D-branes and microscopic description of SYM and SUGRA solitons.

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Outline:

I) Introduction to string theory
  1) Bosonic string (general idea of string theory) A. Polyakov „Gauge fields and strings"
  2) Superstrings

II) Definition of D-branes & their physics.
J. Polchinski

III) D-branes as probes
  1) Probing SUGRA solitons
  Douglas, Polchinski & Strominger
  Akhmedov
  2) Probing SYM solitons
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  Akhmedov

IV) Non-Abelian structures in open string theory and background independence
Akhmedov, Gerasimov & Shatashvili

V) On unification of RR couplings.

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VI) Conclusions

I) Introduction into string theory

1) Action

$$S_{NG} = \boxed{T} \cdot Area = \boxed{T} \int_{\Sigma} d^2\sigma \sqrt{\det(\partial_a X_\mu \partial_b X^\mu)}$$

$$a, b = \overline{1,2}$$
$$\mu = \overline{0, D-1}$$

$$S = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2\sigma \sqrt{h}\, h^{ab} \partial_a X_\mu \partial_b X^\mu$$

$$h^{ab} h_{bc} = \delta^a{}_c$$

On the classical trajectories:

$$h_{ab} \propto \partial_a X^\mu \partial_b X_\mu$$
$$\Downarrow$$
$$S \propto S_{NG}$$

Symmetry: $\sigma_a \longrightarrow f_a(\sigma_b) \Rightarrow$

$$\Rightarrow \quad h_{ab} = \delta_{ab}\, e^{\varphi(\sigma)}$$

$$ds^2 = h_{ab}\, d\sigma^a d\sigma^b = e^{\varphi(z, \bar{z})} dz\, d\bar{z}$$

Quantization of the theory is to find:

$$Z(g_{\mu\nu}, B_{\mu\nu}, \Phi, T, \ldots) = \sum_{g=0}^{\infty} \int DX_\mu \, [\text{moduli}]_g \times$$

$$\times \exp\Big\{ -\frac{1}{2\pi\alpha'}\Big[\int d^2z \, \partial X_\mu \bar\partial X^\mu -$$

$$- \int : g_{\mu\nu}(X) \, \partial X^\mu \bar\partial X^\nu : d^2z -$$

$$- \int : B_{\mu\nu}(x) \, \partial X^\mu \bar\partial X^\nu : d^2 z \Big] -$$

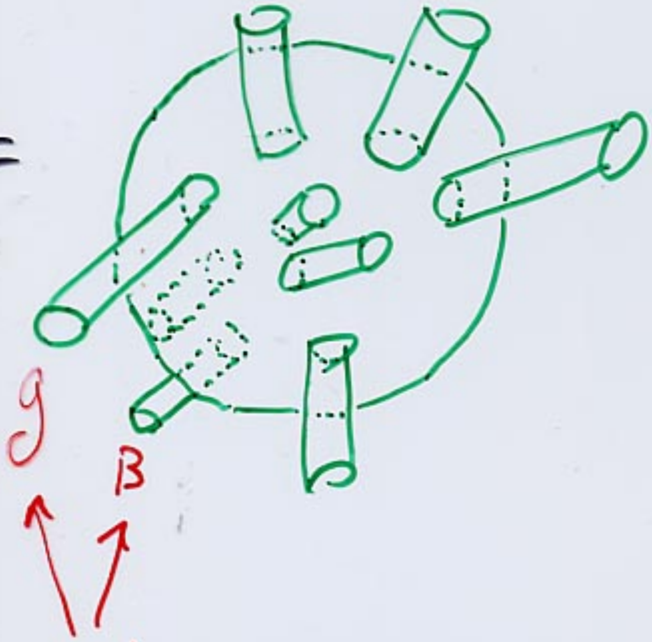
$$- \int : \underbrace{R\sqrt{h}}_{\overset{\shortparallel}{\partial^2\varphi(z,\bar z)}} \, \Phi(x) : d^2z - \frac{1}{2\pi\alpha'}\int : \underset{\overset{\shortparallel}{e^{\varphi(z,\bar z)}}}{\sqrt{h}} \, T(x) : d^2z + \ldots \Big\}$$

$$\ldots \Leftrightarrow \int_{\{\varphi\}} \sum J_{\{q\}} \cancel{(z,\bar z)} \int G_{\{q\}}[\varphi, X] \, d^2 z$$

$$\boxed{:X(z)X(w): \equiv X(z)X(w) + \underset{\substack{\| \\ -G(z,w) \\ \| \\ \langle X(z)X(w)\rangle}}{\ln|z-w|^2}}$$

$$\frac{\delta^{n_1}}{\delta b^{n_1}(k)}\,\frac{\delta^{n_2}}{\delta g^{n_2}(k)}\cdots \underset{\text{Sphere}}{Z}(g, B, \Phi, T, \dots) =$$

$=$



states with definite momenta k in the target space

$=$

$\|$

Operators — States

conformal weights — masses:

$$\left\langle \prod_{i=1}^{n} \int d^2 z_i :\mathcal{O}_i(z_i): \right\rangle =$$

$$= \left\langle \int d^2 z_1 :\mathcal{O}_1(z_1): \int d^2 z_2 :\mathcal{O}_2(z_2): \cdot \right.$$

$$\left. \cdot \prod_{i=\boxed{3}}^{n} \int d^2 z_i :\mathcal{O}_i(z_i): \right\rangle \underset{\boxed{z_1 \to z_2}}{\simeq}$$

$$\simeq \left\langle \overset{\boxed{a}}{\int} d^2 \eta \int d^2 z_2 :\mathcal{O}_1(z_2)::\mathcal{O}(z_2+\eta): \cdot \right.$$

$$\left. \cdot \prod_{i=\boxed{3}}^{n} \int d^2 z_i :\mathcal{O}_i(z_i): \right\rangle + \text{less singular terms}$$

$$= \sum_q C_{12q}^{\neq A_{12q}} \int d^2 \eta \, |\eta|^{-4+\Delta_q} \left\langle \prod_{i=\boxed{2}}^{n} \int d^2 z_i \right.$$

$$\left. :\mathcal{O}_i(z_i): \right\rangle + \ldots =$$

$$= \sum_q A_{12q} A_{q[n]} \frac{1}{-2+\Delta_q} + \ldots$$

$$Z_{\text{sphere}}(G_{\mu\nu} = \delta_{\mu\nu} + g_{\mu\nu}, B_{\mu\nu}, \Phi, T) \propto$$

$$\propto \frac{1}{g_s^2 \alpha'^{12}} \int d^{26}x \sqrt{G}\, e^{-2\Phi} \times$$

$$\times \left[R(G) + 4(\partial_\mu \Phi)^2 - \frac{1}{12}(\partial_{[\mu} B_{\nu\lambda]})^2 + \right.$$

$$\left. + \frac{1}{2}(\partial_\mu T)^2 + \frac{1}{2} m^2 T^2 \right] +$$

$$+ \boxed{\propto \frac{1}{g_s^2 \alpha'^{12}} \alpha' R^2}$$ – σ-model corrections

$$\boxed{g_s = e^{\langle \Phi \rangle}}$$

Tachyonic problems ⇒

⇒ superstrings

2) NSR action:

$$S_{NSR} = \frac{1}{2\pi\alpha'}\int d^2z\Big[\partial x_\mu \partial x^\mu - i\,\psi^\mu \partial \psi_\mu + \text{c.c.}\Big] + \text{ghosts}$$

Vertex operators are the same as in bosonic string (apart from tachyon), plus fermions and bosonic RR states:

$$\sum_n \underset{\text{„sources"}}{\underbrace{F_{\mu\nu\dots\mu_n}}}(x)\ \tilde{Q}\,(\underset{\text{charge conjugation matrix}}{C}\gamma)^{\mu_1\dots\mu_n}\,Q$$

$\tilde{Q}$, Q: supercharges

$$\underset{IIB}{Z}(G, B, \Phi, \{C\}, \text{fermions}) \propto$$

$$\propto \frac{1}{g_s^2\alpha'^4}\int d^{10}x\sqrt{G}\,\Big\{e^{-2\Phi}\Big[R(G) + 4(\partial\Phi)^2 - \frac{1}{12}(\partial B)^2 - e^{-\Phi}\Big[F_\mu^2 + F_{\mu_1\mu_2\mu_3}^2 + \dots\Big]\Big\};$$

II) Open strings and D-branes

Boundary conditions:

$$\partial_{\vec{n}} X_m = 0 \quad (N) \qquad X_\mu = (X_m, X_i)$$

$$\partial_{\vec{t}} X_i = 0 \quad (D) \qquad m = \overline{0,p}$$

$$\Downarrow \qquad\qquad i = \overline{p+1,9}$$

$$X_i = \text{const}$$

$$Z_{\underset{\text{Disc}}{op}}\left(G_{\mu\nu}, B_{\mu\nu}, \bar{\Phi}, A_m, \phi_i, \ldots\right) \equiv \int_{\substack{\text{Bound}\\ \text{cond.}}} DX_\mu \cdot$$

$$\cdot \exp\left\{ -\frac{1}{2\pi\alpha'} \int_{\text{Disc}} d^2 z \left[\ldots G, B, \bar{\Phi} \right] - \right.$$

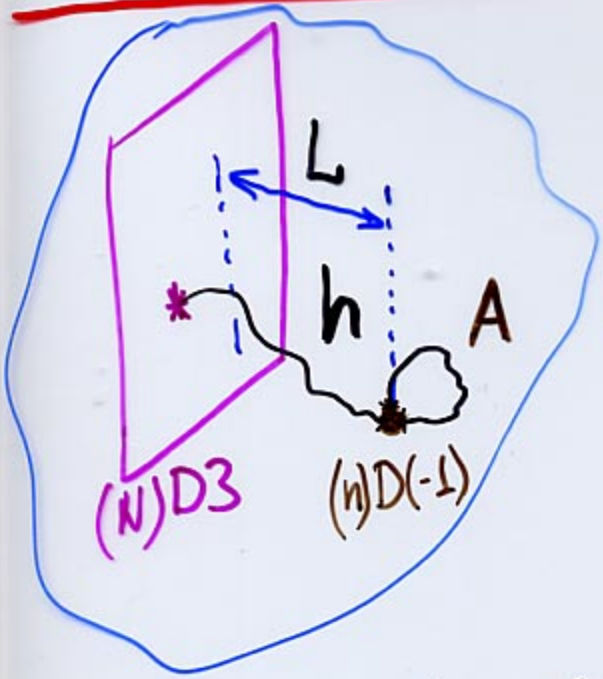
$$- i \oint_{\text{Boundary}} A_m(X_n)\, \partial_{\vec{t}} X_m \, d\tau -$$

$$\left. - \oint_{\text{Boundary}} \phi_i(X_n)\, \partial_{\vec{n}} X_i \, d\tau + \ldots \right\} \propto$$

$$\propto \frac{\alpha'^{\frac{p+1}{2}}}{g_s} \int d^{p+1}x \, e^{-\Phi} \sqrt{\det\left[g_{mn} + b_{mn} + \alpha' F_{mn}\right]}$$

$$+ \frac{\text{const}}{g_s} \int C_{RR} \wedge e^{-(F + B/\alpha')} + \ldots$$

III) D-branes as probes



D3: $\overline{0,3}$

1) Probing GR solitons

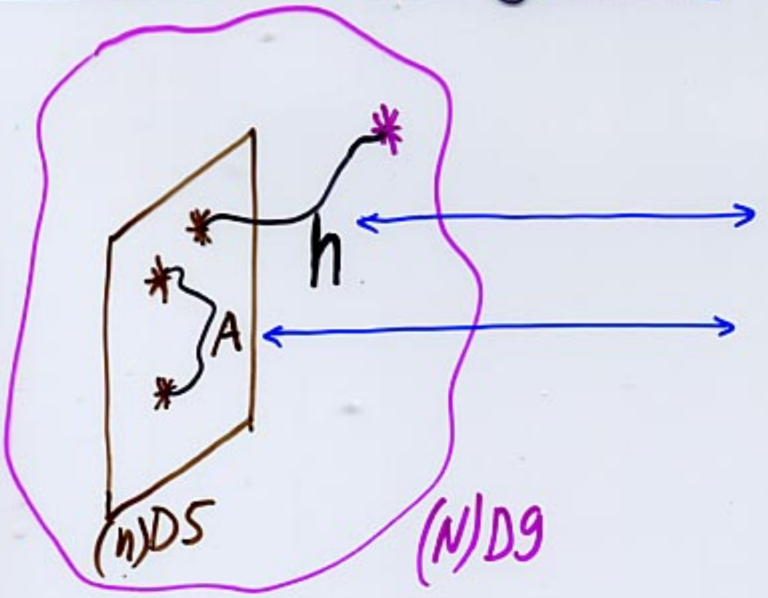
$$m_h \sim L/\alpha'$$

$$S_{inst} = \frac{\text{const}\,\alpha'^2}{g_s}\,\mathrm{tr}\Big\{-G_{\mu\mu'}(A)\,G_{\nu\nu'}(A)\times$$

$$\times\,[A^\mu, A^\nu][A^{\mu'}, A^{\nu'}]\Big\} + \text{superpartners} + O\left(\mathrm{tr}[\,,\,]^3\right),$$

$$A \equiv \mathrm{Diag}\left(A^2_\mu\right).$$

D5 - D9 - brane system:



$h_p^{\dot\alpha}, \rho_p^{\mathcal{A}}$ (c.c.)

A_μ, φ_m

$\lambda_{\dot\alpha}^{\mathcal{A}}, \Psi_\alpha^{\mathcal{A}}$

$$SO(10) \longrightarrow \underset{\substack{m \\ (\alpha,\dot\alpha)}}{SO(4)} \times \underset{\substack{\mu \\ (\mathcal{A})\,(\mathrm{Spin}(6))}}{SO(6)}$$

$$S \sim \frac{1}{g_s\alpha'}\int d^6x \; tr_{(n)} \Big\{ \frac{1}{2} F_{\mu\nu}^2 - \sqrt{2}\,\pi\, \overline{\lambda}_{\dot\alpha}^{\mathcal{A}}\, \hat{D}^{Ad}_{\mathcal{A}\mathcal{B}} \lambda^{\dot\alpha\mathcal{B}} +$$

$$+ \left| D_\mu^{Ad} \varphi_m \right|^2 - \sqrt{2}\,\pi\, \overline{\Psi}^{\alpha\mathcal{A}} \hat{D}^{Ad}_{\mathcal{A}\mathcal{B}} \Psi_\alpha^{\mathcal{B}} +$$

$$+ \frac{1}{2} \left| [\varphi_n, \varphi_m]\, \vec{\eta}_{nm} + \overline{h}_p^{\dot\alpha} h_p^{\dot\beta}\, \vec{\tau}_{\dot\alpha\dot\beta} \right|^2 +$$

$$+ i\pi \left[\varphi_{\alpha\dot\alpha}, \Psi^{\alpha\mathcal{A}} \right] \lambda_{\mathcal{A}}^{\dot\alpha} + \left| D_m^F h_p^{\dot\alpha} \right|^2 -$$

$$- 2\sqrt{2}\, \overline{\rho}_p^{\mathcal{A}} \hat{D}^F_{\mathcal{A}\mathcal{B}} \rho_p^{\mathcal{B}} + i\pi\, \overline{\rho}_p^{\mathcal{A}} h_{p\dot\alpha} \lambda_{\mathcal{A}}^{\dot\alpha} + \text{c.c.} \Big\}$$

Reduction $\Rightarrow$ D(-1) - D3 - brane system.

Reduction to D(-1)-D3-brane system:

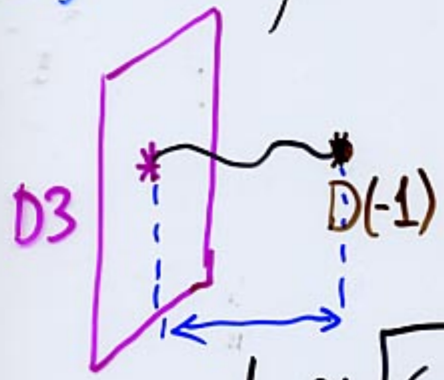
$A_\mu(x) \longrightarrow A_\mu$ etc. $\int d^6x \longrightarrow \alpha'^3$

$$F_{\mu\nu}(x) \longrightarrow [A_\mu, A_\nu]$$

$$\hat{D}_{AB} \longrightarrow A_\mu \gamma^\mu_{AB}$$

We put all D3-branes at $r \equiv \sqrt{x^m x_m} = 0$

If $\langle A_\mu^2 \rangle \neq 0$, hence:



$$L \sim \sqrt{\langle A_\mu^2 \rangle}$$

We take $\sum_\mu \left(A_\mu^2\right)_{ij} = \frac{r_i^2}{\alpha'}\delta_{ij}$ $i = \overline{1,n}$

Integrating out "massive" h & ρ, when background A & Ψ, fermions are slightly non-diagonal we obtain (as $r_i^2 \to r, \forall i$):

$$S^{(eff)}_{inst} = \frac{const\, \alpha'^2}{g_s} \left\{ -\frac{1}{2} \left(1 + \frac{R^4}{r^4} \right)^{-1} tr[A_\mu, A_\nu]^2 - \right.$$

$$\left. - tr[A_\mu, \varphi_m]^2 + \frac{1}{2} \left(1 + \frac{R^4}{r^4} \right)^{+1} tr[\varphi_m, \varphi_n]^2 \right\} +$$

$$+ O(tr[\,,\,]^3)$$, where $R^4 \sim g_s N \alpha'^2$

Hence, the metric of the D3-brane is:

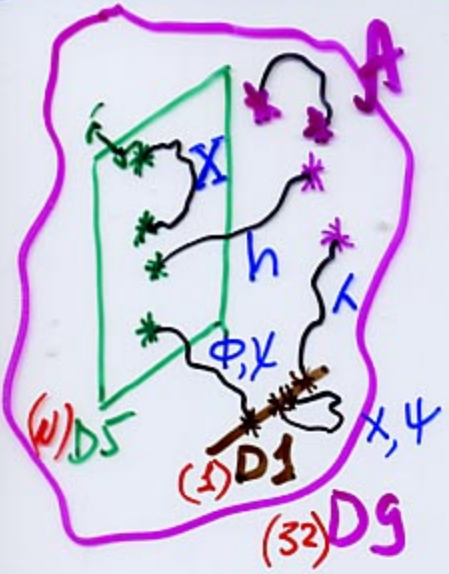
$$ds^2_{D3} = \left(1 + \frac{R^4}{r^4} \right)^{-1/2} d\tilde{\varphi}_m \, d\tilde{\varphi}_m +$$

$$+ \left(1 + \frac{R^4}{r^4} \right)^{1/2} d\tilde{A}_\mu \, d\tilde{A}_\mu$$, where

$$r^2 \equiv \tilde{A}_\mu^2 .$$

2) Probing SYM solitons

a) D1-brane probing D5-brane in type I ST



D1 : $\overline{0,1}$

D5 : $\overline{0,5}$

$$SO(1,9) \to \underset{(+\ -)}{SO(1,1)} \times \underset{(\alpha,\dot\alpha)}{SO_\ell(4)} \times \underset{(\xi,\dot\xi)}{SO_t(4)}$$

D1 − D1 :

- $X_{\xi\dot\xi}$, $\psi_-^{\alpha\dot\xi}$ – transverse to D5
- $X_{\alpha\dot\alpha}$, $\psi_-^{\dot\alpha\xi}$ – along D5 $(X_{\alpha\dot\alpha} \equiv X_m \sigma^m_{\alpha\dot\alpha})$

D1 − D5 :

- $\Phi^{\dot\alpha a}$ $\chi_-^{\xi a}$
- $\chi_+^{\dot\xi a}$; $a = \overline{1,2N}$ $(USp(2N))$

D1 − D9 : λ_+^n

D5 − D5 : $X^{ab}_{\xi\dot\xi}$, superpartners & gauge fields

D5 − D9 : h^{na}_ξ , superpartners.

$$\langle h^n_\xi h^\xi_n \rangle = \rho$$

D1-brane Lagrangian:

$$\mathcal{L} = \partial_{++} x^{\alpha\dot\alpha}\, \partial_{--} x_{\alpha\dot\alpha} + \partial_{++} x^{\zeta\dot\zeta}\, \partial_{--} x_{\zeta\dot\zeta} +$$

$$+ \psi_-^{\alpha\dot\zeta}\, \partial_{++} \psi_{-\alpha\dot\zeta} + \psi_-^{\dot\alpha\zeta}\, \partial_{++} \psi_{-\dot\alpha\zeta} + \lambda_+^{n}\, \partial_{--} \lambda_+^{n} +$$

$$+ \partial_{++} \phi^{\alpha a}\, \partial_{--} \phi_\alpha^{a} + \chi_-^{\zeta a}\, \partial_{++} \chi_{-\zeta}^{a} + \chi_+^{\dot\zeta a}\, \partial_{--} \chi_{+\dot\zeta}^{a} +$$

$$+ \phi^{\alpha a} \phi_\alpha^{b} \left(\mathbf{X}^{ac} - x\, \delta^{ac}\right)_{\zeta\dot\zeta} \left(\mathbf{X}^{cb} - x\, \delta^{cb}\right)^{\zeta\dot\zeta} +$$

$$+ \chi_-^{\zeta a} \chi_+^{\dot\zeta b} \left(\mathbf{X}^{ab}_{\zeta\dot\zeta} - x_{\zeta\dot\zeta}\, \delta^{ab}\right) + \psi_-^{\alpha\dot\zeta} \chi_{+\dot\zeta}^{a} \phi_\alpha^{a} +$$

$$+ \mathbf{h}^{na}_{\zeta}\, \lambda_+^{n} \chi_-^{\zeta a} + \mathbf{h}^{na}_{\zeta}\, \mathbf{h}^{nb\zeta}\, \phi^{\alpha a} \phi_\alpha^{b} .$$

$\mathbf{X}$ & $\mathbf{h}$ are external quasiclassical fields from the point of view of the D1-brane Lagrangian.

$$\mathbf{h}^{na}_{\zeta}\, \mathbf{h}^{nb}_{\eta} + \epsilon^{cd} \epsilon^{\dot\zeta\dot\eta}\, \mathbf{X}^{ac}_{\zeta\dot\zeta}\, \mathbf{X}^{db}_{\eta\dot\eta} = 0 \quad \textcircled{*}$$

RG-flow

On a generic point of the D5-brane moduli space (arbitrary $\mathbf{X}$ & h) we obtain:

Φ are massive

χ_- are massive

x & Ψ are massless

Among χ_+ & λ_+ massless are given by:

$$\left(\mathbf{X}^{ab}_{\xi\dot{\xi}} - x_{\xi\dot{\xi}}\,\delta^{ab}\right)\chi_+^{\dot{\xi}b} + \mathbf{h}^{na}_{\xi}\,\lambda_+^{n} = 0$$

$\Updownarrow$ or $\lambda_+^{P} = (\chi_+^{\dot{\xi}b}, \lambda_+^{n})$

$$\Delta^{aP}_{\xi}\,\lambda_{+P} = 0 \qquad \boxed{**}$$

$$\Delta^{aP}_{\xi}\,\Delta^{b}_{\eta P} = \delta_{\xi\eta}\,\delta^{ab} f^{-1}(x) \Longleftrightarrow \boxed{*}$$

There are 32 solutions to $\boxed{**}$: $V^{n}_{p}(x)$

$$\lambda_+^{P} = \sum_{n=1}^{32} V_n^{P}(x)\,\lambda_+^{n} \qquad \boxed{***}$$

Integrating out massive modes and substituting $\boxed{***}$ into our Lagrangian, we obtain:

$$\mathcal{L} = \partial_{++} X^{\alpha\dot{\alpha}} \partial_{--} X_{\alpha\dot{\alpha}} + \partial_{++} X^{\varsigma\dot{\varsigma}} \partial_{--} X_{\varsigma\dot{\varsigma}} +$$

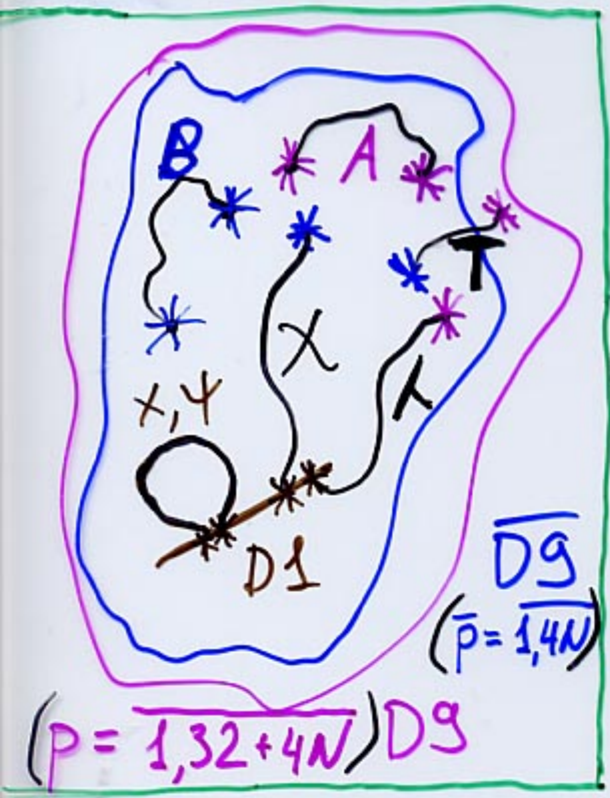
$$+ \psi_{-}^{\alpha\dot{\varsigma}} \partial_{++} \psi_{-\alpha\dot{\varsigma}} + \psi_{-}^{\dot{\alpha}\varsigma} \partial_{++} \psi_{-\dot{\alpha}\varsigma} +$$

$$+ \tilde{\lambda}_{+}^{n} \left(\partial_{--} \delta^{nm} + \partial_{--} X^{\varsigma\dot{\varsigma}} A^{nm}_{\varsigma\dot{\varsigma}}(X) \right) \tilde{\lambda}_{+}^{m},$$

where $A^{nm}_{\varsigma\dot{\varsigma}}(X) = \left(V_p^n\right)^{-1} \left(\frac{\partial}{\partial X^{\varsigma\dot{\varsigma}}} V^{pm} \right)$

Thus, the model under consideration gives a microscopic description of the YM instanton!

6) D1-brane probing D9-$\overline{\text{D9}}$-brane annihilation in type I ST



GS or Twistor formulation of the D1-brane theory:

$$\mathcal{L} = P^{---}_{M}\Big[e^{++}_{m}\big(\partial_m X^{M} - \partial_m \psi^{A}\Gamma^{M}_{AB}\psi^{B}\big) - \varphi_{-}\Gamma^{M}\varphi_{-}\Big] + WZ +$$

$$+ \lambda^{p}\Big[e^{m}_{--}\big(\delta^{pq}\partial_m - \partial_m X^{M} A^{pq}_{M}(X)\big) + \frac{1}{4}F^{pq}_{ML}(X)\,\Gamma^{ML}_{AB}\psi^{A}\psi^{B}\Big]\lambda^{q} +$$

$$+ \chi^{\bar{p}}\Big[e^{m}_{++}\big(\delta^{\bar{p}\bar{q}}\partial_m - \partial_m X^{M} B^{\bar{p}\bar{q}}_{M}(X)\big) + \frac{1}{4}H^{\bar{p}\bar{q}}_{ML}(X)\,\Gamma^{ML}_{AB}\psi^{A}\psi^{B}\Big]\chi^{\bar{q}} + T^{p\bar{p}}\lambda^{p}\chi^{\bar{p}}.$$

Constraints & κ-transformations

$$\boxed{P^{M}_{--} = e^{-4}_{--}\,\varphi_{-}\,\Gamma^{M}\,\varphi_{-}}$$

$$\delta P^{m}_{M} = 0 \; ; \; \delta \varphi_{-} = 0$$

$$\delta \Psi^{A} = 2i\, P^{M}_{--}\, \Gamma^{AB}_{M}\, \kappa_{B++}$$

$$\delta X^{M} = -i\, \delta \Psi^{A}\, \Gamma^{M}_{AB}\, \Psi^{B}$$

$$\boxed{\begin{aligned}
&\delta\left(\mathbf{A}_{M}\partial_{--}X^{M}\right) = \partial_{--}\Lambda_{\kappa} + \left[\Lambda_{\kappa}, \mathbf{A}_{M}\partial_{--}X^{M}\right]\\
&\delta \lambda^{P} = \left(\Lambda_{\kappa}\lambda\right)^{P} \; ; \; \Lambda_{\kappa} = \delta X^{M}\mathbf{A}_{M}\\
&\delta\left(\mathbf{B}_{M}\partial_{++}X^{M}\right) = \partial_{++}\Lambda'_{\kappa} + \left[\Lambda'_{\kappa}, \mathbf{B}_{M}\partial_{++}X^{M}\right]\\
&\delta x^{\bar{P}} = \left(\Lambda'_{\kappa}x\right)^{\bar{P}} \; ; \; \Lambda'_{\kappa} = \delta X^{M}\mathbf{B}_{M}\\
&\delta \mathbf{T}(X) = \partial_{M}\mathbf{T}(X)\,\delta X^{M}
\end{aligned}}$$

Conditions:

$$\delta x^{AB} D_{AB} T(x) \equiv \delta x^{AB} \left\{ \partial_{AB} T^{q\bar p} + T^{q\bar q}(x) B^{\bar q \bar p}_{AB}(x) - A^{qp}_{AB}(x) T^{p\bar p}(x) \right\} = 0 \; ; \; \left(\delta x^{AB}\right)^2 = 0$$

$$\delta x^{(1)} \delta x^{(2)} \left[D^{(1)}, D^{(2)} \right] = 0$$

$(1,2) \to (x_1, x_2)$

The simplest solution to the conditions:

$$T_{[4N+32]\times[4N]} \propto \left(D_{[4N]\times[4N]} \oplus O_{[32]\times[4N]} \right)$$

If $A_\mu = B_\mu = 0$

Gauge invariant expression for the T VEV

$$T^{\bar p p} T^{\bar q}{}_{p} = \delta^{\bar p \bar q}$$

With this T $\chi^{\bar p}$ become massive and $\lambda^{\bar p}$ also (where $\lambda^{p} = (\lambda^{\bar p}, \lambda^{n})$, $n = \overline{1,32}$)

At the same time λ^{n} is massless.

Integrating massive fields out we get Heterotic string Lagrangian.

The Heteratic string Lagrangian

$$\mathcal{L} = \partial x \partial x + \psi \partial \psi +$$

$$+ \lambda_+^n \partial_{--} \lambda_+^n$$

This is superconformal theory.

Hence, chosen $\langle T \rangle$ is correct.

Codimension four soliton:

Consider: $A_m = B_m = 0 \;;\; m=\overline{2,5}$

$$SO(1,1)\times SO(8) \longrightarrow \underset{(+,-)}{SO(1,1)}\times \underset{(\alpha,\dot\alpha)}{SO_e(4)}\times \underset{(\xi,\dot\xi)}{SO_t(4)}$$

Define: $\tilde{A}_{\xi\dot\xi} = A_{\xi\dot\xi} - B_{\xi\dot\xi}$ respecting

$$\boxed{SO(32)\times SU(2)\times USp(2N)}\times\boxed{SU(2)\times USp(2N)}$$

and $A^{[32]\times[32]}_{\xi\dot\xi} = 0$

embed these CP indexes into tangent bundle. $(\xi,\dot\xi)$

Thus (after fixing the LC gauge):

$$X^{\bar P} = X^{\dot\xi a}_{-} \quad , \quad \lambda^{P} = \left(X^{\xi a}_{+}, \lambda^{n}_{+}\right)$$

$a=\overline{1,2N}$

The condition now looks like:

$$d^{\xi\dot\xi}_{(1);(2)}\,\tilde{D}_{\xi\dot\xi}\,T(x) = 0;$$

$$\underset{\substack{\downarrow\\ x_1}}{d^{\xi\dot\xi}_{(1)}}\;\underset{\substack{\downarrow\\ x_2}}{d^{\eta\dot\eta}_{(2)}}\;\tilde{F}_{\xi\dot\xi\eta\dot\eta}(x) = 0. \Longleftrightarrow \text{SD equation}$$

Solutions to the conditions:

Non-trivial $\tilde{A}_{\mu}^{\bar{p}\bar{q}} \sim \left[(\partial_\mu \hat{S})\,\hat{S}^{-1}\right]^{ab}_{\xi\eta}$ as $|x| \to \infty$

where $\hat{S} : S^3 \to USp(2N)_{diagonal}$

Most generic (upto gauge transformation:

$$S^{ab}_{\xi\dot{\xi}} = \Delta^{ac}(x)\left(x_{\xi\dot{\xi}}\delta^{cb} - X^{cb}_{\xi\dot{\xi}}\right)$$

where $(\Delta^{-2})^{ab} = \left\{(x\delta^{ac} - X^{ac})_\mu \times (x\delta^{cb} - X^{cb})_\nu - [X_\mu, X_\nu]^{ab}\right\}\tau^\mu\tau^\nu$

$$X^{\xi\dot{\xi}} = \epsilon^{\xi\eta}\epsilon^{\dot{\xi}\dot{\eta}}X^{*}_{\eta\dot{\eta}}$$

Then as $|x| \to \infty$

$$T^{a}_{p\dot{\xi}} \sim \Delta^{ab}\left\{\left(x_{\xi\dot{\xi}}\delta^{ba} - X^{ba}_{\xi\dot{\xi}}\right) \oplus h^{bn}_{\dot{\xi}}\right\}$$

where $h^{\dot{\xi}an} = \epsilon^{ab}\epsilon^{\dot{\xi}\dot{\eta}}\left(h^{n}_{b\dot{\eta}}\right)^{*}$,

and $T^{\bar{p}p}T^{\bar{q}}_{p} = \delta^{\bar{p}\bar{q}} \Longleftrightarrow$

$$\left(\epsilon^{\xi\eta}X_{\xi\dot{\xi}}X_{\eta\dot{\eta}}\right)^{ab} + h^{an}_{\dot{\xi}}h^{nb}_{\dot{\eta}} = 0$$

RG flow:

$$\mathcal{L} = \partial_{++} X^{\mu} \partial_{--} X_{\mu} + \psi^{\mu}_{-} \partial_{++} \psi_{-\mu} + \chi^{\zeta a}_{-} \partial_{++} \chi^{a}_{-\zeta} + \chi^{\dot\zeta a}_{+} \partial_{--} \chi^{a}_{+\dot\zeta} + \chi^{\zeta a}_{-} \Delta^{ab} \cdot \left\{ \left(\left(X_{\zeta\dot\zeta} \delta^{bc} - \mathbf{X}^{bc}_{\zeta\dot\zeta} \right) \chi^{\dot\zeta c}_{+} + \mathbf{h}^{bn}_{\zeta} \lambda^{n}_{+} \right\}$$

Again massless modes are:

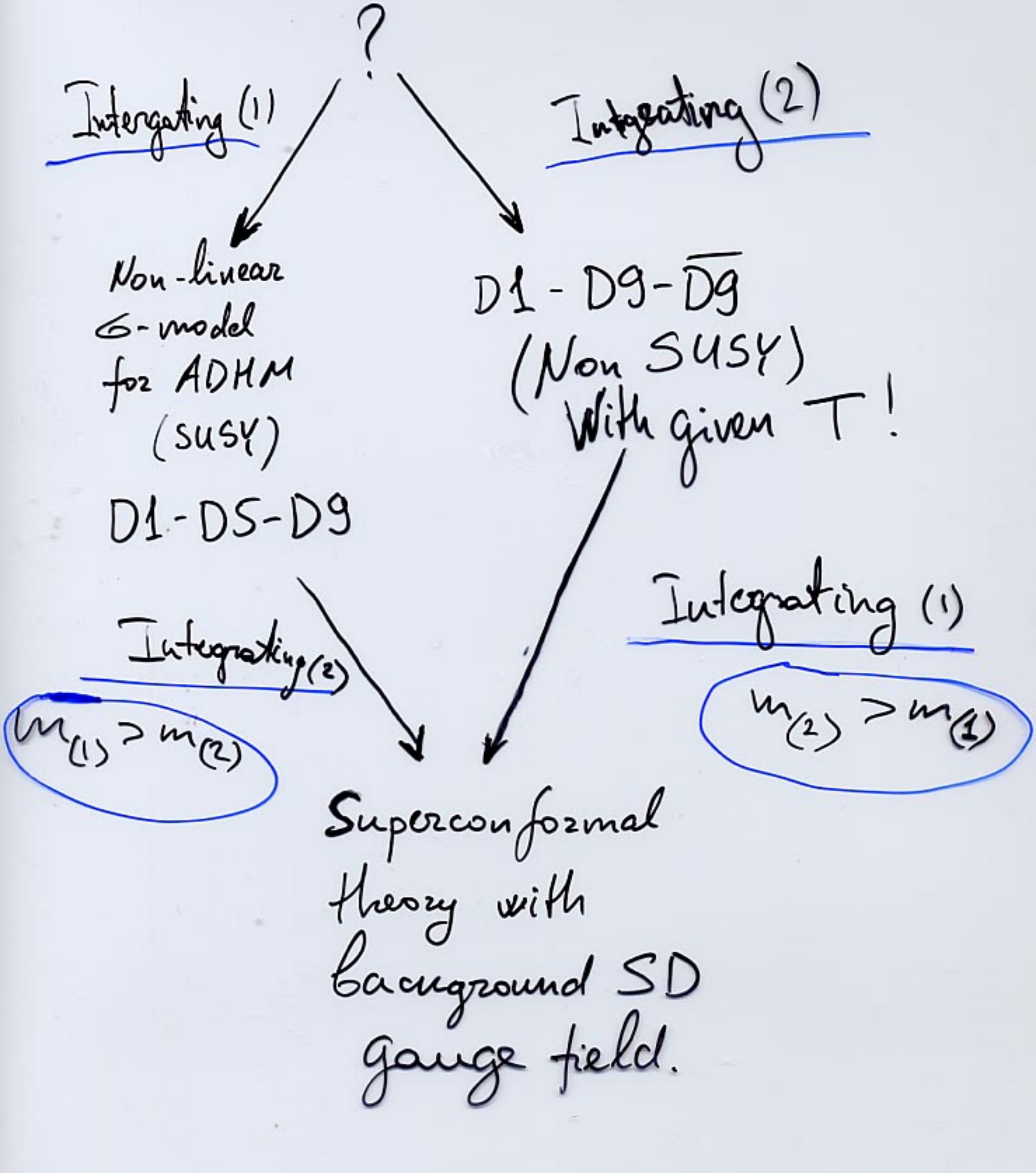
$$\mathbf{T}^{a}_{p\zeta}(X)\, V^{pn}(X) = 0$$

$$\lambda^{p}_{+} = \sum_{n=1}^{32} V^{pn} \tilde\lambda^{n}_{+} .$$ Hence:

$$\mathcal{L} = \partial_{++} X^{\mu} \partial_{--} X_{\mu} + \tilde\lambda^{n}_{+} \left[\partial_{--} \delta^{nm} + \partial_{--} X^{\zeta\dot\zeta} \mathbf{A}^{nm}_{\zeta\dot\zeta}(X) \right] \tilde\lambda^{m}_{+} ,$$ where

$$\mathbf{A}^{nm}_{\zeta\dot\zeta}(X) = \left(V^{n}_{p} \right)^{-1} \frac{\partial}{\partial X^{\zeta\dot\zeta}} V^{m}_{p}$$

Questions:



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Intergating (1)

Non-linear σ-model for ADHM (SUSY)

D1-D5-D9

Integrating (2)

D1 - D9 - $\overline{\text{D9}}$ (Non SUSY) With given T !

Integrating (2)

$m_{(1)} > m_{(2)}$

Integrating (1)

$m_{(2)} > m_{(1)}$

Superconformal theory with background SD gauge field.

IV) Non-Abelian structures is open string theory

1) Approximation

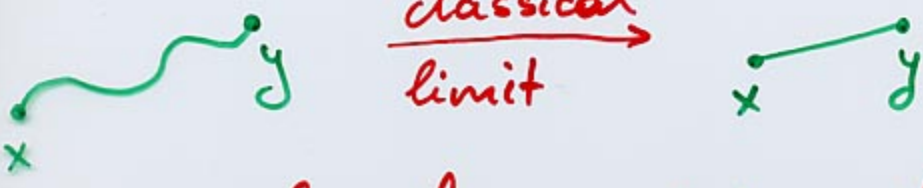
Open string functionals = operators on the space of paths of the target space

$A(l,r) * A'(l'=r,r') = A''(l,r')$ —

— operators acting on the l & r halves of the open strings (halves = paths in the target space)

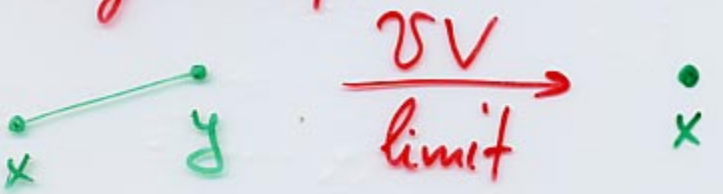
There is no a well defined formalizm to deal with such objects!!! ⇔ Big problems

We take the approximation



$\Rightarrow A(x,y)$ -

- kernel of an integral operator on the target space



$\Rightarrow D_x$ – Diff operator on the target space.

More precisely :

$$S_{2d} = \underset{D \,\leftarrow\, \text{disc}}{\int} \partial_a \tilde{X}_\mu \, \partial_a \tilde{X}_\mu \, d^2\sigma \qquad a = \overline{1,2} \qquad \mu = 0, \ldots, 25$$

$$\tilde{X}_\mu \Big|_{\partial D} = X_\mu(\theta) \quad \Longrightarrow$$

$$\Rightarrow S_{2d}^{cl} = \oint_{\partial D} X_\mu \, \underset{\uparrow}{\partial_n} X_\mu \, d\theta$$

normal derivative
(non-local operator on the boundary)

$$\int DX_\mu \, e^{-S_{2d}^{cl}} = \int DP_\mu DX_\mu \, e^{-S_B} \, \delta\left[P_\mu(0)\right]$$

$$S_B = \oint_{\partial D} \left(P_\mu \, \underset{\uparrow}{\partial_t} X_\mu + \frac{1}{2} P_\mu \, \partial_n P_\mu \right) d\theta$$

[redacted]

tangential derivative

$$\boxed{\text{Similar to } \quad I = \int \left(p\dot{q} + H(p,q) \right) dt}$$

$H(P)$ in our case is non-local.

Moreover, Green function of the ∂_u operator is:

$$G(\tau,\tau') = \ln \|\tau-\tau'\|^2$$

$$\tau = e^{i\theta}, \quad \tau' = e^{i\theta'}$$

$G(\tau,\tau')$ is singular when $\tau \to \tau' \Rightarrow$

$\Rightarrow$ Regularization is necessary.

We take the regularization:

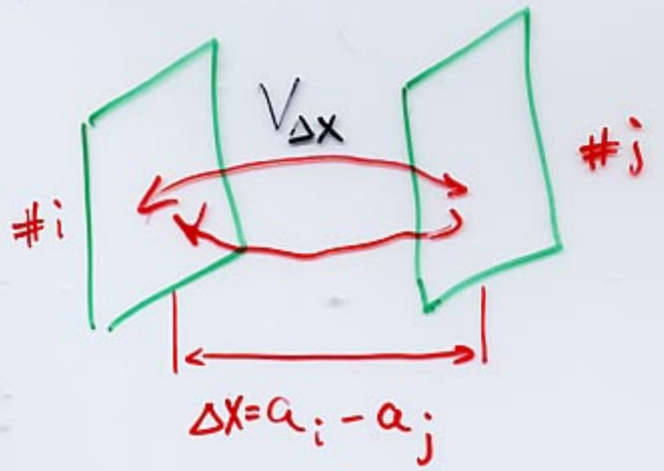
$$\int_{\tau}^{\tau'} d\tau'' e(\tau'') \equiv \|\theta-\theta'\| =$$

$$= \begin{cases} |\theta-\theta'|, & \text{if } |\theta-\theta'| \gg l_R \\ 1, & \text{if } |\theta-\theta'| \ll l_R \end{cases} \text{ — regularization distance}$$

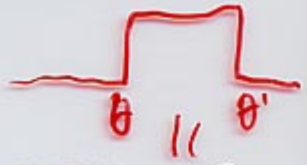
Thus, $G_R(0) = 0$!!!

Hence, $S_B^{Reg} \underset{\text{limit}}{\overset{w}{\simeq}} \oint_{\partial D} P_\mu \partial_\theta X^\mu d\theta$ + correct..

3) Open String vertex operators in this approximation



From S^{cl}_{3D} it follows that



$$V_{\Delta x} = e^{-I\oint_{\partial D} \partial_n X_\mu \Delta X_\mu}$$

in our approximation!

$$\equiv e^{-I\oint_{\partial D} P_\mu \Delta X_\mu}$$

$$\uparrow$$

Translation by ΔX

$$\boxed{V^{ij}_{\Delta x} = e^{I\frac{a^i-a^j}{\alpha'}P}} \quad P = \delta/\delta x, \quad m^2 = |a^i-a^j|^2/\alpha'^2 \;-$$

– mass of the corresponding string field.

Thus, Open string vertex operators become Diff. op. in the target space in our approximation. More concretely, . $GL(\infty)$ of all diff operators is broken down to $GL(N)$ in the presence of N D-branes.

More precisely:

N D-branes in our approximation are defined as $\sum_{i=1}^{N} \delta(x-a_i)$ or by the locus $\partial_x \underset{\prod_{i=1}^{N}(x-a_i)}{\underset{\parallel}{W(x)}} = 0$ (we explain this point more carefully in a few minits)

Let us find operators which respect this structure:

Proj on a_i $$E_{ii}(x)\ \delta(x-a_j) = \delta_{ij}\,\delta(x-a_j)$$

$$E_{ii}(x)\, E_{jj}(x) = \delta_{ij}\, E_{ii}(x) \bmod W(x)$$

$$E_{ij}^{(x)}\delta(x-a_k) = \delta_{jk}\,\delta(x-a_i)$$

Solution:

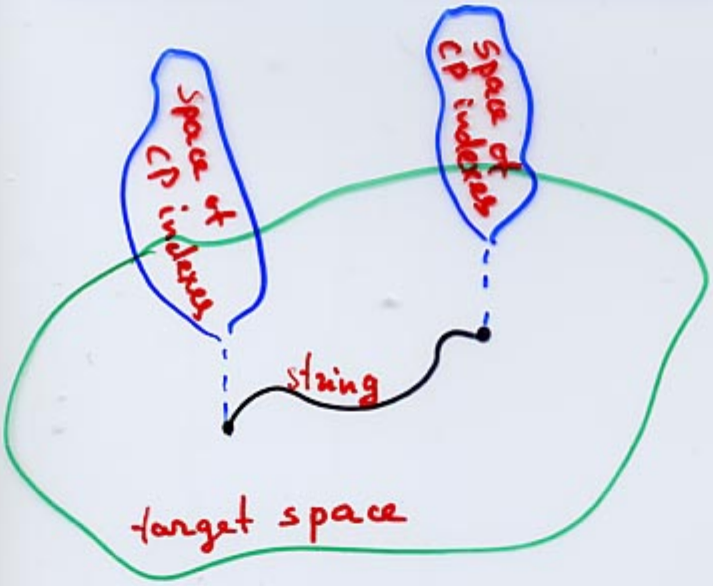
$$E_{ij}^{(x)} = \frac{\prod\limits_{k\neq j}(x-a_k)}{\prod\limits_{k\neq j}(a_i-a_k)}\, e^{(a_i-a_j)\frac{\delta}{\delta x}} =$$

$$= E_{jj}(x)\, e^{(a_i-a_j)\frac{\delta}{\delta x}}$$

Note that $E_{jj}(x=a_j) = 1 \Rightarrow$

$\Rightarrow E_{ij} = V_{\Delta x}^{ij} \Rightarrow$ Generate $GL(N)$ group!!!

3) Non-Abelian structures for multiple D-branes from D-$\bar{D}$-annihilation



We observed that open string vertex operators are Diff. op. on the target space. More precisely:

$$\Phi(X|y,p)$$

target space $\mathcal{X}$ CP space $T^*\mathcal{Y}$

X - coordinate on $\mathcal{X}$

y - coordinate on $\mathcal{Y}$

$p = \frac{\partial}{\partial y}$ & $(y,p) \in T^*\mathcal{Y}$

In toutological situation $\mathcal{X} = \mathcal{Y}$ (above we encountered exactly this situation).

The situation with generic $\mathcal{Y}$ could be mimiced by infinite numer of D25 [redacted] - branes [redacted] in bosonic string theory.

CP indexes = $\infty \times \infty$ matrixes = Diff operators on a space $\mathcal{Y}$

Now we are going to observe how after (∞) D25-brane annihilation (N) Dp-branes appear and as the result, how $GL(\infty)$ braces down to $GL(N)$, etc.

Consider:

$$Z_{D25} = \left\langle \int Dp\, Dy \ldots e^{-i\oint p\dot{y} - \oint T(\underline{X}|p,y) + \ldots} \right\rangle_{BS}$$

Now let us take tachyon profile:

$$T_{\{s\}}(X|y,p) \propto \frac{1}{s_1}|\underline{X}_\mu - y_\mu|^2 + \frac{1}{s_2}|\partial_y W(y_a)|^2 + \frac{1}{s_3}|p_a|^2$$

$\mu = \overline{0,25}$

$a = \overline{p+1, 25}$

localizes $X=Y$

localize on N Dp-branes.

In fact;

$$\lim_{\{s\}\to 0} \int Dy\, Dp \ldots e^{-i\oint p\dot{y} - \oint T(\underline{X}|p,y) \ldots} \propto \sum_{i=1}^{N} \delta(x-a_i)$$

where a_i originate from $\partial_{\underline{X}} W(\underline{X}) = \prod_{i=1}^{N}(\underline{X}-a_i)$

$GL(\infty)$ acting as Hamiltonian transformations on the functionals of p & y braces down to $GL(N)$ acting on the vector $\begin{pmatrix}\delta(x-a_1)\\ \vdots \\ \delta(x-a_N)\end{pmatrix}$

II) Lowest excitations of open strings

Let us now deform the background such that the system $\sum_{i=1}^{N}\delta(x-a_i)$ is respected, i.e. a_i remain inv.!

In particular, we can shift the background by

$$A_\mu(\underline{X}|P,y) = \Big(A_m^g(\underline{X}^m)\,T^g(y^a,P^a),\ \underbrace{0,0,\dots}_{a}\Big)$$

$m=\overline{0,p}$ and $\mu=(m,a)$

$$[T^{g_1},T^{g_2}] = f^{g_1g_2g_3}T^{g_3}$$

$g=\overline{1,N^2}$

and by $\tilde{\Phi}_\mu(\underline{X}|P,y) = \Big(0,\dots,0,\ \tilde{\Phi}_a^{\tilde{g}}(\underline{X}_m)\,T^{\tilde{g}}(P,y)\Big)$

Where $\tilde{g}$ runs over all g except Cartan elements, otherwise a_i would be shifted.

Thus, we act:

$$e^{i\oint_{\partial D}A_\mu\frac{\delta}{\delta P_\mu}+\oint_{\partial D}\Phi_\mu\frac{\delta}{\delta \underline{X}_\mu}}\,Z_{D25} = Z_{D25}^{\text{shifted}}$$

But in our case

$$\frac{\delta}{\delta P_m} = \partial_t X_m \qquad \frac{\delta}{\delta X_a} = \partial_n X_a\ !!!$$

Now, we obtain:

$$\lim_{\{S\}\to 0} Z^{\text{shifted}}_{D25} = \left\langle \operatorname{Tr} P\, e^{i\oint_{\partial D} \hat{A}_m(\underline{X})\partial_+ X^m + \oint_{\partial D} \hat{\varphi}_a(\underline{X})\partial_n X^a} \right\rangle_{BS}$$

$$\equiv Z^{\#N}_{Dp}$$

where

$$\hat{A}_m(\underline{X}) = A^g_m(\underline{X})\,\hat{t}^g \leftarrow$$

$$\hat{\Phi}_a(\underline{X}) = \Phi^{\tilde{g}}_a\,\hat{t}^{\tilde{g}} + \operatorname{diag}(a_1, \dots, a_N)$$

matrix generators of U(N)

If $A = 0$ & $\Phi = 0 \Rightarrow$

$$Z_{D25} = \left\langle \sum_{i=1}^{N} \delta(\underline{X} - a_i) \right\rangle_{BS}$$

Thus, deformations which respect GL(N) are massless, otherwise they are massive open string excitations.

5) Unification of the RR couplings to D-branes in type II String theory.

RR coupling are of anomalous origin and, hence, sensitive to zero modes. Thus, they can be studyed in our approximation exactly.

a) Definition: type IIB string theory

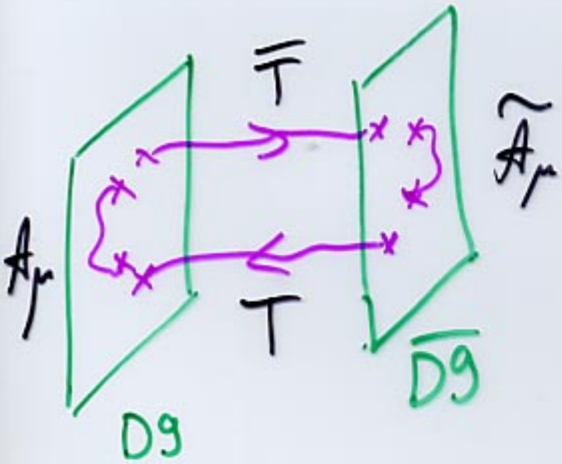
$$S_{RR} = \underset{\substack{\uparrow \\ \text{D9-brane charge}}}{q_9} \int_{\mathcal{X}_{10} \leftarrow \text{target space (flat)}} \Big[\mathbb{C}_{RR} \wedge Ch(\mathbb{A}) \Big]_{\substack{top \\ \uparrow \\ \text{10-form} \\ \text{should be} \\ \text{taken}}}$$

$$\mathbb{C}_{RR} = \sum_{k=1}^{5} C_{(2k)}$$

$$C_{(2k)} = C_{\mu_1 \dots \mu_{2k}} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_{2k}}$$

Let us define now $Ch(\mathbb{A})$ and

$\mathbb{A}$ - superconnection.



$$\mathbb{A} = \begin{pmatrix} \nabla_\mu^+ dx^\mu & T \\ \bar{T} & \nabla_\mu^- dx_\mu \end{pmatrix}$$

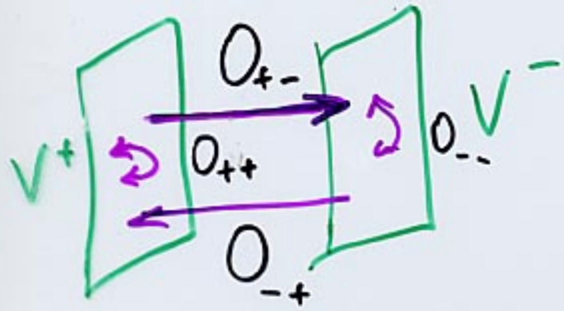
$$\nabla_\mu^+ = \partial_\mu + i A_\mu$$

$$\nabla_\mu^- = \partial_\mu + i \widetilde{A}_\mu$$

$$Ch(\mathbb{A}) \equiv \underset{\uparrow}{Str}\, e^{-\frac{\mathbb{A}^2}{2\pi}}$$

super-trace (do not confuse with symmetric trace)

Definition of Str:



$$O : V \longrightarrow V$$

$$V = V_+ \oplus V_-$$ — Z_2-graded bundle

$$O = \begin{pmatrix} O_{++} & O_{+-} \\ O_{-+} & O_{--} \end{pmatrix}$$

$$Str_V O = Tr_V \underset{\uparrow}{\tau} O = tr_{V_+} O_{++} - tr_{V_-} O_{--}$$

Z_2-grading operator

To set the notations and explain the idea
we start with:

b) Dp-branes from finite number of D9-$\overline{D9}$-branes.

k D-instantons: We should start with 16k - D9-branes and 16k $\overline{D9}$-branes

16 indexes we embedd into $S_\pm$ -spinor bundle of the target space.

I.e.
$$V = V_+ \oplus V_- = \left(S_+ \otimes E_k\right) \oplus \left(S_- \otimes E_k\right)$$

E_k is a k-dimensinal vector bundle

We choose it trivial, i.e. $\nabla = \nabla^+ \oplus \nabla^- = d.$

But
$$T_s = \frac{1}{\sqrt{s}}\, x^\mu \sigma_\mu \otimes 1\!\!1_{k\times k}$$

parameter

16×16 10-dimensional σ-matrixes

Witten & Sen & Horava

$$\boxed{|x|}$$

Then:

$$\mathbb{A}_s^2 = \begin{pmatrix} \frac{1}{s}|x|^2 & \frac{1}{\sqrt{s}} dx^\mu \sigma_\mu \\ \frac{1}{\sqrt{s}} dx^\mu \bar{\sigma}_\mu & \frac{1}{s}|x|^2 \end{pmatrix} \otimes \mathbb{1}_{k\times k} =$$

$$= \left(\frac{1}{\sqrt{s}} dx^\mu \gamma_\mu + \frac{1}{s}|x|^2 \right) \otimes \mathbb{1}_{k\times k}$$

For our choise of V, $\tau = \left(\begin{array}{c|c} 1 & \\ \hline & -1 \end{array} \right)$

$$\mathrm{Str}\, O = \mathrm{tr}_k \, \underset{\substack{\uparrow \\ \text{trace} \\ \text{over} \\ \gamma\text{-matrixes}}}{\mathrm{Sp}} \, \gamma^{11} O$$

$\gamma^{11} = i\gamma^0 \ldots \gamma^9$

Then, using $\mathrm{Sp}\, \gamma^{d+1} \gamma^{\mu_1} \ldots \gamma^{\mu_d} = (2i)^{\frac{d}{2}} \epsilon^{\mu_1 \ldots \mu_d}$

and $\lim\limits_{s \to 0} \frac{1}{(\pi s)^{d/2}} e^{-\frac{|y|^2}{s}} = \delta^{(d)}(y)$,

we obtain

$$\mathrm{Ch}(\mathbb{A}) = \lim_{s \to 0} \mathrm{Ch}(\mathbb{A}_s) = k\, \delta^{(10)}(x) \, \mathrm{vol}(X^{10})$$

I.e. $S_{RR} = k \underset{\substack{\uparrow \\ \text{D-instanton charge}}}{q_{-1}} C_{(0)}(x=0)$!!!

Consider deformation of the background T:

$$T = \frac{1}{\sqrt{s}}\left(x^\mu \sigma_\mu \otimes \mathbb{1}_{k\times k} - \underset{\uparrow}{\Phi^\mu} \sigma_\mu \right)$$

appears as the Goldstown mode

Then,

$$\mathbb{A}_s^2 = \frac{1}{\sqrt{s}}\, dx^\mu \gamma_\mu \otimes \mathbb{1}_{k\times k} + \frac{1}{2s}\left[\Phi^\mu, \Phi^\nu\right]\gamma_{\mu\nu} +$$

$$+ \frac{1}{s}\left| x^\mu \otimes \mathbb{1}_{k\times k} - \Phi^\mu \right|^2$$

Now using that

symmetric trace

$$\mathrm{Tr}\, e^{A+B+C} = \mathrm{Sym}\,\mathrm{Tr}\, e^{A+B+C}$$

and

$$\delta^{(d)}\left(x^\mu \otimes \mathbb{1} - \Phi^\mu \right) \equiv \lim_{s\to 0} \frac{1}{(\pi s)^{d/2}} e^{-\frac{1}{s}\left| x^\mu \otimes \mathbb{1} - \Phi^\mu \right|^2},$$

we obtain:

$$S_{RR} = q_{-1}\, \underset{\uparrow}{\mathrm{Sym}\,\mathrm{Tr}}\, e^{-\left[\dot{I}_\Phi, \dot{I}_\Phi\right]/2\pi}\, \mathbb{C}_{RR}^{(\Phi)}$$

exactly Myers term !!!

Note that this is exactly the deformatione we have encountered before...

⋆ Dp-branes:

$\mu = (m, a) \qquad m = \overline{0, p} \qquad a = \overline{p+1, 9}$

Now we take $2^{(9-p)/2} \otimes k$ D9-branes and $2^{(9-p)/2} \otimes k$ $\overline{D9}$-branes

Deformed case:

$$\nabla = \left[\partial_m + i A_m(x_m)\right] dx^m + \partial_a dx^a$$

$$T = \frac{1}{\sqrt{s}} \left[\underset{\substack{\uparrow \\ (9-p)\text{-dimensional} \\ \sigma\text{-matrixes}}}{x^a \sigma_a} \otimes \mathbb{1}_{k \times k} - \Phi^a(x_m) \sigma_a \right]$$

In this case:

$$S_{RR} \overset{s \to 0}{\underset{\text{limit}}{=}} \int_{\mathcal{X}^{(p+1)}} \Big\{ \mathrm{Sym}\,\mathrm{Tr}_2\, C_{RR}(x_m, \varphi_a) \times$$

$$\times\, e^{-\frac{1}{2\pi}\left[F_{(2)}(x_m) + \left[\nabla_{(1)}(x_m), i_\Phi\right] + \left[i_\Phi, i_\Phi\right]\right]} \Big\}_{top}$$

Note that $A_m(x_m)$ originates from ∇ rather than T. To obtain A from T one has to conside infinite number of D9-$\overline{D9}$-branes.

Along this lines we can obtain a most general configuration Dp-$\overline{\text{Dp}}$-brane systems with various p at the same time and <u>all</u> lowest energy excitations on them! Explicite tachyon values are known accordingly.

~~[illegible]~~

c) Dp-branes from infinite number of D9-$\overline{\text{D9}}$-branes

X — target space; T^*Y — „CP space"

$$(\gamma_\mu, \Gamma_\nu, \hat{\Gamma}^\rho) \longleftrightarrow (X_\mu, y_\nu, p^\rho)$$

↑ Clifford algebra corresponding to $X \otimes T^*Y$

$$\{\gamma_\mu, \gamma_\nu\} = \{\Gamma_\mu, \Gamma_\nu\} = \delta_{\mu\nu}; \; \{\hat{\Gamma}^\mu, \hat{\Gamma}^\nu\} = \delta^{\mu\nu}$$

$$\{\Gamma_\mu, \gamma_\nu\} = \{\hat{\Gamma}^\mu, \Gamma_\nu\} = \{\hat{\Gamma}^\mu, \gamma_\nu\} = 0$$

<u>One D9-brane from infinite D9-$\overline{\text{D9}}$-branes:</u>

$$|T(\underline{X}|y, p)|^2 \propto \frac{1}{s_1}|\underline{X}_\mu - y_\mu|^2 + \frac{1}{s_2}|p^\mu|^2$$

in similarity with the case considered above.

To obtain corresponding $T(\underline{X}|p.y)$ & $\mathbb{A}$ we have

$$V_\pm = \underset{\uparrow}{S_\pm} \otimes \underset{\uparrow}{S_\pm} \otimes \underset{\uparrow}{\hat{S}_\pm} \otimes \underset{\uparrow}{\mathcal{H}}$$

spinor bundles corresponding to $\mathcal{X} \otimes T^*\underline{Y}$

a Hilbert space.

Then, $$T(\underline{X}|y,p) = \frac{1}{\sqrt{s_1}}\, \underset{\uparrow}{\sigma_\mu} (\underline{X}^\mu - y^\mu) + \frac{1}{\sqrt{s_2}} \underset{\uparrow}{\hat{\Sigma}^\mu} p_\mu$$

σ-matrixes corresponding to γ_μ & $\hat{\Gamma}^\mu$, respectively

At the same time, again $\nabla = \nabla^+ \oplus \nabla^- = d$

Thus, $$\mathbb{A}^2_{\{s\}} = \frac{1}{s_1}|\underline{X}^\mu - y^\mu|^2 + \frac{1}{s_2}|p_\mu|^2 + \frac{1}{\sqrt{s_1 s_2}}\gamma_\mu \hat{\Gamma}^\mu + \frac{1}{\sqrt{s_1}}\, dx^\mu \gamma_\mu ,$$ and we obtain

$$\lim_{\{s\}\to 0} ch(\mathbb{A}_{\{s\}}) = Tr_{\mathcal{H}}\, \delta^{(10)}(p)\, \delta^{(10)}(\underline{X} - y) = 1.$$

I.e. $$S_{RR} = q/g \int_{\mathcal{X}^{(10)}} C_{(10)}(x)$$

Less trivial situation (say D7-branes with gauge fields and scalars on them)

Let $W(y_a)$ $a=\overline{1,2}$ be a polinomial whose critical points are positions of D7- and $\overline{D7}$-branes: the Hessian $\partial^2 W$ defines the sign of the RR charge.

Consider: $$\mathbb{A}_{\{s\}} = d + \frac{1}{\sqrt{s_1}}\gamma_\mu(X^\mu - y^\mu) + \frac{1}{\sqrt{s_2}}\Gamma_a \partial_a W(y_b) +$$
$$+ \frac{1}{\sqrt{s_3}}\hat{\Gamma}^\mu p_\mu$$

Then $$\lim_{\{s\}\to 0} Ch(\mathbb{A}_{\{s\}}) = \mathrm{Tr}_{\mathcal{H}}\Big\{\delta^{(10)}(p)\,\delta^{(10)}(X - y)\times$$
$$\times \det\left[\partial_a\partial_b W(y)\right]\,\delta^{(2)}\left[\partial_a W(y)\right]\Big\}$$

Hence,
$$S_{RR} = q_7 \int_{\mathcal{X}^{(8)}} C_{(8)}(X)\left\{\sum_{i=1}^{k_+}\delta(X^b - a^b_{i+}) - \right.$$
$$\left. - \sum_{j=1}^{k_-}\delta(X^b - a^b_{j-})\right\}$$

$k_\pm$ - numbers of D7- and $\overline{D7}$-branes.

Now let us deforme the $\mathbb{A}_{\{S\}}$ under consideration. There are many ways of deformation. We are interested in those which respect the D7-brane sysmem (we choose $\kappa_- = 0$)

$$\mathbb{A}_{\{S\}} = d + \frac{1}{\sqrt{S_1}} \gamma_m \left(\cancel{X}^m - y^m \right) + \frac{1}{\sqrt{S_1}} \gamma_a \left\{ X^a - y^a - \tilde{\Phi}^a (y^n | y^a, p^a) \right\} + \frac{1}{\sqrt{S_2}} \Gamma_a \partial^a W(y) + \frac{1}{\sqrt{S_3}} \hat{\Gamma}^m \left(p_m + i A_m (y^n | y^a, p_a) \right\} + \frac{1}{\sqrt{S_2}} \hat{\Gamma}^a p_a$$

Where $m = \overline{0,7}$ & $A_m, \tilde{\Phi}^a$ are defined as above!

Thus,

$$S_{RR} = q_7 \int_{X^{(7)}} \mathrm{SymTr} \left[e^{-\frac{1}{2\pi} \left(F_{(2)} + [\nabla_{(1)}, \dot{I}_\Phi] + [\dot{I}_\varphi, \dot{I}_\Phi] \right)} C_{RR}^{(X^m, \Phi^a)} \right]_{top}$$

$\uparrow$

Myers term

VI) Conclusions

1) D-branes give the microscopic description of SYM & SUGRA solitons.

2) D-branes show how non-Abelian structures (part of large symmetry group of string theory) appear in string theory.

3) D-$\bar{D}$-systems help to partialy resolve the problem of background independence.

In particular: all RR couplings can be treated from the same perspective.

4) There are still a lot of problems to solve: despite the resent „tremendous progress" in string theory we dot understand basically anything about this theory.