The polarization effects in the neutrino-electron scattering in a magnetic field

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I. Introduction

In this paper we give the general expression for the cross section of the neutrino-electron scattering in a magnetic field with allowance for the longitudinal polarizations of both the initial and the final electrons.

The process of the neutrino-electron scattering is responsible for a significant fraction of the energy and momentum exchange between the neutrinos and the stellar matter [1-5].

Strong magnetic fields exist in compact objects in the Milky Way Galaxy [e.g , 9-10]. For example, magnetic fields of neutron stars can be as large as $H \geq H_0 = m_e^2 c^3 / e\hbar = 4.41 \times 10^{13} G$ [e.g., 7]. Strong magnetic fields of $H \sim 10^{15} - 10^{17} G$ are generated inside the astrophysical cataclysms like a supernova explosion or a coalescence of neutron stars [6-10]. Such strong magnetic fields influence the neutrino-electron scattering by modifying the motion of the electrons. As it was indicated in [11] the presence of an external magnetic field provides a preferred direction in space and it opens the way for parity violating effects to produce an asymmetry in cross section of neutrino-electron scattering.

As it was noted in [12-14] in neutrino-electron scattering the presence of a strong magnetic field could lead on the one hand to anisotropy and asymmetry in the heating of the stellar matter and on the other hand to the anisotropy and asymmetry of the subsequent explosion of the outer layers of the collapsing stellar core.

To explain the above indicated and other astrophysical phenomena it is important to investigate the polarization effects in the neutrino-electron scattering in a magnetic field. The results of the investigations of the polarization effects in an inverse muon decay and in an annihilation of a muon neutrino and an electron antineutrino into a muon-positron pair in a magnetic field [15,16] show that P and C nonconservation in weak interactions and the choice of kinematical conditions lead to the asymmetric dependence of the differential cross section on the angular and spin variables. The dependence of the differential cross section of the neutrino-electron scattering in a magnetic field on the field strength and the angular variable was investigated by a number of authors [11-14, 17-23]. To clarify the anisotropy and the asymmetry arising in astrophysical phenomena connected with the neutrino-electron scattering it is important to investigate the dependence of the differential cross section of the considered process in a magnetic field on the spin variable.

II. The general expressions for the amplitude and the differential cross section of the neutrino-electron scattering

The amplitude of the process $\nu_l e \to \nu_l e$ $(\nu_l = \nu_e, \nu_\mu, \nu_\tau)$ in a magnetic field in the framework of the Weinberg-Salam model is given by

$$M = \left[\frac{g}{2\cos\theta_w}\right]^2 \int d^4x d^4x' D^{\alpha\beta}(x'-x)\left[\overline{\psi}_{\nu'}(x')\gamma^L_\alpha \psi_\nu(x')\right]\times \\ \times\left[\overline{\psi}_{e'}(x)\gamma_\beta\left(g_V + g_A\gamma^5\right)\psi_e(x)\right], \tag{1}$$

where g is the weak coupling constant, $g\sin\theta_w = e$, e is the elementary electric charge θ_w is the Weinberg angle, $\gamma^L_\alpha = \gamma_\alpha\left(1+\gamma^5\right)/2$, γ_α are the Dirac matrices, $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$,

$g_V = -\frac{1}{2} + 2\sin^2\theta_w$, $g_A = -\frac{1}{2}$ for $\nu_\mu e(\nu_\tau e)$- scattering, $g_V = \frac{1}{2} + 2\sin^2\theta_w$, $g_A = \frac{1}{2}$ for $\nu_e e$- scattering [24], ψ_ν , $\psi_{\nu'}$ are the wave functions of the initial and final neutrinos, x and x' are 4-coordinates, $\overline{\psi} = \psi^+\gamma_0$,

$$D^{\alpha\beta}(x'-x) = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x'-x)} D^{\alpha\beta}(q), \tag{2}$$

$$D^{\alpha\beta}(q) = -\frac{g^{\alpha\beta} - q^\alpha q^\beta / m_z^2}{q^2 - m_z^2 + io} \tag{3}$$

is the propagator of the Z-boson with the 4-momentum $q = k - k'$, $k(k')$ is the 4-momentum of the initial (final) neutrino with the energy $\omega(\omega')$, m_z is the mass of the Z boson, $\psi_e(x)$ $(\psi_{e'}(x))$ is the exact solution of the Dirac equation for an electron in a constant uniform magnetic field [25]

$$\psi_e(x) = \frac{h^{1/4}}{L} \exp\left(-iE_{np_z}t + ip_y y + ip_z z\right)U , \tag{4}$$

$$U = \begin{bmatrix} c_1 u_{n-1}(\eta) \\ ic_2 u_n(\eta) \\ c_3 u_{n-1}(\eta) \\ ic_4 u_n(\eta) \end{bmatrix}, \tag{5}$$

where $h = eH$, H - is the strength of the magnetic field, L is the normalization length, the variable $\eta = h^{1/2}\left(x + \frac{p_y}{p}\right)$, $E_\perp = \sqrt{E^2 - p_z^2} = \sqrt{m_e^2 + 2hn}$, $E_{np_z} = E = \sqrt{m_e^2 + 2hn + p_z^2}$ is the energy of the initial electron $p_y(p_z)$ is y- component (z-component) of the momentum of the initial electron, m_e is the mass of an electron, n is the principle quantum number of the electron in a magnetic field, c_1, c_2, c_3, c_4 are the spin coefficients of the electron.

The primed corresponding notations that will be met soon belong to the final electron. It is well known that the Hermite function $u_n(\eta)$ is defined as follows

$$u_n(\eta) = \left(2^n n! \pi^{1/2}\right)^{-1/2} e^{-\eta^2/2} H_n(\eta), \tag{6}$$

where

$$H_n(\eta) = (-1)^n e^{\eta^2} \frac{d^n e^{-\eta^2}}{d\eta^n} \tag{7}$$

is the Hermite polynomial.

The gauge of a 4-potential is chosen as

$$A^{\mu} = (0,0,xH,0) \tag{8}$$

and $\mathbf{H}$ is directed along the axis Oz.

Here we have dealings with a massless neutrino. The relation $\frac{1+\gamma^5}{2}u(k) = u(k)$ is satisfied for a left-handed neutrino.

We use the pseudo-Euclidean metric with signature (+---) and the system of units where $\hbar = c = 1$.

Using (1-5) it is easy to write the amplitude

$$M = \frac{g^2}{8\cos^2\theta_w (\omega\omega')^{1/2} V} \left[\overline{u}(k')\gamma_\alpha^L u(k)\right] C_\beta(q) D^{\alpha\beta}(q), \tag{9}$$

where $u(k)$, $u(k')$ are the bispinors of the initial and final neutrinos

$$C_\beta(q) = \int d^4 x e^{-iqx} \left[\overline{\psi}_{e'}(x)\gamma_\beta\left(g_V + g_A\gamma^5\right)\psi_e(x)\right] \tag{10}$$

is the electron current. This current can be given by

$$C_\beta(q) = 2\pi\delta(E' + \omega' - E - \omega) j_\beta(\mathbf{q}), \tag{11}$$

where

$$j_\beta(\mathbf{q}) = \int d^3 x e^{-i\mathbf{qr}} j_\beta(\mathbf{r}), \tag{12}$$

$$j_\beta(\mathbf{r}) = \overline{\psi}_{e'}(\mathbf{r})\gamma_\beta\left(g_V + g_A\gamma^5\right)\psi_e(\mathbf{r}), \tag{13}$$

The calculations give the following expression for the current

$$j_\beta(\mathbf{q}) = N_0 J_\beta, \tag{14}$$

where

$$N_0 = \frac{4\pi^2}{L^2}\delta(p'_y + k'_y - p_y - k_y)\delta(p'_z + k'_z - p_z - k_z)e^{-i\alpha}e^{i(n-n')(\lambda+\pi/2)} \tag{15}$$

and J_β is the transition amplitude of the four-current for the neutrino-electron scattering in a magnetic field. The components of J_β will be given below. The structure of the components of J_β depends on the kind of the polarizations of the electrons. In the expression (15)

$$\tan\lambda = \frac{q_y}{q_x}, \tag{16}$$

$$\alpha = \frac{1}{2h}q_x(p_y + p'_y). \tag{17}$$

By using of the expressions (9), (11), (14) and (15), we represent the amplitude of the neutrino-electron scattering in the general form

$$\begin{aligned} M = {} & \frac{g^2\pi^3}{\cos^2\theta_w(\omega\omega')^{1/2}VL^2}\delta(E' + \omega' - E - \omega)\delta(p'_y + k'_y - p_y - k_y)\times \\ & \delta(p'_z + k'_z - p_z - k_z)e^{-i\alpha}e^{i(n-n')\lambda'}\left[\bar{u}(k')\gamma^L_\alpha u(k)\right]J_\beta D^{\alpha\beta}. \end{aligned} \tag{18}$$

For the low-energy limit

$$\lim_{m_z^2 >> q^2}\left[-\frac{g^{\alpha\beta} - q^\alpha q^\beta / m_z^2}{q^2 - m_z^2 + i\varepsilon}\right] = \frac{g^{\alpha\beta}}{m_z^2}. \tag{19}$$

In this case the matrix element of the process can be written

$$\begin{aligned} M = {} & \frac{8G_F\pi^3}{\sqrt{2}(\omega\omega')^{1/2}VL^2}\delta(E' + \omega' - E - \omega)\delta(p'_y + k'_y - p_y - k_y)\delta(p'_z + k'_z - p_z - k_z)e^{-i\alpha}e^{i(n-n')\lambda'}\times \\ & \times\left[\bar{u}(k')\gamma^L_\alpha u(k)\right]J^\alpha \end{aligned} \tag{20}$$

In [12] the authors derived the general expression for the cross section of the neutrino-electron scattering in dense, hot stellar matter, in the presence of strong magnetic fields. However, in [12] the authors did not investigate the polarization effects. In this work we take into account the longitudinal polarizations of the initial and final electrons. We use the standard general formalism presented in [12].

The differential probability of the process can be written in the form

$$dw = \frac{G_F^2}{(2\pi)^2 \omega\omega' V} R\delta(E' + \omega - E - \omega)\delta(p'_y + k'_y - p_y - k_y)\delta(p'_z + k'_z - p_z - k_z)dp'_y dp'_z d\mathbf{k} \ , \qquad (21)$$

where

$$R = (kJ^*)(k'J) + (k'J^*)(kJ) - (kk')(JJ^*) - i\varepsilon^{\alpha\beta\mu\nu} J^*_\alpha J_\beta k_\mu k'_\nu \qquad (22)$$

and $\varepsilon^{\alpha\beta\mu\nu}$ is the antisymmetric unit tensor.

Dividing (21) by neutrino flux equal to V^{-1} we obtain the differential cross section of the neutrino-electron scattering in the form

$$d\sigma = \frac{G_F^2}{(2\pi)^2 \omega\omega'} R\delta(E' + \omega - E - \omega)\delta(p'_y + k'_y - p_y - k_y)\delta(p'_z + k'_z - p_z - k_z)dp'_y dp'_z d\mathbf{k} \ . \qquad (23)$$

Here we have the following expression for the quantity R appearing in (23):

$$\begin{aligned} R = {} & (\omega\omega' + k_x k'_x + k_y k'_y + k_z k'_z)J_0^2 + (\omega\omega' + k_x k'_x - k_y k'_y - k_z k'_z)|J_1|^2 + (\omega\omega' - k_x k'_x + k_y k'_y - k_z k'_z)|J_2|^2 + \\ & + (\omega\omega' - k_x k'_x - k_y k'_y + k_z k'_z)J_3^2 - 2(\omega k'_x + \omega' k_x)\operatorname{Re} J_0 J_1 - 2(\omega k'_y + \omega' k_y)\operatorname{Re} J_0 J_2 - 2(\omega k'_z + \omega' k_z)J_0 J_3 + \\ & + 2(k_x k'_y + k'_x k_y)\operatorname{Re} J_1^* J_2 + 2(k_x k'_z + k'_x k_z)\operatorname{Re} J_1 J_3 + 2(k_y k'_z + k'_y k_z)\operatorname{Re} J_2 J_3 - \\ & - 2\Big[(k_y k'_z - k_z k'_y)\operatorname{Im} J_0 J_1 + (k_z k'_x - k_x k'_z)\operatorname{Im} J_0 J_2 - (\omega' k_z - \omega k'_z)\operatorname{Im} J_1^* J_2 + \\ & + (\omega' k_x - \omega k'_x)\operatorname{Im} J_2 J_3 - (\omega' k_y - \omega k'_y)\operatorname{Im} J_1 J_3\Big] \end{aligned} \qquad (24)$$

After performing averaging over the initial electron states and summation over the final electron states the differential cross section can be written in the form

$$\begin{aligned} d\sigma = {} & \frac{G_F^2 eH}{32\pi^4 \omega\omega'} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dp_z \sum_{n'=0}^{\infty} dp'_y \int_{-\infty}^{\infty} dp'_z f(1-f')R\delta(E' + \omega' - E - \omega)\delta(p'_y + k'_y - p_y - k_y)\times \\ & \times \delta(p'_z + k'_z - p_z - k_z)dp'_y dp'_z d\mathbf{k}' \end{aligned} \qquad (25)$$

where

$$f = f(E) = \left[e^{(E-\mu)/T} + 1\right]^{-1} \qquad (26)$$

is the Fermi-Dirac distribution of the initial electrons, $f' = f'(E')$ is the Fermi-Dirac distribution of the final electrons, μ is the electron chemical potential, T is the temperature of the matter.

After taking into account that $d\mathbf{k}' = \omega'^2 d\omega' d\Omega'$ the differential cross section of the neutrino-electron scattering can be given by the following general expression

$$\frac{d\sigma}{d\omega' d\Omega'} = \frac{G_F^2 eH\omega'}{32\pi^4 \omega} \sum_{n,n'=0}^{\infty} \int_{-\infty}^{\infty} dp_z \delta(E' + \omega' - E - \omega) f(1 - f') R . \quad (27)$$

III. The case of longitudinal polarizations of the electrons

In the case of longitudinal polarization of the electrons we have dealings with the generalized helicity operator [25]

$$\mathbf{\Sigma} \cdot \mathbf{P} = \gamma^5 \left(m_e \gamma^0 - E\right), \quad (28)$$

$$(\mathbf{\Sigma} \cdot \mathbf{P})\psi = \zeta p \psi , \quad (29)$$

where $p = \left(E^2 - m_e^2\right)^{1/2}$ and the value $\zeta = +1(-1)$ corresponds to right-hand (left-hand) helicity.

In the case of longitudinal polarization of the electrons the spin coefficients are [25]

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} A_1 B_1 \\ A_1 B_2 \\ A_2 B_1 \\ A_2 B_2 \end{pmatrix}, \quad (30)$$

where

$$A_1 = \left(1 + \frac{m_e}{E}\right)^{1/2}, \quad A_2 = \zeta \left(1 - \frac{m_e}{E}\right)^{1/2}, \quad (31)$$

$$B_1 = \left(1 + \zeta \frac{p_z}{\sqrt{E^2 - m_e^2}}\right)^{1/2}, \quad B_2 = \zeta \left(1 - \zeta \frac{p_z}{\sqrt{E^2 - m_e^2}}\right)^{1/2}. \quad (32)$$

By means of (4), (5), (12-15) and (30-32) we find the general expressions for the current in the neutrino-electron scattering in a magnetic field with allowance for the longitudinal polarizations of the electrons

$$J_\beta = \begin{pmatrix} (g_V \tilde{s}_0 + g_A \tilde{s})\tilde{f}_0 \\ (g_V \tilde{s} + g_A \tilde{s}_0)\tilde{f}_1 \\ (g_V \tilde{s} + g_A \tilde{s}_0)\tilde{f}_2 \\ (g_V \tilde{s} + g_A \tilde{s}_0)\tilde{f}_3 \end{pmatrix}, \tag{33}$$

where

$$\begin{aligned} \tilde{f}_0 &= B_2 B_2' I_{nn'} + B_1 B_1' I_{n-1,n'-1}, \\ \tilde{f}_1 &= B_1' B_2 e^{i\lambda} I_{n,n'-1} + B_1 B_2' e^{-i\lambda} I_{n-1,n'}, \\ \tilde{f}_2 &= -iB_1' B_2 e^{i\lambda} I_{n,n'-1} + iB_1 B_2' e^{-i\lambda} I_{n-1,n'}, \\ \tilde{f}_3 &= B_2 B_2' I_{nn'} - B_1 B_1' I_{n-1,n'-1} \end{aligned} \tag{34}$$

and

$$\begin{aligned} \tilde{s}_0 &= \frac{1}{4}(A_1 A_1' + A_2 A_2'), \\ \tilde{s} &= -\frac{1}{4}(A_1 A_2' + A_1' A_2). \end{aligned} \tag{35}$$

If we take into account the components of the current J_β in the formulae (24-27), we obtain the general expression for the differential cross section of the neutrino-electron scattering in a magnetic field with allowance for the longitudinal polarizations of the initial and final electrons

$$\frac{d\sigma}{d\omega' d\Omega'} = \frac{G_F^2 eH\omega'}{32\pi^4 \omega} \sum_{n,n'=0}^{\infty} \int_{-\infty}^{\infty} dp_z \delta(E' + \omega' - E - \omega) f(1 - f')R, \tag{36}$$

where

$$R=\omega\omega'\Big[d_1\Big(r_1I_4^2+r_2I_3^2+2r_3I_3I_4\Big)+2d_2\Big(r_4I_1^2+r_5I_2^2\Big)+d_3\Big(r_6I_4^2+r_7I_3^2+2r_8I_3I_4\Big)+4d_4r_9I_1I_2-$$
$$-2d_5\Big(r_{10}I_4^2-r_{11}I_3^2\Big)+2d_6\Big(r_4I_1^2-r_5I_2^2\Big)-2d_7\Big(r_{12}I_1I_4+r_{13}I_2I_4+r_{14}I_1I_3+r_{15}I_2I_3\Big)-$$
$$-2d_8\Big(r_{16}I_1I_4-r_{17}I_1I_3-r_{18}I_2I_4+r_{19}I_2I_3\Big)+2d_9\Big(r_{16}I_1I_4-r_{17}I_1I_3+r_{18}I_2I_4-r_{19}I_2I_3\Big)+$$
$$+2d_{10}\Big(r_{12}I_1I_4-r_{13}I_2I_4+r_{14}I_1I_3-r_{15}I_2I_3\Big)\Big], \quad (37)$$

$$r_1=\frac{1}{4}[g_+(\mu_1-\mu_2\zeta-\mu_3\zeta'+\mu_4\zeta\zeta')+2g_\perp(\mu_5-\mu_6\zeta-\mu_7\zeta'+\mu_8\zeta\zeta')],$$
$$r_2=\frac{1}{4}[g_+(\mu_1+\mu_2\zeta+\mu_3\zeta'+\mu_4\zeta\zeta')-2g_\perp(\mu_5+\mu_6\zeta+\mu_7\zeta'+\mu_8\zeta\zeta')],$$
$$r_3=\frac{1}{4}g_+\delta\delta'\beta\beta'\zeta\zeta',$$
$$r_4=\frac{1}{8}[g_+(\mu_1'-\mu_2'\zeta+\mu_3'\zeta'+\mu_4'\zeta\zeta')-g_-\delta\delta'(1-\upsilon\zeta+\upsilon'\zeta'-\upsilon\upsilon'\zeta\zeta')-2g_\perp(-\mu_5'+\mu_6'\zeta+\mu_7'\zeta'+\mu_8'\zeta\zeta')],$$
$$r_5=\frac{1}{8}[g_+(\mu_1'+\mu_2'\zeta+\mu_3'\zeta'+\mu_4'\zeta\zeta')-g_-\delta\delta'(1+\upsilon\zeta-\upsilon'\zeta'-\upsilon\upsilon'\zeta\zeta')+2g_\perp(\mu_5'-\mu_6'\zeta-\mu_7'\zeta'+\mu_8'\zeta\zeta')],$$
$$r_6=\frac{1}{4}g_-\delta\delta'(1-\upsilon\zeta-\upsilon'\zeta'+\upsilon\upsilon'\zeta\zeta'),$$
$$r_7=\frac{1}{4}g_-\delta\delta'(1+\upsilon\zeta+\upsilon'\zeta'+\upsilon\upsilon'\zeta\zeta'),$$
$$r_8=\frac{1}{4}[g_+\beta\beta'(\sigma\sigma'+\zeta\zeta')-2g_\perp(\nu_8\zeta'+\nu_8'\zeta)],$$
$$r_9=\frac{1}{8}[g_+\beta\beta'(\sigma\sigma'+\zeta\zeta')-g_-\delta\delta'\beta\beta'\zeta\zeta'-2g_\perp(\nu_8\zeta'+\nu_8'\zeta)],$$
$$r_{10}=\frac{1}{8}[g_+(\mu_5-\mu_6\zeta-\mu_7\zeta'+\mu_8\zeta\zeta')+2g_\perp(\mu_1-\mu_2\zeta-\mu_3\zeta'+\mu_4\zeta\zeta')],$$
$$r_{11}=-\frac{1}{8}[g_+(\mu_5+\mu_6\zeta+\mu_7\zeta'+\mu_8\zeta\zeta')-2g_\perp(\mu_1+\mu_2\zeta+\mu_3\zeta'+\mu_4\zeta\zeta')],$$
$$r_{12}=-\frac{1}{8}[g_+(\nu_1'-\nu_5'\zeta-\nu_5\zeta'+\nu_2'\zeta\zeta')+2g_\perp(\nu_7'-\nu_3'\zeta-\zeta'+\nu_4'\zeta\zeta')],$$
$$r_{13}=-\frac{1}{8}[g_+(\nu_1-\nu_6'\zeta-\nu_6\zeta'+\nu_2\zeta\zeta')+2g_\perp(\nu_7-\zeta-\nu_3\zeta'+\nu_4\zeta\zeta')], \quad (38)$$
$$r_{14}=-\frac{1}{8}[g_+(\nu_1+\nu_6'\zeta+\nu_6\zeta'+\nu_2\zeta\zeta')-2g_\perp(\nu_7+\zeta+\nu_3\zeta'+\nu_4\zeta\zeta')],$$
$$r_{15}=-\frac{1}{8}[g_+(\nu_1'+\nu_5'\zeta+\nu_5\zeta'+\nu_2'\zeta\zeta')-2g_\perp(\nu_7'+\nu_3'\zeta+\zeta'+\nu_4'\zeta\zeta')],$$
$$r_{16}=\frac{1}{8}[g_+(-\nu_7'+\nu_3'\zeta+\zeta'-\nu_4'\zeta\zeta')-g_-\delta\delta'\beta'\zeta'(1-\zeta\upsilon)-2g_\perp(\nu_1'-\nu_5'\zeta-\nu_5\zeta'+\nu_2'\zeta\zeta')],$$
$$r_{17}=\frac{1}{8}[g_+(\nu_7+\zeta+\nu_3\zeta'+\nu_4\zeta\zeta')-g_-\delta\delta'\beta\zeta(1+\zeta'\upsilon')-2g_\perp(\nu_1+\nu_6'\zeta+\nu_6\zeta'+\nu_2\zeta\zeta')],$$
$$r_{18}=\frac{1}{8}[g_+(-\nu_7+\zeta+\nu_3\zeta'-\nu_4\zeta\zeta')-g_-\delta\delta'\beta\zeta(1-\zeta'\upsilon')-2g_\perp(\nu_1-\nu_6'\zeta-\nu_6\zeta'+\nu_2\zeta\zeta')],$$

$$r_{19}=\frac{1}{8}[g_+(\nu'_7+\nu'_3\zeta+\zeta'+\nu'_4\zeta\zeta')-g_-\delta\delta'\beta'\zeta'(1+\zeta\upsilon)-2g_\perp(\nu'_1+\nu'_5\zeta+\nu_5\zeta'+\nu'_2\zeta\zeta')],$$

$$\begin{aligned}&\mu_1(\mu'_1)=1\pm\sigma\sigma'\upsilon\upsilon',\ \mu_2(\mu'_2)=\upsilon\pm\upsilon'\sigma\sigma',\\&\mu_3(\mu'_3)=\upsilon'\pm\upsilon\sigma\sigma',\ \mu_4(\mu'_4)=\sigma\sigma'\pm\upsilon\upsilon',\\&\mu_5(\mu'_5)=\sigma\upsilon\pm\sigma'\upsilon',\ \mu_6(\mu'_6)=\sigma\pm\sigma'\upsilon\upsilon',\\&\mu_7(\mu'_7)=\sigma'\pm\sigma\upsilon\upsilon',\ \mu_8(\mu'_8)=\sigma\upsilon'\pm\sigma'\upsilon,\end{aligned}\tag{39}$$

$$\begin{aligned}&\nu_1=\sigma\beta\ ,\ \nu'_1=\sigma'\beta',\ \nu_2=\sigma'\beta,\ \nu'_2=\sigma\beta',\ \nu_3=\sigma\sigma'\beta,\ \nu'_3=\sigma\sigma'\beta',\ \nu_4=\upsilon'\beta,\ \nu'_4=\upsilon\beta',\\&\nu_5=\sigma\upsilon\beta',\ \nu'_5=\sigma'\upsilon\beta',\ \nu_6=\sigma\upsilon'\beta,\nu'_6=\sigma'\upsilon'\beta,\nu_7=\sigma\sigma'\upsilon'\beta,\ \nu'_7=\sigma\sigma'\upsilon\beta',\\&\nu_8=\beta\beta'\sigma,\ \nu'_8=\beta\beta'\sigma',\ \nu_9=\beta\beta'\sigma\sigma'.\end{aligned}\tag{40}$$

İn (37-40) $g_\pm=g_V^2\pm g_A^2$, $g_\perp=g_Vg_A$

and

$$\upsilon=p_z/p\ ,\ \upsilon'=p'_z/p'\ ,\ \delta=m_e/E\ ,\ \delta'=m_e/E'\ ,$$

$$\beta=(1-\upsilon^2)^{1/2},\ \beta'=(1-\upsilon'^2)^{1/2},\tag{41}$$

$$\sigma=(1-\delta^2)^{1/2},\ \sigma'=(1-\delta'^2)^{1/2},$$

$$I_1=I_{n,n'-1},\ I_2=I_{n-1,n'},\ I_3=I_{n-1,n'-1},\ I_4=I_{nn'}\tag{42}$$

where

$$I_{nn'}(x)=(n'!/n!)^{1/2}e^{-x/2}x^{(n-n')/2}L_{n'}^{n-n'}(x);\tag{43}$$

is the Laguerre function.

Averaging over the initial electron polarization and summation over the final electron polarization in (36) leads to the result similar to that obtained in [12].

IV. Discussion and conclusions

We have derived the general expression for the cross section of the neutrino-electron scattering in a magnetic field with allowance for the longitudinal polarizations of the initial and final electrons.

Using (36-40) we can come to the conclusion that the cross sections of the neutrino-electron scattering for the left-handed electrons and for the right-handed electrons are different. It means that the left-handed electrons and the right-handed electrons are not heated by the neutrinos equally. It leads to the anisotropy in heating of the stellar matter. The analyses of the cross section show that the final neutrinos and electrons are emitted asymmetrically.

The expression for the cross section obtained in this paper can be applied in the future in explanation of anisotropies and asymmetries arising in astrophysical phenomena connected with the neutrino-electron scattering in a magnetic field.

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REFERENCES

[1] M. Rampp and H. T. Janka, astro-ph/ 0005438.

[2] S. W. Bruenn, Astrophys. J. Suppl. **58**, 771 (1985).

[3] S. W. Bruenn, Astrophys. Space Sci. **143**, 15 (1988).

[4] E. S. Myra, Phys. Reports **163**, 127 (1988).

[5] A. Mezzacappa and S. W. Bruenn, Astrophys. J. **410**, 740 (1993).

[6] J. LeBlanc and J. R. Wilson, Astrophys. J. **161**, 541 (1970).

[7] G. S. Bisnovatiy – Kogan, Yu. P. Popov and A. A. Samochin, Astrophys. Space Sci. **41**, 287 (1976).

[8] E. Mueller and H. Hellebrandt, Astron. Astrophys. **80**, 147 (1979).

[9] V. M. Lipunov, Astrophysics of Neutron Stars, Heldelberg, Springer-Verlag, (1987).

[10] G. G. Raffelt, Stars as Laboratories for Fundamental Physics, Univ. of Chicago Press, Chicago, (1996).

[11] C. J. Benesh and C. J. Horowitz, astro-ph/ 9708033.

[12] V. G. Bezchastnov and P. Haensel, Phys. Rev. D **54,** 3706 (1996).

[13] N. N. Chugai, Sov. Astron. Lett. **10**, 87 (1984).

[14] A. Vilenkin, Ap. J **451**, 700 (1995).

[15] A. V. Borisov, V. A. Guseinov, and O. S. Pavlova, Yad. Fiz. **61**, 103 (1998); Phys. At. Nucl. **61**, 94 (1998).

[16] A. V. Borisov, V. A. Guseinov and N. B. Zamorin, Yad. Fiz. **63**, 2041 (2000); Phys. At. Nucl. **63**, 1949 (2000).

[17] V. A. Lyul' ka, Yad. Fiz. **39**, 680 (1984); Sov. J. Nucl. Phys. **39**, 431 (1984).

[18] V. M. Zakhartsov, Y. M. Loskutov and K. V. Parfenov, Teor. Mat Fiz. (Theoretical and Mathematical Physics) **81**, 215 (1989).

[19] V. A. Guseinov, J. Phys. G, **26**, 1313 (2000).

[20] W. R. Yueh and R. Buchler, Astrophys. Sp. Sci. **39**, 429 (1976).

[21] P. Schinder and S. L. Shapiro, Astrophys. J. Suppl. **50**, 23 (1982).

[22]A. V. Borisov, M. K. Nanaa, I.M. Ternov, Vestnik Moskovskogo universiteta. Fizika. Astronomiia, **48(2)**, 15(1993).

[23] A. V. Kuznetsov and N. V. Mikheev, Phys. Lett. B **394**, 123 (1997).

[24] L. B. Okun, Leptons and Quarks, Nauka, Moscow, (1990); North-Holland, Amsterdam, (1984).

[25] A. A. Sokolov and I. M. Ternov, Radiation from Relativistic Electrons, Nauka, Moscow, (1983); New York, AIP, (1986).