General helicity formalism for single and double spin asymmetries in $pp \longrightarrow \pi + X$

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Outline

- ◇ Spin asymmetries: naive pQCD prediction vs experiment
- ◇ Partonic transverse momenta
- ◇ Helicity Formalism: helicity density matrix
- ◇ Polarized hadronic cross section for $pp \to \pi + X$
- ◇ Transverse SSA
- ◇ Double longitudinal spin asymmetry A_{LL}
- ◇ Conclusions

Naive Collinear pQCD prediction

◇ Collinear pQCD predicts no sizable spin asymmetries

Let's rewrite a transverse spin state in a helicity state

$$|\uparrow/\downarrow\rangle = \frac{1}{\sqrt{2}}[|+\rangle \pm |-\rangle]$$

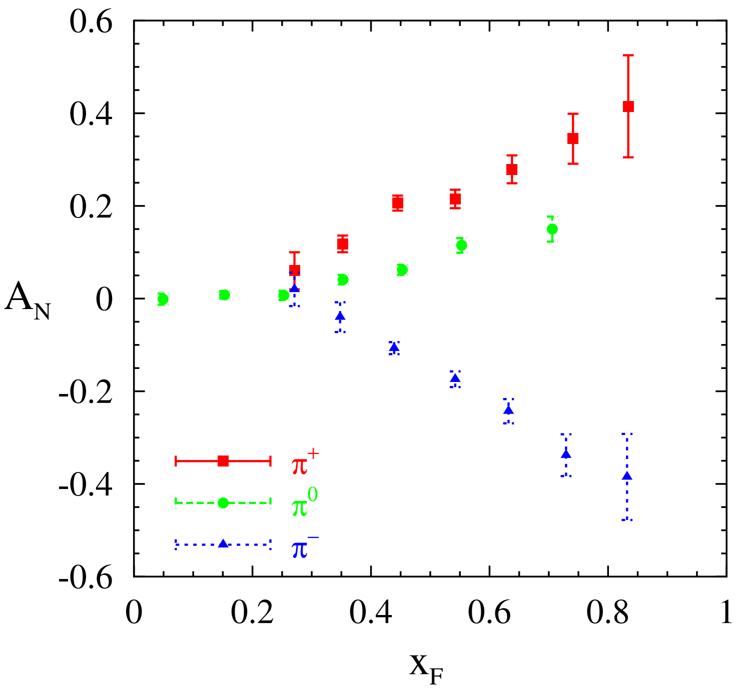
So schematically:

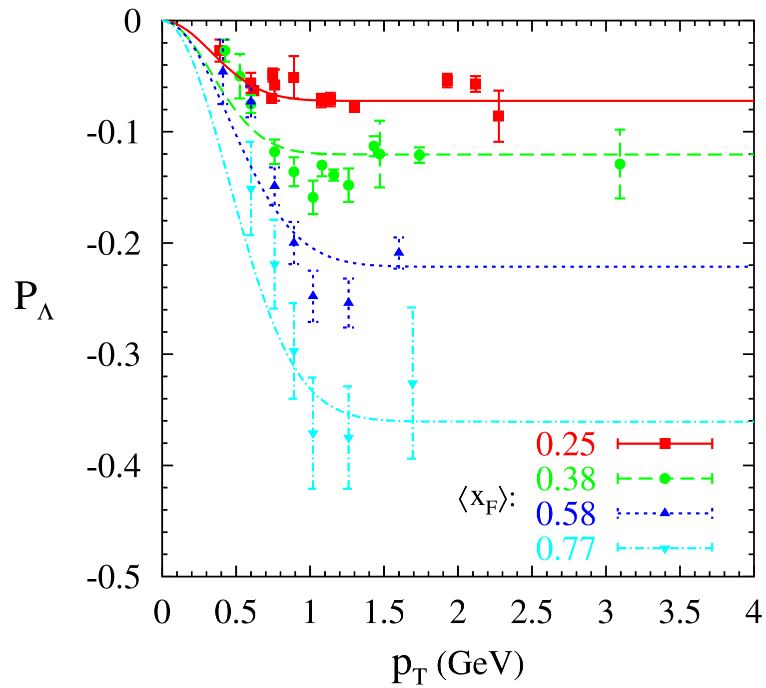
$$A_N \sim \frac{\langle\uparrow|\uparrow\rangle - \langle\downarrow|\downarrow\rangle}{\langle\uparrow|\uparrow\rangle + \langle\downarrow|\downarrow\rangle} \sim \frac{2\mathrm{Im}\langle+|-\rangle}{\langle+|+\rangle + \langle-|-\rangle}$$

- $\langle+|-\rangle$ No spin flip for massless collinear QCD $\Rightarrow A_N \propto \frac{m_q}{E_q}$
- Im: Relative phase only at higher order $\Rightarrow A_N \propto \alpha_s$

$$A_N \propto \alpha_s \frac{m_q}{E_q} \qquad (1)$$

...experiment





$p^\uparrow p \to \pi X$ at $\sqrt{s}$=19.4 GeV, p_T=1.5 GeV/c

Collection of data $pBe \to \Lambda^\uparrow X$ at $\sqrt{s}$=27-38.8 GeV

Partonic intrinsic transverse momenta

➢ A possible description for SSA in pQCD can be obtain by introducing:

☞ 1) the *partonic intrinsic transverse momentum* $\boldsymbol{k}_\perp$.

2) a $k_\perp$-factorization for cross section (ansatz).

3) the helicity formalism and a new class of spin and $\boldsymbol{k}_\perp$ dependent parton distribution and fragmentation functions.

➠ We have to generalize the usual partonic functions into $\boldsymbol{k}_\perp$ dependent functions. For instance, the parton distribution function is generalized as:

$$f_{q/h}(x_q) \to \hat{f}_{q/h}(x_q, \boldsymbol{k}_{\perp q})$$

$$\text{with } f_{q/h}(x_q) = \int \mathrm{d}^2\boldsymbol{k}_{\perp q}\hat{f}_{q/h}(x_q, \boldsymbol{k}_{\perp q})$$

x_q light-cone momentum fraction of parton q inside hadron h

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From collinear to transverse configuration (I)

◇In the usual collinear pQCD factorization formula reads for the inclusive process $AB \to CX$ as:

$$\mathrm{d}\sigma^{AB\to CX} = \sum_{a,b,c,d} f_{a/A}(x_a, Q^2) \otimes f_{b/B}(x_b, Q^2) \otimes \mathrm{d}\hat{\sigma}^{ab\to cd}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) \otimes D_{C/c}(z, Q^2)$$

◇Introducing transverse momenta we have:

$$\mathrm{d}\sigma^{AB\to CX} = \sum_{a,b,c,d} \hat{f}_{a/A}(x_a, \boldsymbol{k}_{\perp a}; Q^2) \otimes \hat{f}_{b/B}(x_b, \boldsymbol{k}_{\perp b}; Q^2) \otimes \mathrm{d}\hat{\sigma}^{ab\to cd}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) \otimes \hat{D}_{C/c}(z, \boldsymbol{k}_{\perp C}; Q^2)$$

For complete kinematics: *U.D'Alesio, F.Murgia*: Phys.Rev.D70:074009,2004

Partonic intrinsic transverse momenta

➢ A possible description for SSA in pQCD can be obtain by introducing:

1) the *partonic intrinsic transverse momentum* $\boldsymbol{k}_\perp$.

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From unpolarized to polarized cross section (I)

☞ Now we introduce inside the factorization scheme **the helicity density matrices**, which describe the parton spin states, in order to obtain a polarized cross section:

$$\mathrm{d}\sigma^{(A,S_A)+(B,S_B)\to C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) \otimes \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b}) \otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^{*}_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \otimes \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

- λ's denote the helicity indexes
- S_A and S_B denote the polarization state of $A(B)$
- $\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}$ is the helicity density matrix of parton a inside hadron A with polarization S_A (similarly for parton b inside hadron B)

From unpolarized to polarized cross section (II)

$$\mathrm{d}\sigma^{(A,S_A)+(B,S_B)\to C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) \otimes \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b}) \otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \otimes \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

• $\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$'s are the helicity amplitudes for process $ab \to cd$ and are related to $\mathrm{d}\hat{\sigma}^{ab\to cd}$.

• $\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$'s are defined in the hadron c.m. frame and they are related to the usual helicity amplitudes defined in the partonic c.m. frame, $\hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$, in a non trivial way by proper phases from rotations and boost.

☞The partonic scattering process is not on the same plane of the hadronic one.

$$\hat{M}_{++;++} = \hat{M}^0_1 e^{i\varphi_1} \qquad \hat{M}_{-+;-+} = \hat{M}^0_2 e^{i\varphi_2} \qquad \hat{M}_{-+;+-} = \hat{M}^0_3 e^{i\varphi_3} \qquad (2)$$

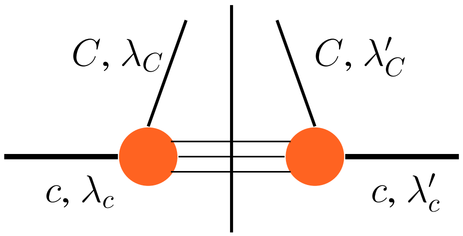
see *M.Anselmino, M.Boglione, U.D'Alesio, E.Leader, F.Murgia*: Phys.Rev.D71:014002,2005

From unpolarized to polarized cross section (III)

$$\mathrm{d}\sigma^{(A,S_A)+(B,S_B)\to C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\,\hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\otimes \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\,\hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})\otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\,\hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\otimes \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

- $\hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$ is the product of **fragmentation amplitudes** for the $c \to C + X$ process:

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c} = \;{\int\!\!\!\!\!\!\sum}_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_X,\lambda_C;\lambda_c}\,\hat{\mathcal{D}}^*_{\lambda_X,\lambda'_C;\lambda'_c} \qquad (3)$$



where ${\int\!\!\!\!\!\!\sum}_{X,\lambda_X}$ stands for a spin sum and phase integration over all undetected particles X. The usual fragmentation function is related to this product by:

$$D_{C/c}(z) = \frac{1}{2}\sum_{\lambda_c,\lambda_C}\int d^2\boldsymbol{k}_{\perp C}\,\hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda_c}(z,\boldsymbol{k}_{\perp C})$$

Master formula

$$\mathrm{d}\sigma^{(A,S_A)+(B,S_B)\to C+X} = \sum_{a,b,c,d,\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) \otimes \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b}) \otimes \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \otimes \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

This expression contains all possible combinations of different spin and $\boldsymbol{k}_\perp$-dependent distribution/fragmentation functions

Quark helicity density matrices

$$\boxed{\rho^{a/A,S_A}_{\lambda_a,\lambda'_a} \;\; \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})}$$

$$\boxed{\frac{1}{2}\begin{pmatrix} 1+P^a_z & P^a_x - iP^a_y \\ P^a_x + iP^a_y & 1-P^a_z \end{pmatrix} \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})}$$

$$(P^a_i\;\hat{f}_{a/A,S_Y}) = \Delta\hat{f}^a_{s_i/S_Y} = \hat{f}^a_{s_i/\uparrow} - \hat{f}^a_{-s_i/\uparrow} \equiv \Delta\hat{f}^a_{s_i/\uparrow}(x_a,\boldsymbol{k}_{\perp a})$$

$$(P^a_i\;\hat{f}_{a/A,S_Z}) = \Delta\hat{f}^a_{s_i/S_Z} = \hat{f}^a_{s_i/+} - \hat{f}^a_{-s_i/+} \equiv \Delta\hat{f}^a_{s_i/+}(x_a,\boldsymbol{k}_{\perp a})$$

$$(\hat{f}_{a/A,S_Y}) = \hat{f}_{a/A}(x_a,\boldsymbol{k}_{\perp a}) + \frac{1}{2}\Delta\hat{f}_{a/S_Y}(x_a,\boldsymbol{k}_{\perp a})$$

Sivers Function

$$\begin{aligned}\Delta\hat{f}_{a/S_Y}\,(x_a,\boldsymbol{k}_{\perp a}) &= \hat{f}_{a/S_Y}\,(x_a,\boldsymbol{k}_{\perp a}) - \hat{f}_{a/-S_Y}\,(x_a,\boldsymbol{k}_{\perp a}) \\ &= \Delta^N\hat{f}_{a/A^\uparrow}\,(x_a,k_{\perp a})\,(\hat{\boldsymbol{p}}_A\times\hat{\boldsymbol{k}}_{\perp a})\cdot\boldsymbol{P}^A \qquad (4)\\ &= -2\frac{k_{\perp a}}{M}f_{1T}^{\perp}(x_a,k_{\perp a}) \qquad (5)\end{aligned}$$

Boer-Mulders Function

$$\Delta\hat{f}^a_{s_y/S_Y} = \Delta\hat{f}^a_{s_y/A} + \Delta^-\hat{f}^a_{s_y/S_Y} \qquad (6)$$

with

$$\Delta\hat{f}^a_{s_y/A}(x_a,\boldsymbol{k}_{\perp a}) = -\frac{\boldsymbol{k}_{\perp a}}{M}h_1^{\perp}(x_a,k_{\perp a}) \qquad (7)$$

and

$$\Delta^-\hat{f}^a_{s_y/S_Y} \equiv \frac{1}{2}\left[\Delta\hat{f}^a_{s_y/\uparrow} - \Delta\hat{f}^a_{s_y/\downarrow}\right] \qquad (8)$$

Transverse SSA; Channel: $q_a q_b \to q_c q_d$

$$
\begin{aligned}
& d\sigma(A^{\uparrow}B \to C+X) - d\sigma(A^{\downarrow}B \to C+X) \propto \\
& \qquad \frac{1}{2}\, \Delta\hat{f}_{a/A^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})\, \hat{f}_{b/B}(x_b, k_{\perp b}) \left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2 \right] \hat{D}_{C/c}(z, k_{\perp C}) \\
& + \quad 2 \left[\Delta^- \hat{f}^a_{s_y/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \cos(\varphi_3 - \varphi_2) - \Delta\hat{f}^a_{s_x/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \sin(\varphi_3 - \varphi_2) \right] \\
& \qquad \times \Delta\hat{f}^b_{s_y/B}(x_b, \boldsymbol{k}_{\perp b})\, \hat{M}_2^0\, \hat{M}_3^0\, \hat{D}_{C/c}(z, k_{\perp C}) \qquad (9) \\
& + \quad \left[\Delta^- \hat{f}^a_{s_y/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \cos(\varphi_1 - \varphi_2 + \phi_C^H) - \Delta\hat{f}^a_{s_x/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \sin(\varphi_1 - \varphi_2 + \phi_C^H) \right] \\
& \qquad \times \hat{f}_{b/B}(x_b, k_{\perp b})\, \hat{M}_1^0\, \hat{M}_2^0\, \Delta^N \hat{D}_{C/c^{\uparrow}}(z, k_{\perp C}) \\
& + \quad \frac{1}{2}\, \Delta\hat{f}_{a/A^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a}) \Delta\hat{f}^b_{s_y/B}(x_b, \boldsymbol{k}_{\perp b}) \cos(\varphi_1 - \varphi_3 + \phi_C^H)\, \hat{M}_1^0\, \hat{M}_3^0\, \Delta^N \hat{D}_{C/c^{\uparrow}}(z, k_{\perp C})
\end{aligned}
$$

• Sivers Effect • Boer-Mulders Effect • Collins Effect •Phases

Gluon helicity density matrices

$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a} \; \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})$$

$$\frac{1}{2}\begin{pmatrix} 1+P^g_z & \mathcal{T}^g_1 - i\mathcal{T}^g_2 \\ \mathcal{T}^g_1 + i\mathcal{T}^g_2 & 1-P^g_z \end{pmatrix} \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})$$

$$(\mathcal{T}^g_1 \, \hat{f}_{g/A,S_Y}) \equiv \Delta\hat{f}^g_{\mathcal{T}_1/\uparrow}(x_g,\boldsymbol{k}_{\perp g}) = \Delta\hat{f}^g_{\mathcal{T}_1/A} + \Delta^-\hat{f}^g_{\mathcal{T}_1/\uparrow}$$

$$(\mathcal{T}^g_2 \, \hat{f}_{g/A,S_Y}) \equiv \Delta\hat{f}^g_{\mathcal{T}_2/\uparrow}(x_g,\boldsymbol{k}_{\perp g})$$

$$(\mathcal{T}^g_1 \, \hat{f}_{g/A,S_Z}) \equiv \Delta\hat{f}^g_{\mathcal{T}_1/+}(x_g,\boldsymbol{k}_{\perp g})$$

$$(\mathcal{T}^g_2 \, \hat{f}_{g/A,S_Z}) \equiv \Delta\hat{f}^g_{\mathcal{T}_2/+}(x_g,\boldsymbol{k}_{\perp g})$$

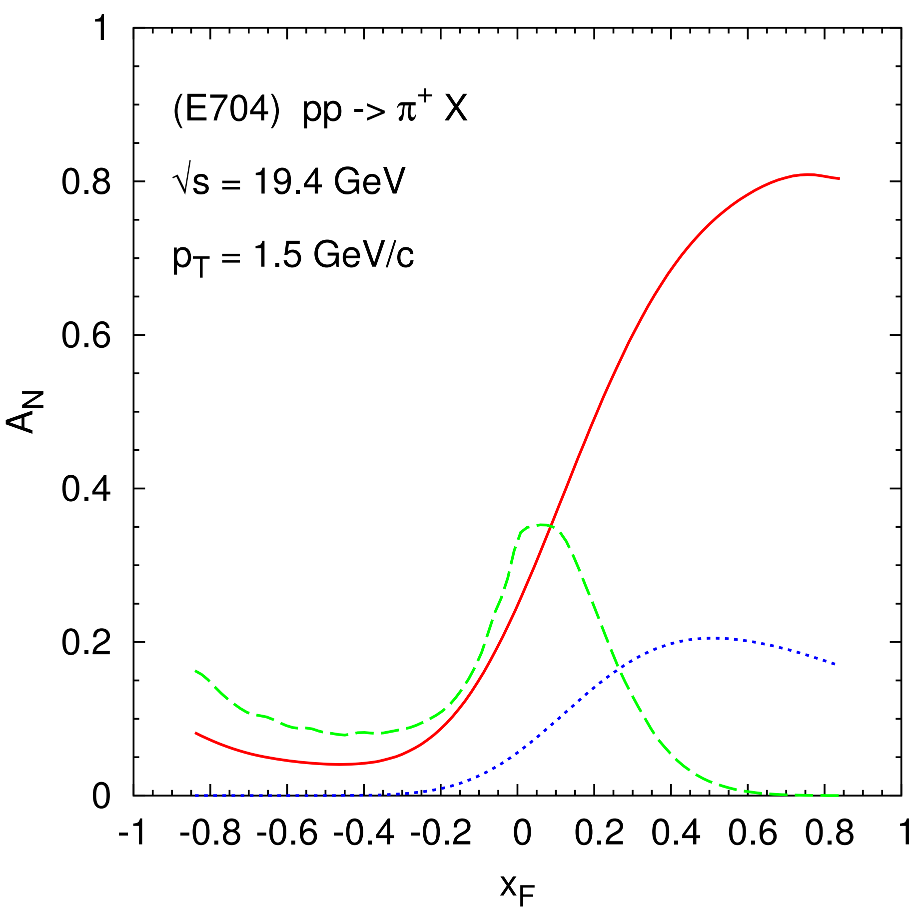
Linearly polarized gluon inside an unpolarized hadron

$\Delta\hat{f}^g_{\mathcal{T}_1/A} \longrightarrow H^\perp$ Mulders& Rodriguez

Transverse SSA; Channel: $g_a g_b \to g_c g_d$

$$
\begin{aligned}
& d\sigma(A^{\uparrow}B \to C + X) - d\sigma(A^{\downarrow}B \to C + X) \propto \\
& \quad \frac{1}{2}\, \Delta \hat{f}_{g/A^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})\, \hat{f}_{g/B}(x_b, k_{\perp b}) \left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2 \right] \hat{D}_{C/g}(z, k_{\perp C}) \\
& + \quad 2 \left[\Delta^- \hat{f}^g_{\mathcal{T}_1/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \cos(\varphi_3 - \varphi_2) + \Delta \hat{f}^g_{\mathcal{T}_2/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \sin(\varphi_3 - \varphi_2) \right] \\
& \quad \times \Delta \hat{f}^g_{\mathcal{T}_1/B}(x_b, \boldsymbol{k}_{\perp b})\, \hat{M}_2^0\, \hat{M}_3^0\, \hat{D}_{C/g}(z, k_{\perp C}) \\
& + \quad \left[\Delta^- \hat{f}^g_{\mathcal{T}_1/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \cos(\varphi_1 - \varphi_2 + 2\phi_C^H) + \Delta \hat{f}^g_{\mathcal{T}_2/\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \sin(\varphi_1 - \varphi_2 + 2\phi_C^H) \right] \\
& \quad \times \hat{f}_{g/B}(x_b, k_{\perp b})\, \hat{M}_1^0\, \hat{M}_2^0\, \Delta^N \hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C}) \\
& + \quad \frac{1}{2}\, \Delta \hat{f}_{g/A^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})\, \Delta \hat{f}^g_{\mathcal{T}_1/B}(x_b, \boldsymbol{k}_{\perp b}) \cos(\varphi_1 - \varphi_3 + 2\phi_C^H)\, \hat{M}_1^0\, \hat{M}_3^0\, \Delta^N \hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C})
\end{aligned}
$$

• Sivers Effect • "Boer-Mulders"-like Effect • "Collins"-like Effect •Phases



- Gaussian $k_\perp$ dependence for all distribution functions with $\langle k_\perp \rangle = 0.8$ GeV/c for PDF
- All unknown polarized distribution functions have been replaced with the corresponding unpolarized distributions. In some cases this is certainly an overestimate: for the transversity distribution it violates the Soffer bound
- The Sivers and Collins functions have been chosen saturating their positivity bounds:

$$\Delta^N \hat{f}_{a/A^\uparrow}(x_a, k_{\perp a}) = 2\, \hat{f}_{a/A}(x_a, k_{\perp a})$$

$$\Delta^N \hat{D}_{C/q^\uparrow}(z, k_{\perp C}) = 2\, \hat{D}_{C/q}(z, k_{\perp C})$$

- Same sign for all flavours

Different contributions to A_N, plotted as a function of x_F, for $p^\uparrow p \to \pi^+ X$ processes and E704 kinematics. *solid line* = quark Sivers mechanism alone; *dashed line* = gluon Sivers mechanism alone; *dotted line* = transversity $\otimes$ Collins. All other contributions are much smaller.

A_{LL}; Channel: $q_a q_b \to q_c q_d$

$$d\sigma(A^+B^+ \to C+X) - d\sigma(A^+B^- \to C+X) \propto$$

$$\begin{aligned}
& \Delta \hat{f}^a_{s_z/+}(x_a, \boldsymbol{k}_{\perp a})\, \Delta \hat{f}^b_{s_z/+}(x_b, \boldsymbol{k}_{\perp b}) \left[|\hat{M}^0_1|^2 - |\hat{M}^0_2|^2 - |\hat{M}^0_3|^2 \right] \hat{D}_{C/c}(z, k_{\perp C}) \\
+ \quad & 2\Delta \hat{f}^a_{s_x/+}(x_a, \boldsymbol{k}_{\perp a})\, \Delta \hat{f}^b_{s_x/+}(x_b, \boldsymbol{k}_{\perp b}) \cos(\varphi_3 - \varphi_2) \hat{M}^0_2\, \hat{M}^0_3\, \hat{D}_{C/c}(z, k_{\perp C}) \\
+ \quad & 2\Delta \hat{f}^a_{s_y/A}(x_a, \boldsymbol{k}_{\perp a})\, \Delta \hat{f}^b_{s_x/+}(x_b, \boldsymbol{k}_{\perp b}) \sin(\varphi_3 - \varphi_2)\, \hat{M}^0_2\, \hat{M}^0_3\, \hat{D}_{C/c}(z, k_{\perp C}) \\
- \quad & \hat{f}_{a/A}(x_a, k_{\perp a})\, \Delta \hat{f}^b_{s_x/+}(x_b, \boldsymbol{k}_{\perp b})\, \hat{M}^0_1\, \hat{M}^0_3 \sin(\varphi_1 - \varphi_3 + \phi^H_C)\, \Delta^N \hat{D}_{C/c^\uparrow}(z, k_{\perp C})
\end{aligned}$$

• Helicity PDF • Boer-Mulders Effect • Collins Effect •Phases

Conclusions and working in progress

◇ We have developed a complete formalism for single and double spin asymmetries in $pp \to \pi + X$ with a non collinear kinematics

◇ We have given a partonic interpretation on our $k_\perp$ dependent functions

◇ Gluon's PDF and FF

◇ We have developed a numerical analysis on the role of phases for SSA showing the suppression of some effects

In progress:

◇ Numerical analysis on double SA in particular A_{LL}

◇ Formalism and Numerical analysis on Lambda production