}.
\end{aligned}
$$

Joint dressing of two fermions of opposite parities

Substituting all into general solution, we have the dressed matrix propagator

$$G = \Lambda^+ \begin{pmatrix} \frac{-m_2 - E - \bar{\Sigma}^2_{22}}{\Delta_1} & 0 \\ 0 & \frac{-m_1 - E - \bar{\Sigma}^2_{11}}{\Delta_2} \end{pmatrix} + \Lambda^- \begin{pmatrix} \frac{-m_2 + E - \bar{\Sigma}^1_{22}}{\Delta_2} & 0 \\ 0 & \frac{-m_1 + E - \bar{\Sigma}^1_{11}}{\Delta_1} \end{pmatrix} +$$
$$+ \Lambda^+ \gamma^5 \begin{pmatrix} 0 & -\frac{\bar{\Sigma}^3_{12}}{\Delta_1} \\ -\frac{\bar{\Sigma}^3_{21}}{\Delta_2} & 0 \end{pmatrix} + \Lambda^- \gamma^5 \begin{pmatrix} 0 & -\frac{\bar{\Sigma}^4_{12}}{\Delta_2} \\ -\frac{\bar{\Sigma}^4_{21}}{\Delta_1} & 0 \end{pmatrix}. \quad (2)$$

Here

$$\Delta_1 = \big(-m_1 + E - \bar{\Sigma}^2_{11}\big)\big(-m_2 - E - \bar{\Sigma}^2_{22}\big) - \bar{\Sigma}^3_{12}\bar{\Sigma}^4_{21},$$
$$\Delta_2 = \big(-m_1 - E - \bar{\Sigma}^1_{11}\big)\big(-m_2 + E - \bar{\Sigma}^1_{22}\big) - \bar{\Sigma}^4_{12}\bar{\Sigma}^3_{21} = \Delta_1\big(E \to -E\big).$$

The appearance of nilpotent operators in decomposition (2) is an indication for transitions between states of different parities.

In search of analogy

Summarizing our consideration of the dressing of Dirac fermions:

1) We found very convenient the using of the projection operators $\Lambda^{\pm} = \left(1 \pm \hat{p}/\sqrt{p^2}\right)/2$ to solve the Dyson-Schwinger equations.

2) $\Lambda^{\pm}$ are very useful in another aspect: its coefficients have the definite parity. There is such correspondence: the parity of the field Ψ is the parity of „alive“ component Λ^{+}, which has the pole $1\big/(E-m)$.
Another component Λ^{-} which has the pole $1\big/(-E-m)$ demonstrates the opposite parity.

3) In contrast to boson case, even if the interactions conserve the parity, the loop transitions between different parity fields are not zeroth.

4) The joint dressing of two fermions has different picture depending of parities of mixing fields.

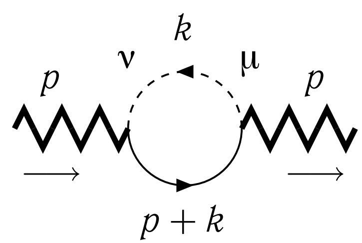
$J^P = 1/2^{\pm}$	$\Leftrightarrow$	$J^P = 1/2^{\pm}$	$J^P = 1/2^{\pm}$	$\Leftrightarrow$	$J^P = 1/2^{\mp}$
Λ^{+}	$\Longleftrightarrow$	Λ^{+}	Λ^{+}	$\Longleftrightarrow$	Λ^{-}
Λ^{-}	$\Longleftrightarrow$	Λ^{-}	Λ^{-}	$\Longleftrightarrow$	Λ^{+}

Spin-parity of the Rarita-Schwinger field

Studying the dressing of Dirac fermions gives us some hint: presence of the nilpotent operators $\mathcal{P}_7$—$\mathcal{P}_{10}$ in decomposition of propagator $G^{\mu\nu}(p)$ is an indication on the transitions between components of opposite parities $1/2^{\pm}$.
To make sure in this conclusion, we can calculate loop contributions in propagator.
As an example we will take the standard interaction lagrangian $\pi N \Delta$

$$L_{int} = g_{\pi N \Delta} \overline{\Psi}^{\mu}(x)(g^{\mu\nu} + z\gamma^{\mu}\gamma^{\nu})\Psi(x) \cdot \partial_{\nu}\phi(x) + h.c. \,. \qquad (3)$$

Here z is arbitrary parameter.
The one-loop self-energy contribution is



$$J^{\mu\nu}(p) = -ig^2_{\pi N\Delta} \int \frac{d^4k}{(2\pi)^4}(g^{\mu\rho}+z\gamma^{\mu}\gamma^{\rho})k^{\rho}\frac{1}{\hat{p}+\hat{k}-m_N}k^{\lambda}(g^{\lambda\nu}+z\gamma^{\lambda}\gamma^{\nu})\frac{1}{k^2-m_{\pi}^2}.$$

Spin-parity of the Rarita-Schwinger field

Discontinuity of the loop contribution in $\hat{p}$ basis.

$$\begin{aligned}
\Delta J_1 &= -ig^2 I_0 \frac{m_N}{12s} \lambda(s, m_N^2, m_\pi^2), \\
\Delta J_2 &= -ig^2 I_0 \frac{1}{24s^2} (s + m_N^2 - m_\pi^2)\lambda, \\
\Delta J_3 &= -ig^2 I_0 \frac{m_N}{12s} (\lambda + 6z\lambda - 36z^2 m_\pi^2 s), \\
\Delta J_4 &= -ig^2 I_0 \frac{1}{24s^2} [(s + m_N^2 - m_\pi^2)\lambda + 12zs\lambda + 36z^2 s(s^2 - m_\pi^2 s - 2m_N^2 s - m_\pi^2 m_N^2 + m_N^4)], \\
\Delta J_5 &= ig^2 I_0 \frac{m_N}{4s} [(s - m_N^2 + m_\pi^2)^2 + 2z(s - m_N^2 + m_\pi^2)^2 + 4z^2 m_\pi^2 s], \\
\Delta J_6 &= ig^2 I_0 \frac{1}{8s^2} [(s + m_N^2 - m_\pi^2)(s - m_N^2 + m_\pi^2)^2 + 4zs(s - m_N^2 + m_\pi^2)(s - m_N^2 - m_\pi^2) + \\
&\quad 4z^2 s(s^2 - m_\pi^2 s - 2m_N^2 s - m_\pi^2 m_N^2 + m_N^4)], \\
\Delta J_7 &= ig^2 I_0 \sqrt{\frac{3}{s}} \cdot \frac{1}{24s} [(s - m_N^2 + m_\pi^2)\lambda + 4zs(2s^2 - m_\pi^2 s - 4m_N^2 s + 2m_N^4 - m_N^2 m_\pi^2 - m_\pi^4) + \\
&\quad 12z^2 s(s^2 - m_\pi^2 s - 2m_N^2 s - m_N^2 m_\pi^2 + m_N^4)], \\
\Delta J_8 &= -ig^2 I_0 \sqrt{\frac{3}{s}} \cdot \frac{z m_N}{6s} [(s^2 + 4m_\pi^2 s - 2m_N^2 s + m^4 - 2m^2 m_\pi^2 + m_\pi^4) + 6zs m_\pi^2], \\
\Delta J_9 &= \Delta J_7 \\
\Delta J_{10} &= -\Delta J_8.
\end{aligned}$$

Spin-parity of the Rarita-Schwinger field

Here I_0 is the same base integral, arguments of function λ are the same anywhere, but are shown only in first expression.
We saw for case of Dirac fermions that the propagator decomposition in basis of projection operators demonstrates the definite parity. We can expect the same property for Rarita-Schwinger field in Λ-basis. Let us verify it by calculating the threshold behavior of imaginary part.
Indeed, after some calculations:

$$
\begin{aligned}
\Delta\bar{J}_1 &= \Delta J_1 + E\Delta J_2 \sim q^3 \\
\Delta\bar{J}_2 &= \Delta J_1 - E\Delta J_2 \sim q^5 \\
\Delta\bar{J}_3 &= \Delta J_3 + E\Delta J_4 \sim q^3 \\
\Delta\bar{J}_4 &= \Delta J_3 - E\Delta J_4 \sim q \\
\Delta\bar{J}_5 &= \Delta J_5 + E\Delta J_6 \sim q \\
\Delta\bar{J}_6 &= \Delta J_5 - E\Delta J_6 \sim q^3.
\end{aligned}
$$

Such a behavior indicates that the components $\bar{J}_1, \bar{J}_2$ exhibit the spin-parity $3/2^+$, while the pairs of coefficient $\bar{J}_3, \bar{J}_4$ and $\bar{J}_5, \bar{J}_6$ correspond to $1/2^+$, $1/2^-$ contributions respectively.

Conclusions

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- ✓ We found that the nearest analogy for dressing in the spin-1/2 sector is the joint dressing of two Dirac fermions of opposite parities. Calculation of the self-energy contributions in case of Δ isobar confirms that in the Rarita-Schwinger field besides the leading spin-3/2 contribution there are also two spin-1/2 components of opposite parities.

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- ✓ We suppose that an accurate dressing (and renormalization) of the Rarita-Schwinger field propagator is the more adequate approach for description of data on Δ production. As for renormalization – work in progress.