Present matter content in the Universe

Multicomponent !

$\sim 95\%$ of all matter not yet discovered in laboratory !

1. Usual matter — $\Omega_b = 0.045 \pm 0.005$

   p, n, e^-

   a) BBN

   b) acoustic peaks in $\frac{\Delta T}{T}$

$\Omega_{lum} < 0.01$ → the problem of dark baryonic matter

2. Hot dark matter — $10^{-3} \lesssim \Omega_\nu < 0.03$

   massive neutrinos

   $P(k)$

   $\sum_a m_{\nu a} \lesssim 0.7\ eV$

3. Cold dark matter — $\Omega_m = 0.27 \pm 0.04$

   mostly non-baryonic

   non-relativistic

   grav. clustered

   a) $\Delta T/T$

   b) rotation curves

   c) LSS

4. Dark energy — $\Omega_m = 0.73 \pm 0.04$

   relativistic, $p<0$, $|p| \sim \varepsilon$

   grav. unclustered

   a) $\Delta T/T$, $P(k)$

   b) SN Ia

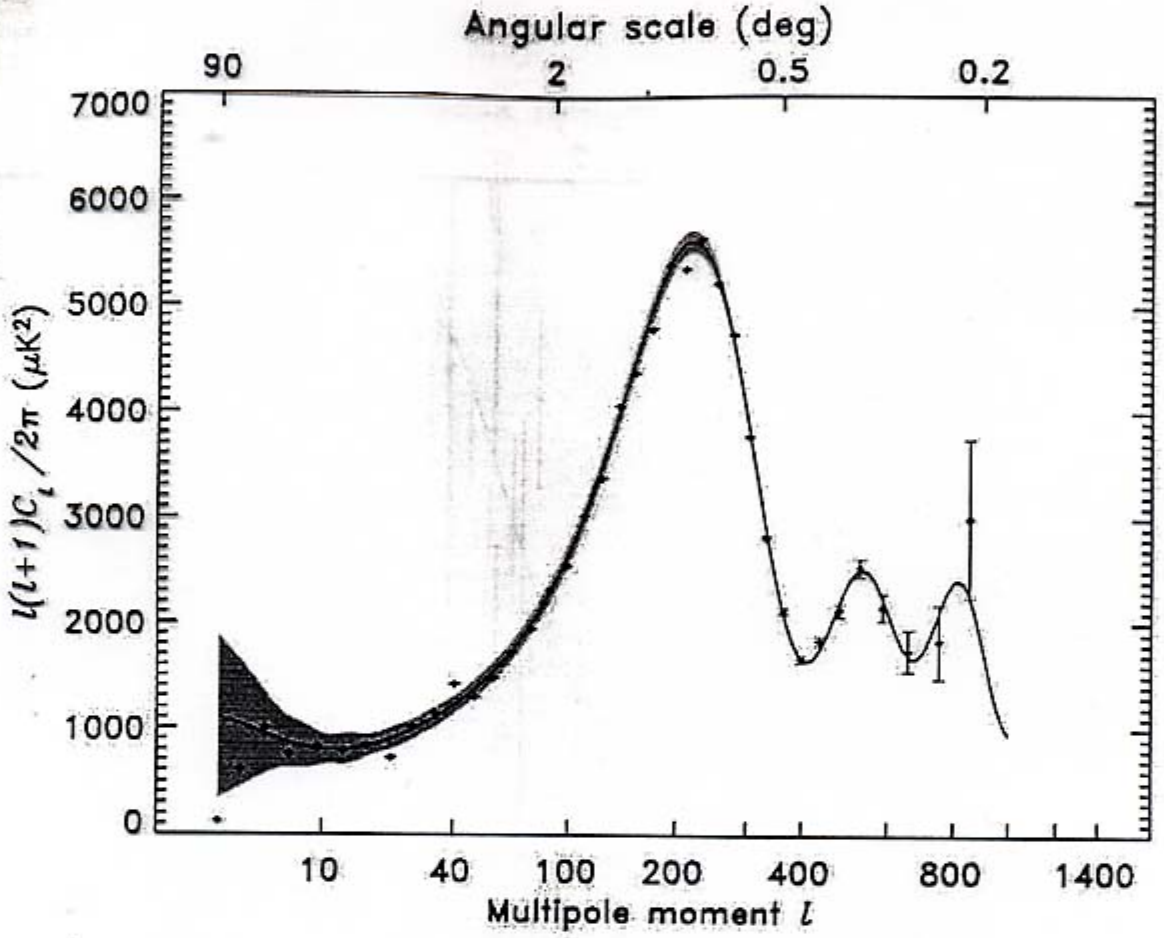


Fig. 8.— The final angular power spectrum, $l(l+1)C_l/2\pi$, obtained from the 28 cross-power spectra, as described in §5. The data are plotted with 1σ measurement errors only which reflect the combined uncertainty due to noise, beam, calibration, and source subtraction uncertainties. The solid line shows the best-fit ΛCDM model from Spergel et al. (2003). The grey band around the model is the 1σ uncertainty due to cosmic variance on the cut sky. For this plot, both the model and the error band have been binned with the same boundaries as the data, but they have been plotted as a splined curve to guide the eye. On the scale of this plot the unbinned model curve would be virtually indistinguishable from the binned curve except in the vicinity of the third peak.

Theoretical Overview of Cosmology

Qualitatively, the present situation in cosmology is similar to that in the particle physics: we have a very simple phenomenological model

SCM – the Standard Cosmological Model

$\Lambda CDM + (\underbrace{n_S = 1}_{\text{adiabatic}})$

which correctly describes all existing observational data

Typical accuracy of data ~ 10% (at 95% c.l.)

However, nobody thinks that SCM is the last word. Certainly, it is only the zero approximation and there is much new physics beyond it.

On the other hand, the matter content of SCM and the standard model of particle physics are practically not intersecting!

1. Non-relativistic gravitationally clustered dark matter ($p \geq 0$, $p \ll \varepsilon$)
~25%
2. Relativistic gravitationally unclusted dark energy ($p<0$, $|p| \sim \varepsilon$)
~70%

$$-1.38 < \frac{p}{\varepsilon} < -0.78$$

SCM

Cosmological parameters: (26)

$\Omega_\Lambda, \Omega_{CDM}, \Omega_b, \underset{HDM}{\Omega_\nu}, \Omega_\gamma, A$

at $H = H_0 = 72 \pm 7$ km/s/Mpc

with $\sum_i \Omega_i = 1$

5 parameters

$\Omega_\Lambda = 0.7 \pm 0.1$ $\qquad \Omega_m = \Omega_{CDM} + \Omega_b + \Omega_\nu$

dust-like matter

$\Omega_b h^2 = 0.022 \pm 0.002$ $\qquad (\Omega_b \approx (0.04 - 0.05))$

$h = H_0/100$

$$\Omega_\gamma h^2 = 2.49 \cdot 10^{-5} \left(\frac{T}{2.73 K}\right)^4$$

$$\frac{n_b}{n_\gamma} = 2.72 \cdot 10^{-8} \Omega_b h^2 \left(\frac{2.73}{T}\right)^3$$

$$\Omega_\nu h^2 = \frac{\sum_i m_{\nu i}\,(eV)}{93.6} \left(\frac{T}{2.73}\right)^3$$

$10^{-3} \leq \Omega_\nu \lesssim 0.015$ $\qquad \sum_i m_{\nu i} < 0.7\,eV$

$\tilde{A} \simeq 2.9 \cdot 10^{-5}$ $\qquad \langle \Phi^2_{in} \rangle = \tilde{A}^2 \int \frac{dk}{k}$

(10% accuracy) $\qquad$ at the MD stage

$$\rho_\Lambda = 6.44 \cdot 10^{-30} \cdot \frac{\Omega_\Lambda}{0.7} \cdot \left(\frac{h}{0.7}\right)^2 \cdot \frac{6.673 \cdot 10^{-8}}{G}$$

$g \cdot cm^{-3}$

$$\frac{\varepsilon_\Lambda}{M_{\varkappa}^4} = \frac{G^2 \hbar \, \varepsilon_\Lambda}{c^7} = 1.25 \cdot 10^{-123} \cdot (\ldots\ldots)$$

Cosmological numbers in SCM:

$\frac{\varepsilon_\Lambda}{M_{\varkappa}^4}$, $\frac{n_b}{n_\gamma}$, $\frac{n_{CDM}}{n_\gamma}$, A 4 numbers

(I don't include $\frac{n_\nu}{n_\gamma}$ since it is calculable and $= 3 \cdot \frac{3}{11}$)

In principle, all these numbers may be calculable from QFT/TOE.

Necessary conditions ("coincidences")

1. $A^{3/4} \sim \frac{n_\gamma}{n_{CDM}} \cdot \frac{\varepsilon_\Lambda^{1/4}}{m_{CDM}}$ $(A \sim \Omega_\gamma)$

The moment when $\frac{\delta\rho}{\rho} \sim 1$ at the cluster scale coincides with the moment when $\varepsilon_{CDM} \sim \varepsilon_\Lambda$

This is not a consequence of the anthropic principle (the latter would give $A^{3/4} \gtrsim \ldots$)

2. From the weak anthropic principle:

$$\frac{t_0}{t_{\mathcal{G}e}} \sim \left(\frac{M_{\mathcal{G}e}}{m_p}\right)^3$$

Using $t_0 \sim \Lambda^{-1/2}$ from observations:

$$\frac{\varepsilon_\Lambda}{M_{\mathcal{G}e}^4} \sim \left(\frac{m_p}{M_{\mathcal{G}e}}\right)^6$$

Regarding the anthropic principle itself: it should be taken into account (as one of existing data), but:

all conclusions based on it are biased!

("anthropic variance")

Models of dark matter

1. SUSY particles (neutralino et al.) — $m \sim 100$ GeV
2. Axion — $m \sim 10^{-5}$ eV
3. Ultra-light particles — $m \sim 10^{-23}$ eV (Gruzinov; Arbey et al. ....)
4. Brane tachyon condensate — No particle-like excitations

The latter has problems with caustics (singularities) formation

(see Frolov, Kofman, A.S., hep-th/0204187
Felder, Kofman, A.S., hep-th/0208019)

Lesson: generically, theories of the $\mathcal{L} = f(\varphi, \varphi_{,\mu}\varphi^{,\mu})$ type ("k-essence, k-matter") with a non-trivial dependence on $\varphi_{,\mu}\varphi^{,\mu}$ are incomplete, further derivatives should be taken into account

TACHYON CONDENSATE COSMOLOGY

Decay of an unstable D-brane

$$L = - V(T)\sqrt{1 - T_{,\mu}T^{,\mu}} \qquad \text{(Sen, 2002)}$$

I. FRW tachyon cosmology

$$\varepsilon = \frac{V(T)}{\sqrt{1-\dot{T}^2}}, \qquad p = -V(T)\sqrt{1-\dot{T}^2}$$

$$V(T) = V_0 = \text{const} \Rightarrow p = -\frac{V_0^2}{\varepsilon}$$

Chaplygin gas

$$H^2 = \frac{8\pi G\varepsilon}{3}, \qquad H = \frac{\dot{a}}{a}$$

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V_{,T}}{V} = 0$$

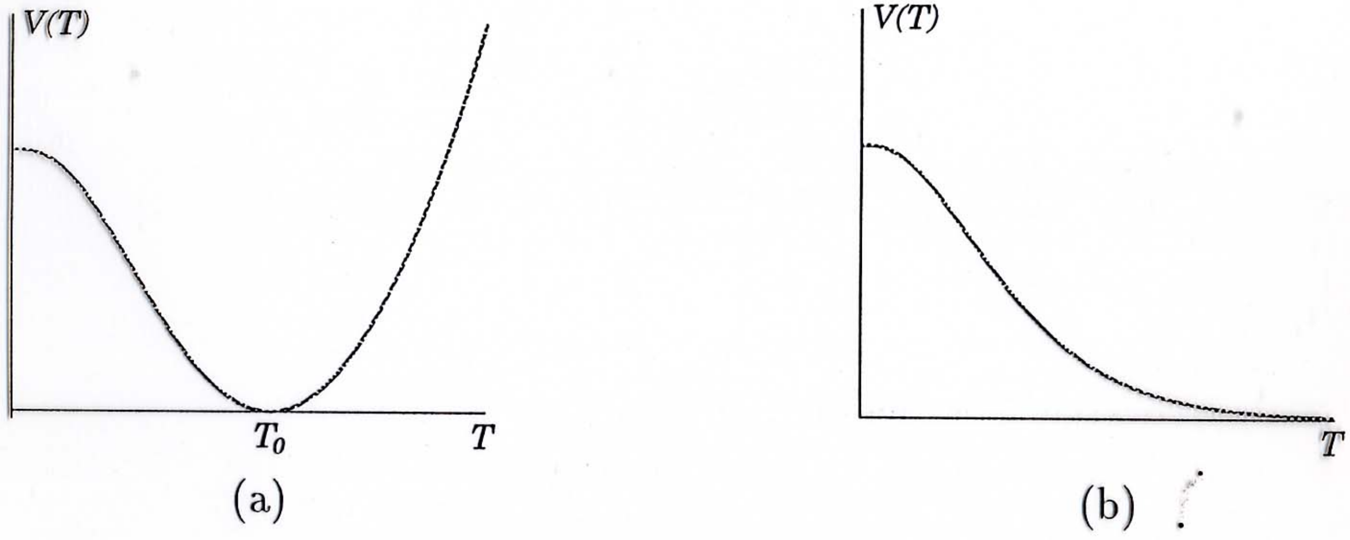


FIG. 1. Tachyon matter potentials with minima at finite (a) and infinite (b) values of the field. The potentials near minimum are taken to be: (a) $V(T) = \frac{1}{2}m^2(T - T_0)^2$, (b) $V(T) = V_0 e^{-T/T_0}$.

$V(T) \geqslant 0$

1. Zero minimum at finite $T = T_0$

$$V = \frac{1}{2} m^2 (T - T_0)^2$$

Oscillations $\left\langle \frac{p}{\varepsilon} \right\rangle = -\frac{1}{3}$

$\bar{\varepsilon} \sim a^{-2} \Rightarrow$ excluded by observations

2. No minimum at finite T

Let $\lim_{T \to \infty} T^2 V(T) = 0$

Theoretical calculations in string theory : $V(T) \sim e^{-\frac{T}{T_0}}$ or $V(T) \sim e^{-\frac{T^2}{T_0^2}}$

$(T \to \infty)$

$T = t + \theta, \quad |\theta| \ll t$

$$\dot{\theta} = -\frac{1}{2} \left(\frac{a}{a_0}\right)^6 \frac{V^2(T=t)}{V_0^2}$$

$$\varepsilon = V_0 \left(\frac{a_0}{a}\right)^3, \quad p = -\left(\frac{a}{a_0}\right)^3 \frac{V^2(T=t)}{V_0}$$

$|p| \ll \varepsilon$ – non-relativistic dark matter if $\lim_{T \to 0} V(T) T^2 = 0$

OTHER ACTIONS

1. BSFT (Kutasov et al., 2000)

$$\mathcal{L} = -\sqrt{2}\,\tau_p\, e^{-\frac{T^2}{2\alpha'}} F(y) \qquad y = T_{,\mu}T^{,\mu}$$

$$F(y) = \frac{\sqrt{\pi}\,\Gamma(1-y)}{\Gamma(\frac{1}{2}-y)}$$

$t \to \infty$: $T \to t$, $y \to 1$

Dust-like behaviour with positive

$p \to 0$, $p \ll \varepsilon$

2. The Lambert-Sachs model (2001)

$$F(y) = e^{y} + \sqrt{\pi(-y)}\,\mathrm{erf}(\sqrt{-y}) \qquad y<0$$

$$= e^{y} - 2\sqrt{y}\int_0^{\sqrt{y}} e^{x^2}dx \qquad y>0$$

$t \to \infty$: $T \sim \exp\left(\frac{t}{\sqrt{2\alpha'}}\right)$, $y \to \infty$

Dust-like behaviour with positive

$p \to 0$, $p \ll \varepsilon$

Evolution of inhomogeneities

1. Linear regime

$$ds^2 = (1+2\Phi)\,dt^2 - a^2(t)(1-2\Psi)\,d\vec{x}^{\,2}$$

$$\Phi = \Psi$$

$$\Phi_k(t) = \text{const}\,, \quad T_k(t) = \Phi_k \cdot t$$

↑ as in dust-like matter

2. Non-linear regime

$$T(\vec{x}, t) = t + u(\vec{x}, t) + f(\vec{x}, t)\left(\frac{a}{a_0}\right)^6 \frac{V^2(T=t)}{V_0^2}$$

$$\dot{u} = \Phi + \frac{1}{2a^2}\left(\nabla_{\vec{x}}\, u\right)^2$$

Cosmic Bernoulli equation for dust-like matter

Generically – caustics formation!

$\varepsilon \to \infty$ ("Zeldovich pancakes")

How to determine a solution beyond them?

Non-linear gravitational instability in Burgers approximation and beyond

Non-relativistic collisionless particles - - dark matter and baryons (gasodynamic effects are not taken into account)

$$f(\vec{r},\vec{q},t)\ ;\quad \vec{q}=a\vec{v}$$

$\Omega=1,\ a(t)\propto t^{2/3}$

$\rho_0=\frac{1}{6\pi G t^2}$

$$\rho=\frac{1}{a^3}\int f(\vec{r},\vec{q},t)\,d^3q$$

$$\rho\vec{u}=\frac{1}{a^4}\int \vec{q}\,f(\vec{r},\vec{q},t)\,d^3q$$

$$\begin{cases}\frac{1}{a^2}\Delta\varphi=4\pi G(\rho-\rho_0)\\ \frac{\partial f}{\partial t}+\frac{1}{a^2}\vec{q}\frac{\partial f}{\partial\vec{r}}-\nabla\varphi\frac{\partial f}{\partial\vec{q}}=0\end{cases}$$

Initial conditions:

$\Phi=\varphi_0(\vec{r}),\ f=f_0(|\vec{q}|),\ \rho_0=\frac{1}{a^3}\int f_0(\vec{q})\,d^3\vec{q}$

No vorticity ; pure potential motion initially.

Hydrodynamical approximation - - valid: a) for $L\gg L_{freestr.}$

b) before the formation of caustics

$$\begin{cases}\frac{1}{a^2}\Delta\varphi=4\pi G(\rho-\rho_0)\\ \frac{1}{a^3}\frac{\partial}{\partial t}(a^3\rho)+\frac{1}{a}\operatorname{div}(\rho\vec{u})=0\\ (a\vec{u})^{\cdot}+(\vec{u}\nabla)\vec{u}=-\nabla\varphi\end{cases}$$

$f_0(|\vec{q}|)=\delta(|\vec{q}|)$

CONCLUSION

Tachyon condesate may be an interesting model of dark matter (not dark energy), but the existing theory is internally incomplete

(one cannot make predictions in the case of generic inhomogeneous evolution)

A. Frolov, L. Kofman, A. S., hep-th/0204187 Phys. Lett. B545(2002)8

G. Felder, L. Kofman, A. S., hep-th/0208019 JHEP 0209(2002)026

MODELS OF DARK ENERGY

No preferred and completely satisfactory model.

No explanation of the value of $\frac{\varepsilon_{DE}}{M_P^4}$.

I. Exactly constant ε_{DE}.

$$a(t) = a_0 \sinh^{2/3}\left(\frac{3}{2} H_1 t\right) \qquad \varepsilon_{DE} = \frac{3H_1^2}{8\pi G}$$

$$H_1 = H_0\sqrt{1-\Omega_m}$$

$$q(t) \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{3}{2}\Omega_m - 1$$

$$\Omega_m(t) = \frac{8\pi G\varepsilon_m(t)}{3H^2(t)}, \quad H(t) = \frac{\dot{a}}{a}$$

$$r(t) \equiv \frac{\dddot{a}a^2}{\dot{a}^3} = 1$$

since the beginning of the matter dominated stage $(z < 3000)$

Another useful quantity

$$s(t) \equiv \frac{r-1}{3\left(q-\frac{1}{2}\right)} = 0$$

(r_0, s_0) – the "Statefinder" (Sahni et al., astro-ph/0201498; JETP Lett. 77, 201 (2003))

Present vision of dark energy

Two alteratives

1. Cosmological constant ≡ vacuum energy

Pure dark energy

2. Variable Λ-term:

a new kind of non-baryonic

dark matter

(Λ-matter ≡ quintessence)

Specific features (differing from non-relativistic CDM):

1. Non-clustered gravitationally (at least, up to scales $\sim 30h^{-1}$ Mpc)

2. Strongly negative effective pressure: $p<0$, $|p| = (0.8-1.4)\varepsilon$

UDM - unclustered dark matter

CLASSICAL COSMOLOGICAL TESTS AND THEIR INVERSION (RECONSTRUCTION OF H(z))

1. High-z supernovae test

$$D_L(z) = a_0(\eta_0 - \eta)(1+z), \qquad \eta = \int_0^t \frac{dt}{a(t)}$$

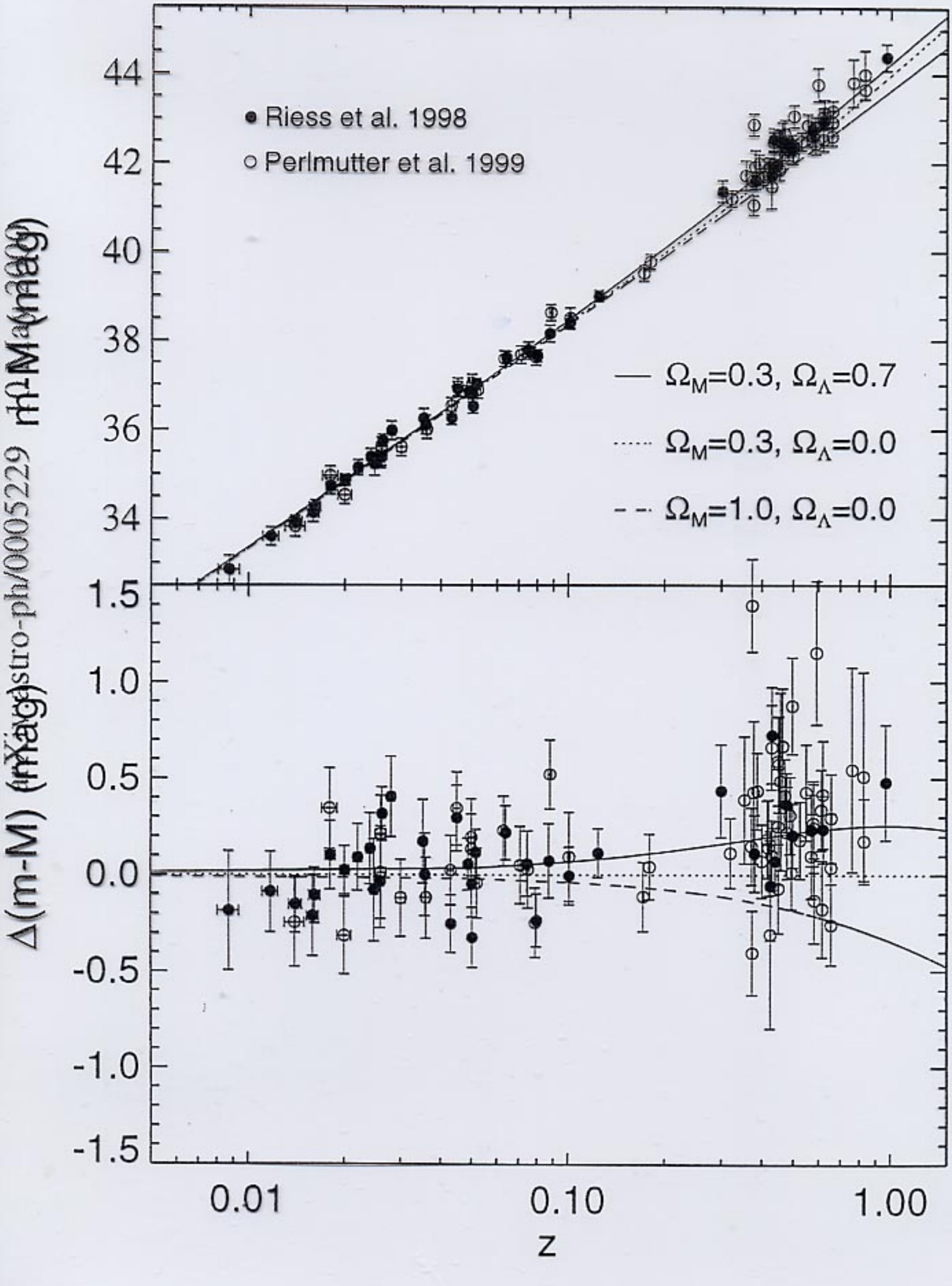
$$H(z) = \frac{da}{a^2 d\eta} = -(a_0 \eta')^{-1} = \left[\left(\frac{D_L(z)}{1+z} \right)' \right]^{-1}$$

2. Angular size test

$$\theta(z) = \frac{d}{a(\eta)(\eta_0 - \eta)} = \frac{d(1+z)}{a_0(\eta_0 - \eta)}$$

$$H(z) = -(a_0 \eta')^{-1} = \left[d \left(\frac{1+z}{\theta(z)} \right)' \right]^{-1}$$

$$m = M + 25 + 5 \log D_L \,(\mathrm{Mpc})$$



3. Volume element test

$$\frac{dN}{dz\,d\Omega} \propto \frac{dV}{dz\,d\Omega} = a^3 r^2 \left|\frac{dr}{dz}\right| =$$

$$= a^3 (r_0 - r)^2 \left|\frac{dr}{dz}\right| = f_V(z)$$

$$f_V(z) = \frac{1}{(1+z)^3 H(z)} \left(\int_0^z \frac{dz}{H(z)} \right)^2$$

$$H^{-1}(z) = \frac{d}{dz} \left\{ \left(3 \int_0^z f_V(z') (1+z')^3 dz' \right)^{1/3} \right\}$$

4. Ages of old objects at high z

$$T(z) > t_i(z)$$

$$T(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

$$H(z) = -\left((1+z) \frac{dT(z)}{dz} \right)^{-1}$$

1) Cosmological constant ("vacuum energy")
2) Casimir energy or vacuum polarization from additional compact or curved non-compact spatial dimensions $D = 4 + d$

$d = 2$ flat compact
$d = 1$ AdS

$$\varepsilon_{DE} = \frac{C}{R_d^4}$$

$R_d < 0.02$ cm $\Longrightarrow$ $\underline{0 < C < 40}$

from the absence of deviations from the Newton law at small distances → the most crucial test for this class of models

II. Variable ε_{DE}

Models borrowed from the inflationary scenario:

1) Quintessence ≡ minimally coupled scalar field with $V(\varphi)$

$$V(\varphi) = V_0 \left(\frac{M_P}{\varphi}\right)^n$$ "tracker" potential

For a "tracker" initial condition $\varphi_{in} \ll M_P$, now: $\varphi(t_0) \sim M_P$, $\varepsilon_{DE}(t_0) \sim V_0$.

$\underline{n < 1.5}$ – from comparison with data

New (actually, very old) way of introducing the inflationary paradigm

Physical

In some period in the past, matter in the Universe was <u>qualitatively</u> the same as the main part of matter in the present Universe

Geometrical

Evolution of the Universe – – transition between two maximally symmetric states (space-times, in particular)

$\dashrightarrow$ De Sitter $\Rightarrow$ FRW $\Rightarrow$ De Sitter $\dashrightarrow$

"Quintessence – inflaton today"

Another lesson from the inflationary scenario for models of dark energy:

Hydrodynamical models do not work, field-theoretical description is needed

Failure of the "Chaplygin gas" model

$$p = -\frac{\varepsilon_0^2}{\varepsilon}$$

$$\varepsilon_{DE} = \sqrt{A + B\left(\frac{a_0}{a}\right)^6}$$

Seems to describe both the MD stage in the past and the transition to the Λ-dominated stage today.

However, λ_J is too large!

$\lambda_J \sim v_s t \sim t^3$ at the MD stage

Perturbations stop growing for

$z \sim 3$ in the scale 100 Mpc

$z \sim 14$ 1 Mpc

Wrong $P(k)$ today!

However, typical inflationary prediction for φ_{in} (after the end of inflation) is $\varphi_{in} \gg M_P$. Then

$$\varphi \approx const, \quad \varepsilon_{DE} \approx const$$

In this model, inflation in the early Universe explains the constancy of dark energy in the present Universe

$\longleftrightarrow$

The quintessence model works if and only if the WEC is satisfied for dark energy

If so, the unambiguous reconstruction of $V(\varphi)$ from observational data is possible, in principle (A.S., 1998)

$$q_0 = -1 + \left.\frac{d\ln H}{d\ln(1+z)}\right|_{z=0}$$

$$\Lambda = \text{const} \rightarrow H^2(z) = H_0^2\left(1-\Omega_m+\Omega_m(1+z)^3\right)$$

$$\rightarrow \quad q_0 = \frac{3}{2}\Omega_m - 1$$

II. Reconstruction of $V(\varphi)$ from $H(a)$

$$\begin{cases} 8\pi G V = aH\dfrac{dH}{da} + 3H^2 - \dfrac{3}{2}\Omega_m H_0^2 \cdot \left(\dfrac{a_0}{a}\right)^3 \\ 4\pi G\, a^2 H^2 \left(\dfrac{d\varphi}{da}\right)^2 = -aH\dfrac{dH}{da} - \dfrac{3}{2}\Omega_m H_0^2\left(\dfrac{a_0}{a}\right)^3 \end{cases}$$

Necessary condition $(\varepsilon_\Lambda + p_\Lambda \geq 0)$

$$\frac{dH^2}{dz} \geq 3\Omega_m H_0^2(1+z)^2$$

$$H^2 \geq H_0^2\left(1+\Omega_m(1+z)^3 - \Omega_m\right)$$

In particular: $\quad q_0 \geq \frac{3}{2}\Omega_m - 1$

No such a condition in case of non-minimal coupling

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

Can dark energy be decaying?

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Received 15 February 2003
Accepted 25 March 2003
Published
Online at stacks.iop.org/JCAP/2003

JCAP 03(2003)001

Abstract. We explore the fate of the Universe given the possibility that the density associated with 'dark energy' may decay slowly with time. Decaying dark energy is modelled by a homogeneous scalar field which couples minimally to gravity and whose potential has *at least one* local quadratic maximum. Dark energy decays as the scalar field rolls down its potential, consequently the current acceleration epoch is a transient. We examine two models of decaying dark energy. In the first, the dark energy potential is modelled by an analytical form which is generic close to the potential maximum. The second potential is the cosine, which can become negative as the field evolves, ensuring that a spatially flat Universe collapses in the future. We examine the feasibility of both models using observations of high redshift type Ia supernovae. A maximum likelihood analysis is used to find allowed regions in the $\{m, \phi_0\}$ plane (m is the tachyon mass modulus and ϕ_0 the initial scalar field value; $m \sim H_0$ and $\phi_0 \sim M_P$ by order of magnitude). For the first model, the time for the potential to drop to half its maximum value is larger than ~ 8 Gyr. In the case of the cosine potential, the time left until the Universe collapses is always greater than ~ 18 Gyr (both estimates are presented for $\Omega_{0\mathrm{m}} = 0.3$, $m/H_0 \sim 1$, $H_0 \simeq 70$ km s^{-1} Mpc^{-1}, and at the 95.4% confidence level).

Keywords: dark energy theory, supernova type Ia

 PII: S1475-7516(03)61302-9 1475-7516/03/000001+13$30.00

$$V = V_0 - \frac{m^2 \varphi^2}{2}$$

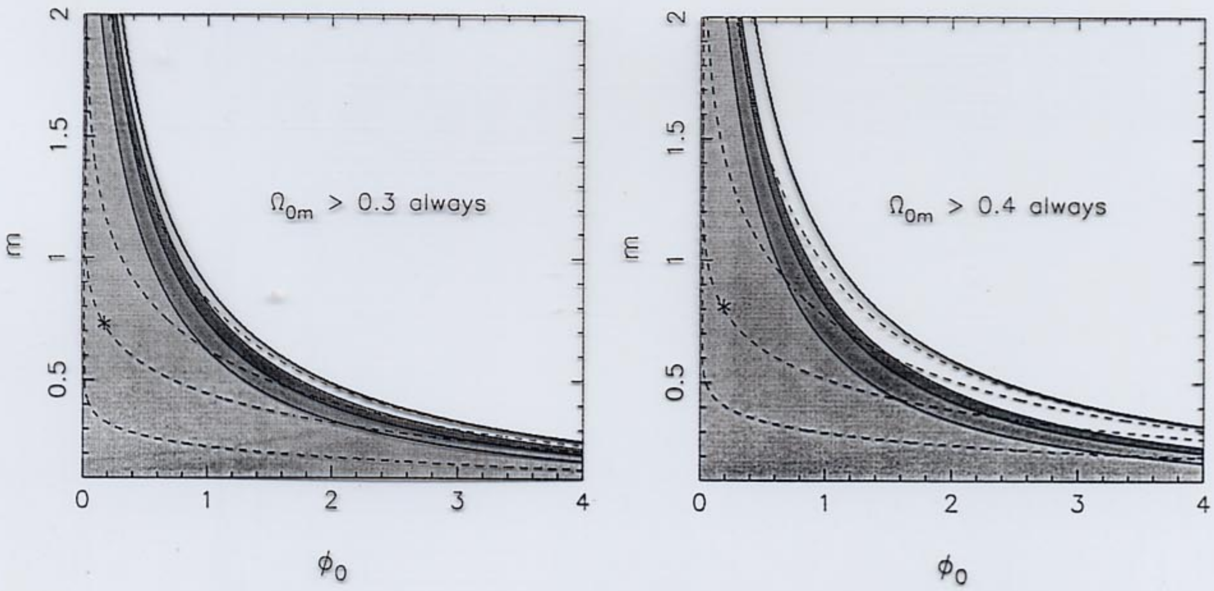


Figure 2. A magnified part of figure 1 with (dashed) lines of constant $\Delta T_{1/2}$ added. Again, in the left panel, the present value of the matter density is $\Omega_{0m} = 0.3$, and in the right panel it is $\Omega_{0m} = 0.4$. $\Delta T_{1/2}$ is the time, measured from the present epoch, to when the DDE potential has dropped to half its maximum value: $V(\phi) = V_0/2$. The values of $\Delta T_{1/2}$ for the dashed curves (from top to bottom) are listed in table 1 (from left to right). For both $\Omega_{0m} = 0.3$ and $\Omega_{0m} = 0.4$, the minimum time elapsed before the potential drops to half its maximum value is $\Delta T_{1/2} \simeq 0.6 H_0^{-1} \simeq 8$ Gyr ($H_0 = 70$ km s^{-1} Mpc^{-1}) at the 95.4% confidence level. In the region to the right of the thick solid curve, parameter values are such that the matter density never reaches its present value. This region is therefore disallowed by observations.

Table 1. Time taken for the potential (1) to drop to half its maximum value.

Ω_{0m}	$\Delta T_{1/2}$ (Gyr)			
0.3	8.3	42.0	139.7	712.8
0.4	8.3	34.9	153.6	712.8

virtually indistinguishable from ΛCDM. For $\Omega_{0m} = 0.3$, we have $\chi^2_{\rm dof} = 1.053$ at the best-fit point of $m = 0.74, \phi_0 = 0.23$, and for $\Omega_{0m} = 0.4$, $\chi^2_{\rm dof} = 1.047$ for the best-fit model having $m = 0.79, \phi_0 = 0.24$. As in the previous analysis, ΛCDM is marginally preferred over the best-fit DDE for $\Omega_{0m} < 0.3$.

In figure 4, we examine the results for this potential more closely by focusing on a smaller region in parameter space. In this region we show the lines of constant $\Delta T_{\rm end}$, which is the time left until the Universe ceases to accelerate. (In terms of the deceleration parameter, $q(t_0 + \Delta T_{\rm end}) = 0$.) We see that, at the 95.4% confidence level, the Universe will continue accelerating *for at least* ~ 10 Gyr. We also plot the lines of constant $\Delta T_{\rm coll}$, which is the time left until the Universe collapses: $H(t_0 + \Delta T_{\rm coll}) = 0$. We find that the

$V = V_0 \cos \frac{\varphi}{f}$, $f = \frac{\sqrt{V_0}}{m}$

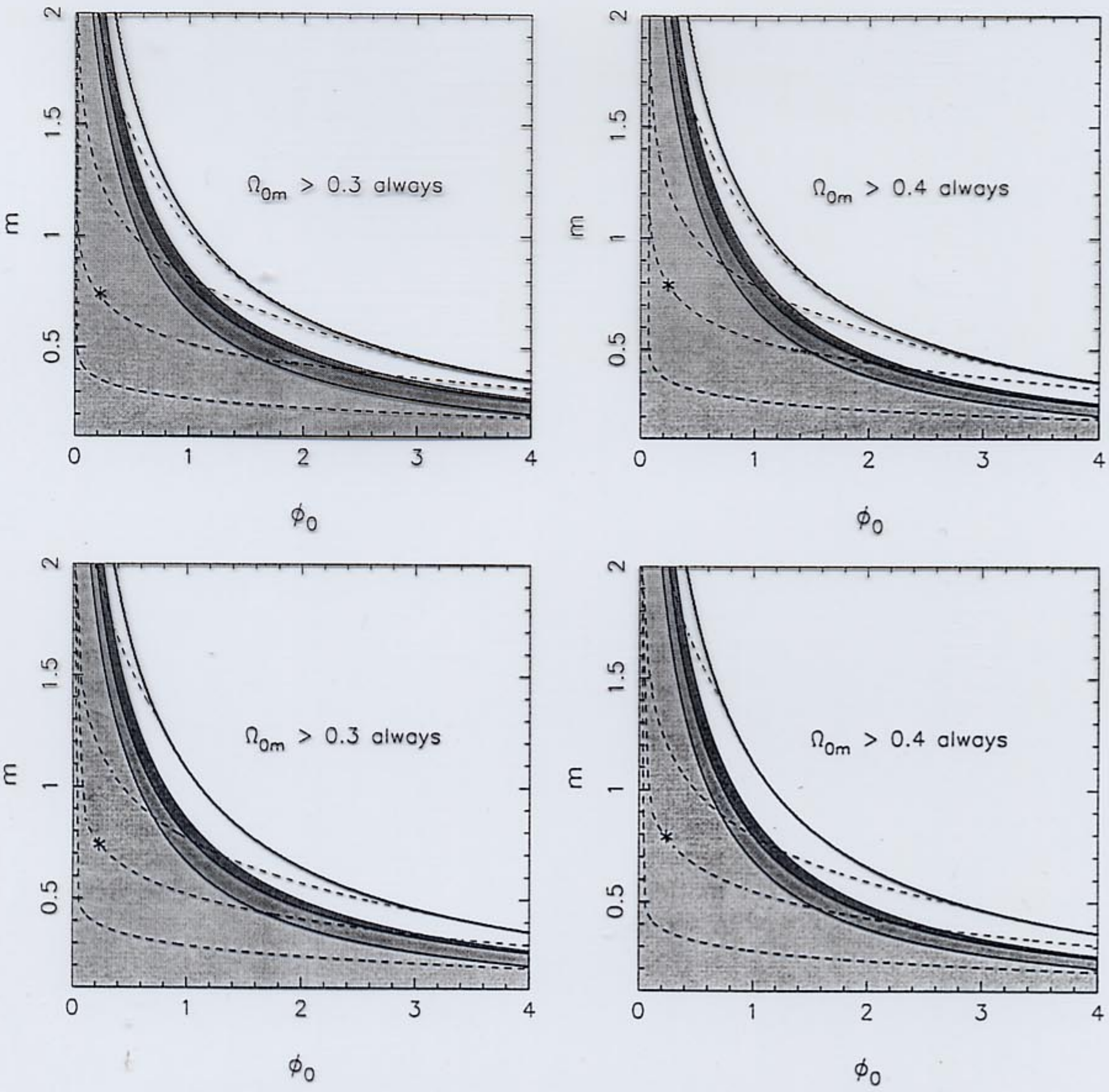


Figure 4. A magnified part of figure 3 with (dashed) lines of constant $\Delta T_{\rm end}$ (upper panels) and constant $\Delta T_{\rm coll}$ (lower panels) added. In the left panels, the present value of the matter density is $\Omega_{\rm 0m} = 0.3$, and in the right panels it is $\Omega_{\rm 0m} = 0.4$. In the upper panels, the time $\Delta T_{\rm end}$ is measured from the present epoch to when the Universe stops accelerating: $q(t_0 + \Delta T_{\rm end}) = 0$. The values of $\Delta T_{\rm end}$ for the dashed curves (from top to bottom) are listed in table 2 (from left to right). For both $\Omega_{\rm 0m} = 0.3$ and $\Omega_{\rm 0m} = 0.4$, the minimum time taken for the deceleration parameter to rise to zero is $\Delta T_{\rm end} \simeq 0.7 H_0^{-1} \simeq 10$ Gyr (at the 95.4% confidence level). For the lower panel, the dashed lines correspond to the time $\Delta T_{\rm coll}$ until the Universe collapses: $H(t_0 + \Delta T_{\rm coll}) = 0$. The values of $\Delta T_{\rm coll}$ for the dashed curves (from top to bottom) are listed in table 2 (from left to right). For both $\Omega_{\rm 0m} = 0.3$ and $\Omega_{\rm 0m} = 0.4$, the minimum time to collapse is $\Delta T_{\rm coll} \simeq 1.3 H_0^{-1} \simeq 18$ Gyr at the 95.4% confidence level (we assume $H_0 = 70$ km s^{-1} Mpc^{-1}). In the region to the right of the thick solid curve the matter density never reaches its present value of $\Omega_{\rm 0m} = 0.3$ (left panels) and $\Omega_{\rm 0m} = 0.4$ (right panels), therefore this region is disallowed by observations.

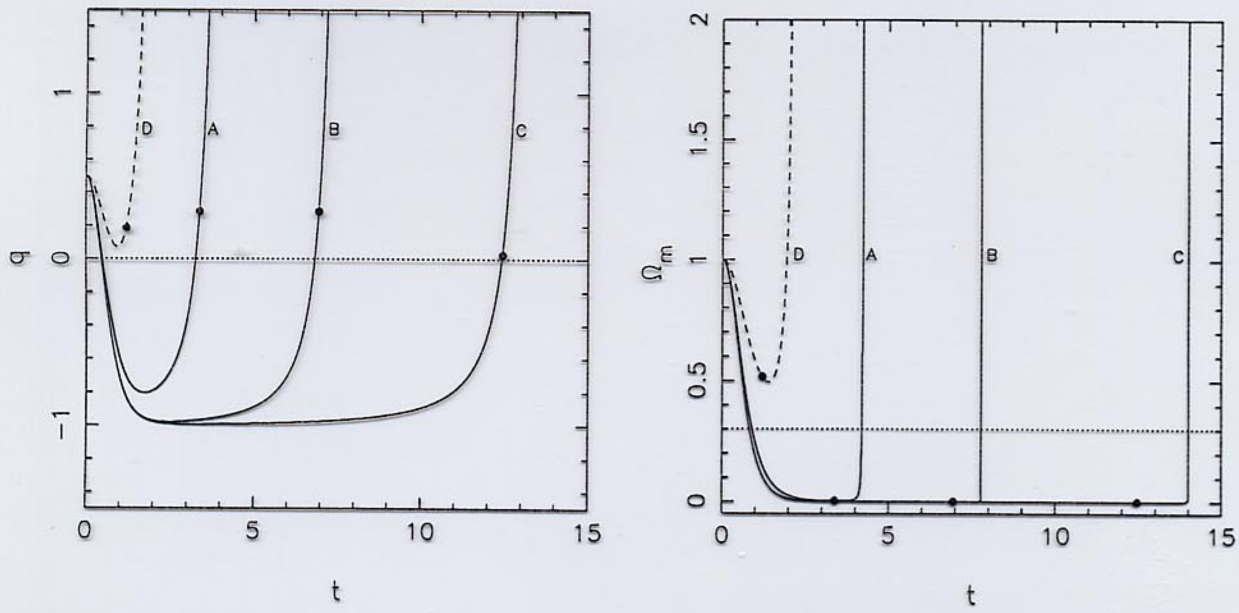


Figure 5. The evolution of the deceleration parameter q and the matter density $\Omega_{\rm m}$ is shown for four different DDE models corresponding to different choices of m and ϕ_0 in the DDE potential $V(\phi) = V_0\cos(m\phi/\sqrt{V_0})$ ($\Omega_{0\rm m} = 0.3$). Time t is in units of $\sqrt{3/8\pi G V_0}$. The models have parameter values: $m = 1.0, \phi_0 = 0.6$ (A), $m = 1.0, \phi_0 = 0.2$ (B), $m = 0.74, \phi_0 = 0.23$ (C), $m = 1.0, \phi_0 = 1.2$ (D). Models A,B,C are allowed by supernova observations at the 95.4% confidence level. The dashed line D in both panels shows the time evolution of q and $\Omega_{\rm m}$ for a DDE model with $m = 1.0, \phi_0 = 1.2$. This model is disallowed by observations since the matter density always remains larger than 0.3 (see figure 4). The horizontal dotted line in the left panel ($q = 0$) divides this panel into two regions. In the upper region $q > 0$ and the universe decelerates, whereas $q < 0$ in the lower region in which the universe accelerates. The points of intersection of $q = 0$ with A,B,C show the commencement and end of the acceleration epoch in these models. The horizontal dotted line in the right panel marks the present epoch when $\Omega_{0\rm m} = 0.3$. The solid circles in both left and right panels show the epoch when the potential energy of the scalar field falls to zero. Note that this occurs *after* the universe stops accelerating.

2) Dark energy models based on scalar-tensor gravity

$$\mathcal{L} = \frac{1}{2}\left(-F(\Phi)R + \Phi_{,\mu}\Phi^{,\mu}\right) - V(\Phi) + \mathcal{L}_m$$

Become necessary if the WEC is violated for dark energy

Unambiguous reconstruction of $V(\Phi)$ and $F(\Phi)$ is possible, in principle, using two independent cosmological tests (e.g., $D_L(z)$ and $\left(\frac{\delta\rho}{\rho}\right)_m(z)$) (Boisseau et al, 2000)

However, possible violation of WEC is strongly restricted by absence of deviations from the Einstein gravity in Solar system experiments

$$\omega(\Phi_0) \equiv \left.\frac{F(\Phi)}{\left(\frac{dF}{d\Phi}\right)^2}\right|_{\varphi=\varphi_0} > 3500$$

3) Models with "gravity leaking to higher dimensions"

The simplest model (Dvali et al.)

Gravity in the $D=5$ bulk + induced gravity on the brane

$$H^2 = \left(\sqrt{\frac{8\pi G\rho}{3} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right)^2$$

$$H_0 r_c = \frac{1}{1-\Omega_m} \approx 1.4$$

$$a = a_0 \sinh^{2/3} \varphi$$

$$\frac{3t}{2r_c} = \varphi - \frac{e^{-2\varphi}}{2} + \frac{1}{2}$$

$$\Omega_m(t) = e^{-2\varphi}$$

$$q(t) = \frac{2\Omega_m - 1}{1+\Omega_m}$$

$$r(t) = 1 - \frac{9\Omega_m^2(1-\Omega_m)}{(1+\Omega_m)^3}$$

$\Omega_m = 0.3$: $\quad q_0 = -0.31$, $r_0 = 0.74$, $s_0 = 0.11$

This model is on verge to be falsified, but still not!

Features beyond SCM in data

1. Anomalously low $\ell = 2, 3$ multipoles

May occur by chance simply.

A number of ways to explain them by complicating SCM (double inflation, tuned inflation, non-trivial spatial topology etc.)

2. Features in the C_ℓ spectrum at $\ell = 40-50$ and $\ell = 200-220$

Statistical significance is not clear.

Physical mechanisms unknown.

3. Running of n_s

(MAP: $\frac{dn_s}{d\ln k} \approx -0.03$ instead of expected $\sim |n_s - 1|^2 \sim 10^{-3}$)

Disputed!

Does not follow from the MAP data only, and even from the MAP+2dF+ +HST data.

If the Ly-α data are not used, $n_s = \text{const}$ and even $n_s \equiv 1$ is in agreement with all data.

4. Large reonization optical depth $\tau = 0.17^{+0.04}_{-0.04}$

(instead of expected $\tau \approx 0.05$ due to first quasars at $z \sim 6$)

Reonization at $z = 20^{+10}_{-9}$

Interesting and probable.

"Population III stars" formed before galaxies and quasars

However, not clear if this is really "beyond SCM"

A more concrete model of CDM is needed.

5. $n_s - 1 \neq 0$ (though small)

Expected on theoretical grounds (any sign possible)

Degenerate with τ

Some evidence for $\overline{n_s} < 1$ (~ 0.96) but at 1σ level only

CONCLUSION

Many ways to go beyond SCM, but no preferred variant from a more fundamental theory and no hints from data at present.

New level of accuracy (~1% at least) is needed.

TRANS- PLANCKIAN EFFECTS

I. Refraction effects

$$\omega^2(k) = k^2\left(1 + \xi \frac{k}{M_{Pl}} + O\left(\frac{k^2}{M_{Pl}^2}\right)\right)$$

Strong upper bounds on $|\xi|$ from different effects

$$|\xi| < (10^{-1} - \underbrace{2\cdot 10^{3}}_{\text{J. Ellis et al. (2003)}})$$

II. Particle creation effects

1. Trans-Planckian particle creation in the present Universe
2. Possible corrections to inflationary perturbation spectra

Trans-Planckian particle creation in cosmology

1. A.A.S., JETP Lett. 73(2001) 415
(astro-ph/0104043)
2. A.A.S., I. Tkachev, JETP Lett. 76(2002) 235
(astro-ph/0207572)

Expansion of the Universe

$\downarrow$

$$\vec{k} = \frac{\vec{n}}{a(t)} \; ; \; \vec{n} = \text{const} \; ; \; \omega = \frac{|\vec{n}|}{a} \quad \left(\begin{array}{c}\text{for } m=0 \\ \text{and WKB}\end{array}\right)$$

All modes had $\omega > M_p$ at sufficiently early times.

Nothing dangerous in the Lorentz-invariant case ($\omega^2 - \vec{k}^2$ should be less than M_p^2)

However, the Lorentz invariance may be broken for some $\omega = M$.

Reasons.

1. Higher-dimensional models of the Universe with non-flat additional dimensions

2. Condensed matter analogs of gravity (where an underlying microscopic theory has less symmetry than an effective macroscopic theory)

3. "Double special relativity"

A simple model of a Lorentz-non-invariant theory in 4D:

a non-standard dispersion law $\omega(k)$ with respect to some preferred reference frame (identified with the FRW frame) with

$$\omega(k) = k \quad \text{for} \quad k < M$$

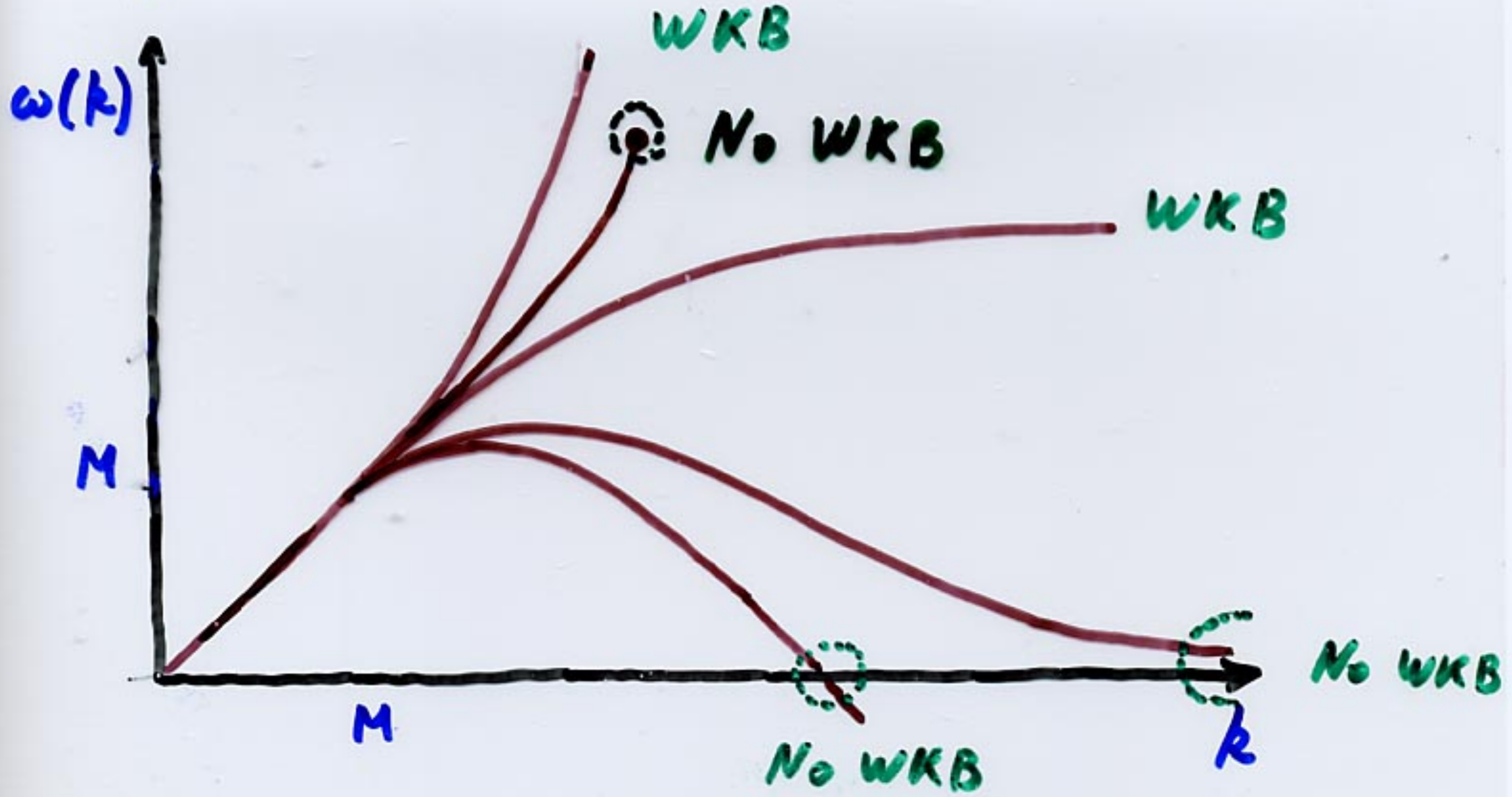
Proposed forms:

1) $\omega(k) = M \tanh^{1/m}\left[(k/M)^m\right]$ (Unruh)

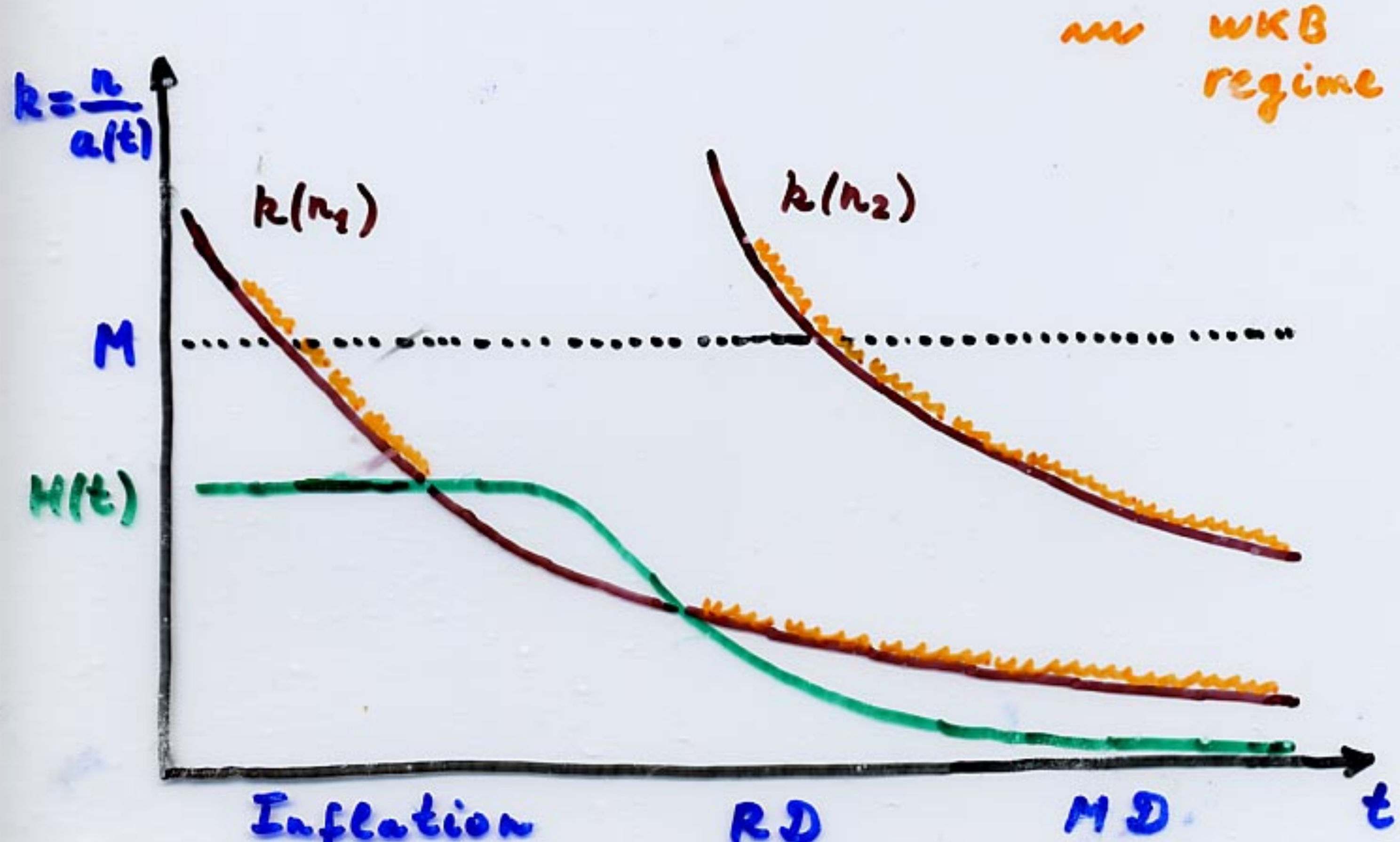
2) $\omega^2 = k^2\left[1 + b_m (k/M)^{2m}\right]$ (Corley & Jakobson, $m=2$)

3) $\omega = M \ln\left(1 + \frac{k}{M}\right)$ (Kowalski-Glikman)

....



ω(k)
WKB
No WKB
WKB
M
M
No WKB
k
No WKB



WKB regime
k = n/a(t)
k(n1)
k(n2)
M
H(t)
Inflation
RD
MD
t

Model: $D_i D^i \varphi = 0$

$$\ddot{\varphi}_n + 3H\dot{\varphi}_n + \omega^2\left(\frac{n}{a}\right)\varphi_n = 0 \qquad H \equiv \frac{\dot{a}}{a}$$

$$\boxed{H \ll M}$$

For $H \ll \omega \ll M$:

$$\varphi_n = \frac{\alpha_n}{\sqrt{2na}} e^{-in\eta} + \frac{\beta_n}{\sqrt{2na}} e^{in\eta}, \quad \eta = \int \frac{dt}{a(t)}$$

WKB solution

All trans-Planckian physics encoded in α_n, β_n

$$\hat{\varphi} = (2\pi)^{-3/2} \int d^3n \left(\varphi_n(t) e^{i\vec{n}\vec{x}} \hat{a}_n + \varphi_n^*(t) e^{-i\vec{n}\vec{x}} \hat{a}_n^+ \right)$$

$$|\alpha_n|^2 - |\beta_n|^2 = 1 \Rightarrow \langle N \rangle = |\beta_n|^2$$

$$\frac{d(\langle \varepsilon \rangle a^4)}{a^4 dt} = \frac{g M^4 H}{2\pi^2} N(n)\Big|_{n = Ma}$$

Two possible cases

1. The WKB condition

$$\frac{H\,|d(1/\omega(k))|}{d\ln k} \ll 1$$

for all $k > M$ $\Rightarrow$ $\beta_n = 0, \ |\alpha_n| = 1$

2. The WKB condition is violated for some $k_0 > M$ (or for $k \to \infty$).

Since $H \ll M$, this typically requires $\omega \to 0$ for $k \to k_0$, or $k \to \infty$

(or if $\frac{d\omega}{dk}$ diverges for $k = k_0$ and finite $\omega(k_0)$)

Then: $N(n) \neq 0$

$$\beta = \beta^{(0)}(n) + \beta^{(1)}(n)\frac{H}{M} +$$

$$N(n) = N^{(0)} + N^{(1)}(n)\frac{H^2}{M^2} + \ldots \quad \left(\begin{matrix}\text{osc. in } n\\ \text{terms}\end{matrix}\right)$$

↑ does not depend on n from time translation invariance

$N^{(0)} = |\beta^{(0)}|^2$

$N^{(1)} = |\beta^{(1)}|^2$

Absence of large amount of created ultrarelativistic particle due to present expansion of the Universe

⇓

$$N^{(0)} \ll \frac{H_0^2}{M^2} \sim 10^{-122} \quad (\text{if } M = M_P)$$

$$N^{(1)} \ll 1$$

As a result: no significant corrections ($< \frac{H^2}{M_P^2} < 10^{-10}$) to inflationary perturbation spectra

An example how the second effect may be produced

A toy model

The "initial" condition for each Fourier mode $\vec{k}$ –

instantaneous minimal energy state at $k(t) = M$

Some trans-Planckian physics is needed to produce it!

Adiabatic vacuum state has a larger energy than this state just at the moment $t(k)$ (by an amount $\sim \frac{\omega(k)}{2} \cdot \frac{H^2(t)}{M_P^2}$), but a smaller energy when average over a time interval $\Delta t \gg k/\omega$

Results

1. Minimally coupled scalar particles (for $\omega = k$) $|\beta_n^{(1)}| = 1/2$

2. Photons $|\beta_n^{(1)}| = \frac{M}{4}\left[\frac{k^2}{\omega^2}\left|\frac{d}{dk}\left(\frac{\omega}{k}\right)\right|\right]_{k=M} \sim 1$

A deviation from the standard dispersion law is required

Oscillations in the inflationary spectrum

$$\frac{\alpha_n e^{-in\eta} + \beta_n e^{in\eta}}{\sqrt{2n}\, a(\eta)} \longrightarrow \frac{i(\alpha_n - \beta_n) H_n}{\sqrt{2n^3}}$$

at $\frac{n}{a} = H$

$\omega \gg H$ (inside Hubble radius)

$\omega \ll H$ (outside Hubble radius)

Relative phase of α_n and β_n becomes important

t or $\eta \longrightarrow k = \frac{n}{a(t)}$, n - fixed

$$\left(\frac{d^2\varphi_n}{d\ln k}\right)^2 + \left(-3 + \frac{d\ln H}{d\ln k}\right)\frac{d\varphi_n}{d\ln k} + \frac{\omega^2(k)}{H^2}\varphi_n = 0$$

$$H \equiv \frac{\dot{a}}{a} = H(k)$$

For $\omega \ll H$, β_n contains the phase factor

$$e^{-i\alpha}, \quad \alpha = \int_0^{\ln k_1} \frac{\omega}{H}\, d\ln k \sim \frac{M}{H_{TP}}$$

at $k = M$

k_1 - point where WKB ends at $\omega \gg M$

Period of oscillations $\Delta \ln k \sim \frac{M_{TC}}{H} \ll 1$ -

Superimposed : $M_{TC} H^{-1} d\ln H / d\ln k$ - unobservable

Upper limits from present UHECR

A.A.S., I. Tkachev, JETP Lett. (2002)
(astro-ph/0207572)

TPPC → UHECR → electromagnetic
$E_{in} = M$

cascades → sub-TeV range

EGRET bound: $S_0 \approx 10^3\, \mathrm{eV\, cm^{-2} s^{-1} sr^{-1}}$

over the range $30\, \mathrm{MeV} < E_\gamma < 100\, \mathrm{GeV}$

$$\mathcal{J} = \frac{d(a^4 \varepsilon)}{a^4 dt} = \frac{g \tilde{N} M^2 H}{2\pi^2} |\beta_n|^2 < \frac{4\pi S_0}{c T_0}$$

$\tilde{N} \sim 10^2 - 10^3$ - number of particle species

$g = 2$ for photons and neutrinos
T_0 - the age of the Universe $(T_0 H_0 \approx 1)$

$$|\beta_n^{(1)}| \lesssim 10^{-6} \frac{1}{\sqrt{\tilde{N}}} \frac{M_g}{M}$$

$$H_0^{-1} \dot{\Omega}_{TPPC} < 10^{-11}$$

The second effect is strongly suppressed, too

CONCLUSIONS

1. The TPPC effect is strongly suppressed (if exists at all).

2. For $M \gtrsim M_{inf}$, no noticeable contributions to primordial spectra of perturbations produced during inflation $\frac{\delta\rho}{\rho} < 10^{-11} \frac{1}{\sqrt{N}} \left(\frac{M_P}{M}\right)^2 \ll 1$

3. Still the effect may be noticed by its possible contribution to UHECR and other recent effects

New possibility for explaining the excess of UHECR with $E > 10^{20}\,eV$ in the AGASA data

(alternative to supermassive particles with a decay time $\gg 10^{18}\,s$)