

	$E$ (A · GeV)	$\sqrt{s_{NN}}$ (GeV)
AGS BNL Au+Au	11	4.7
SPS CERN Pb+Pb	20	6.3
	30	7.6
2003 →	40	8.8
2002 →	80	12.3
1996-2000 →	160	17.4
RHIC BNL Au+Au	$9.0 \times 10^3$	130
2001-...	$2.1 \times 10^4$	200

LHC CERN  $1.3 \times 10^7$  5000

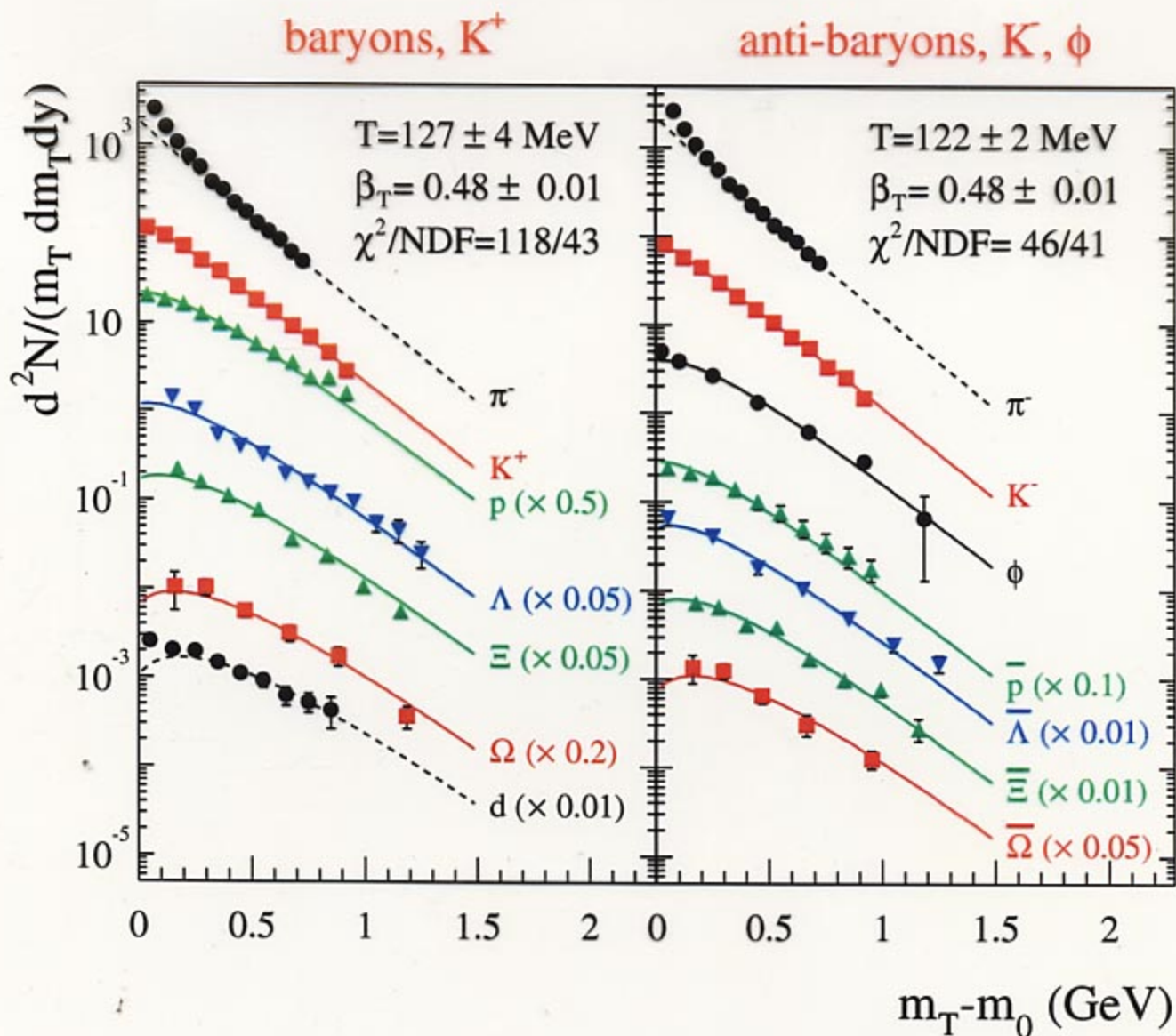
$\hbar = c = 1$   $1 \text{ fm} \approx \frac{1}{200 \text{ MeV}}$

$K = 1$

$m_\pi \approx 140 \text{ MeV}, m_N \approx 940 \text{ MeV}$

$\sqrt{s_{NN}} = (2m_N^2 + 2Em_N)^{1/2}$

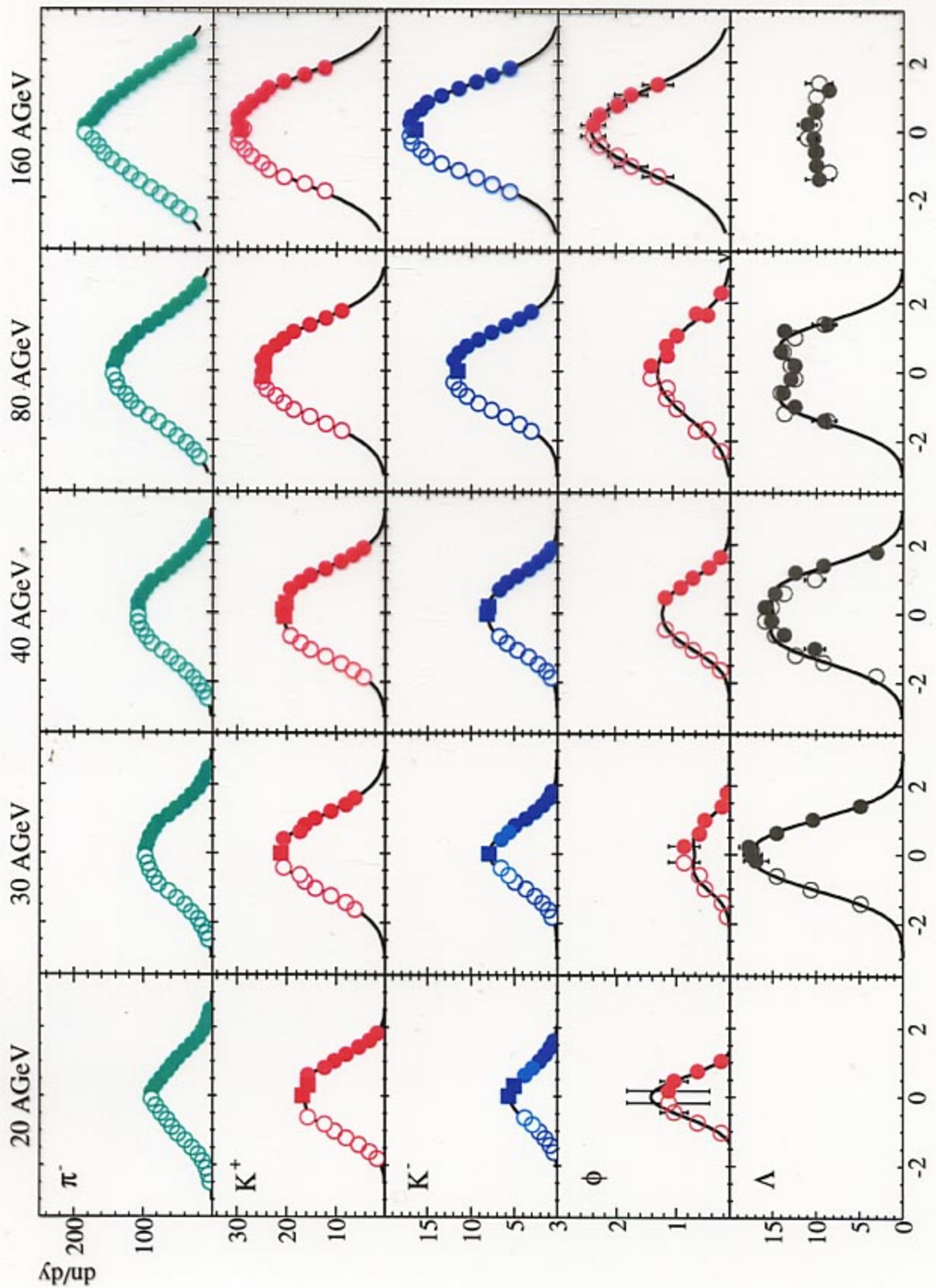
$F \equiv (\sqrt{s'} - 2m_N)^{3/4} / s'^{1/8} \approx (\sqrt{s'})^{1/2}$



$Pb+Pb$ ,  $158 \text{ A}\cdot\text{GeV}$ , central ( $\sim 5\%$ )  
 $y=0$

$$\frac{dN}{m_T dm_T dy} \sim K_1 \left( \frac{m_T \cosh y_T}{T} \right) I_0 \left( \frac{p_T \sinh y_T}{T} \right)$$

$$m_T = (m^2 + p_T^2)^{1/2}; \quad y_T = \text{th}^{-1} v_T, \quad v_T \approx 0.5$$



galaxy  
 $10^{21}$  m



matter  
 $10^{-1}$  m



DNA  
 $10^{-8}$  m



crystal  
 $10^{-9}$  m



atom  
 $10^{-10}$  m



$$\hbar = 1$$

$$c = 3 \times 10^8 \frac{\text{m}}{\text{sec}} = 1$$

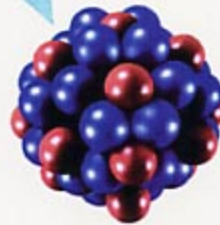
$$1 \text{ fm} = 10^{-15} \text{ m}$$

$$1 \text{ fm}/c = 10^{-23} \text{ sec}$$

nucleon  
 $10^{-15}$  m



atomic nucleus  
 $10^{-14}$  m



electron

$< 10^{-18}$  m



quark



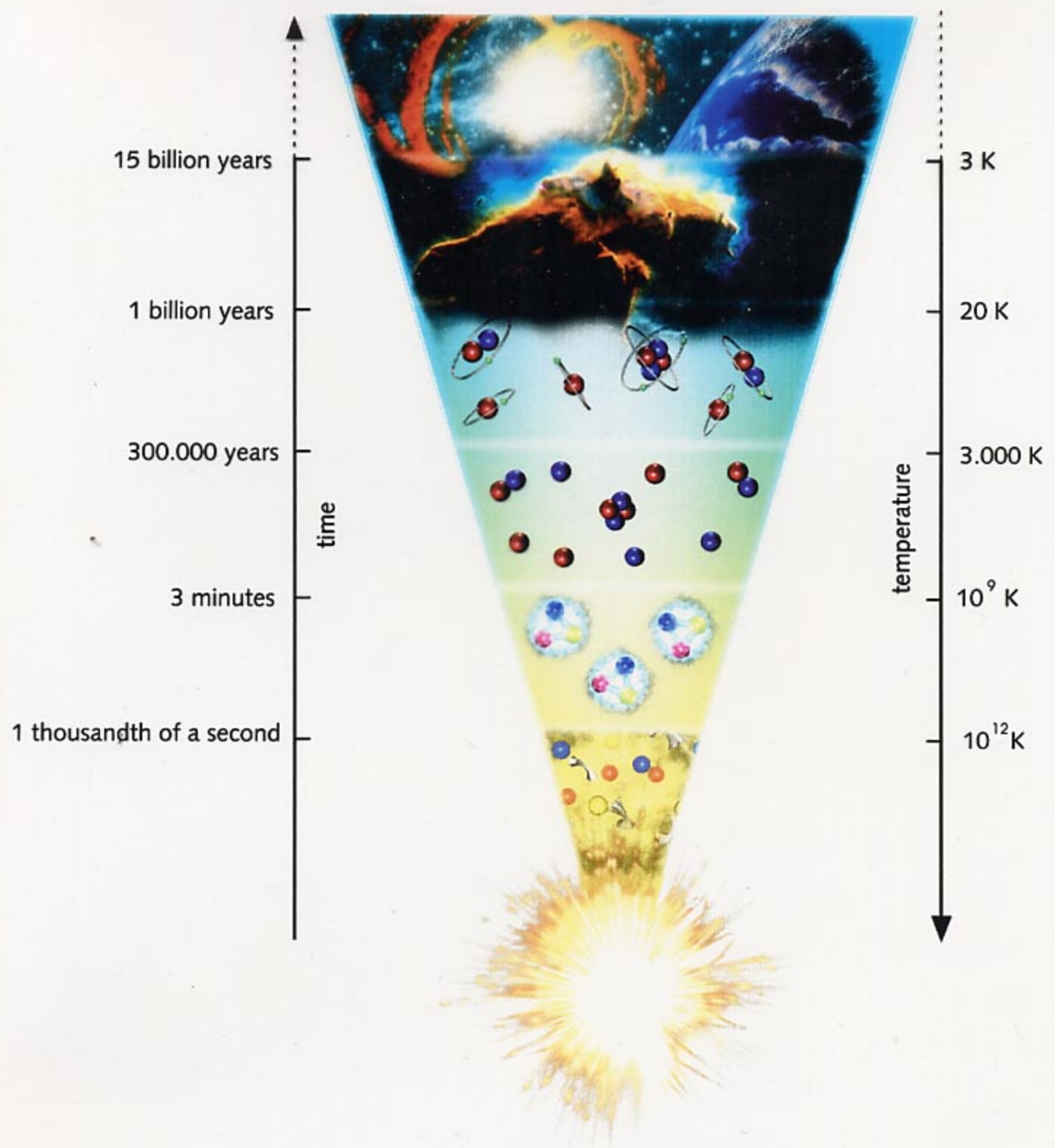
$$1 \text{ fm} \approx \frac{1}{200 \text{ MeV}}$$

$$1 \text{ GeV} = 1000 \text{ MeV}$$

$$m_{\pi} \approx 140 \text{ MeV}$$

$$m_N \approx 940 \text{ MeV}$$

$$T_c \approx 170 \text{ MeV}$$



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# Hadrons

$\pi^+, \pi^0, \pi^-$

$$m_\pi \cong 140 \text{ MeV}$$

.....

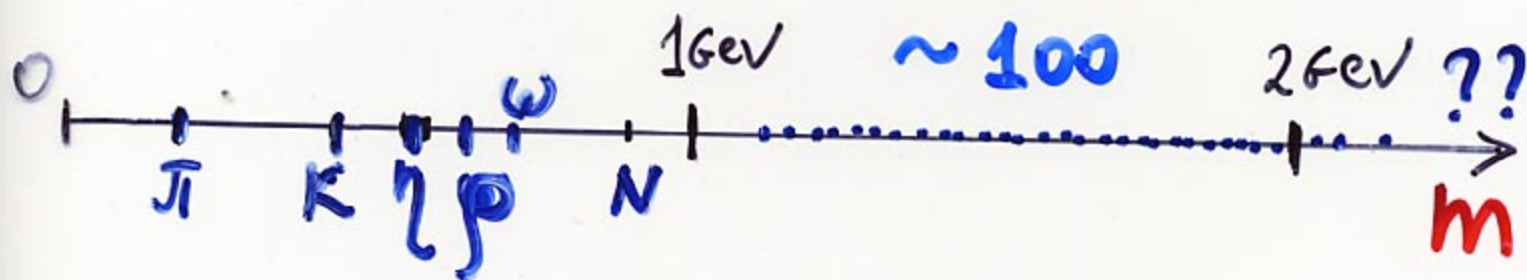
$p, n$

$$m_N \cong 940 \text{ MeV}$$

.....

$\pi, \kappa, \eta, \rho, \omega, \eta', f_0, a_0, \phi, h_1, b_1, \dots$

$N, \Delta, N^*, \Lambda, \Sigma, \Xi, \dots$



**Mesons**  
(Bosons)

**Baryons**  
(Fermions)

$$Q = -1, 0, +1, \pm 2$$

$$B = 1, \cancel{0}, -1$$

$$S = 0, \pm 1, \pm 2, \pm 3$$

Mesons

$$Q = 0, \pm 1$$

$$B = 0$$

$$S = 0, \pm 1$$

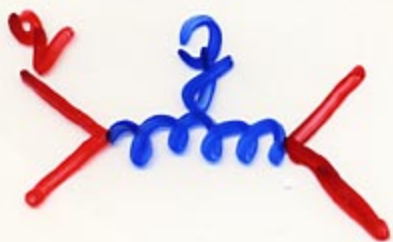
	B	Q	S	C	B	T
u	$1/3$	$2/3$	0	0	0	0
d	$1/3$	$-1/3$	0	0	0	0
s	$1/3$	$-1/3$	-1	0	0	0
c	$1/3$	$2/3$	0	+1	0	0
b	$1/3$	$-1/3$	0	0	-1	0
t	$1/3$	$2/3$	0	0	0	+1

$(Q_1, \bar{Q}_2)$

$$B = 0$$

$$Q = 0, \pm 1$$

$$S = 0, \pm 1$$



$(Q_1, Q_2, Q_3)$

$(\bar{Q}_1, \bar{Q}_2, \bar{Q}_3)$

$$B = \pm 1$$

$$Q = 0, \pm 1, \pm 2$$

$$S = 0, \pm 1, \pm 2, \pm 3$$

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# Quarks

c, b, t

	u	d	s
B	1/3	1/3	1/3
Q	2/3	-1/3	-1/3
S	0	0	-1

$q\bar{q}$

mesons

$qqq$

baryons

$\bar{q}\bar{q}\bar{q}$

antibaryons

$\pi^+(u\bar{d}), \pi^-(d\bar{u}), K^+(u\bar{s}), K^-(\bar{u}s)...$

$p( uud ), n( udd ), \Delta^{++}( uuu ), ...$

$\Lambda^0( uds ), ... , \Xi^0( uss ), ... , \underline{\Omega^-}( sss ), ...$

Colour

$q_a$

$a = 1, 2, 3$

QCD

Fundamental Theory  
of Strong Interactions

$g_c$

$c = 1, \dots, 8$

## Asymptotic Freedom

### QGP at $T \rightarrow \infty$

"ideal gas" of quarks  
and gluons



# I. Confinement

# II. Asymptotic Freedom

QCD



Hadrons

Baryons  $(q_1 q_2 q_3)$   $B = 1$

Mesons  $(q_1 \bar{q}_2)$   $B = 0$

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln \frac{Q^2}{\Lambda^2}}$$

$$\Lambda \approx 200 \text{ MeV}$$

$q \bar{q}$   $B = 0; S = \pm 1, 0; Q = \pm 1, 0$

$qqq$  }  $B = \pm 1; S = 0, \pm 1, \pm 2, \pm 3$

$\bar{q}\bar{q}\bar{q}$  }  $Q = 0, \pm 1, \pm 2$

$$f_i = \frac{1}{\exp\left[\frac{(\kappa^2 + m_i^2)^{1/2} - \mu_i}{T}\right]} \quad \begin{matrix} - \\ + \\ \downarrow \end{matrix}$$

$$f_i \approx e^{\mu_i/T} \exp\left[-\frac{(\kappa^2 + m_i^2)^{1/2}}{T}\right]$$

$$\rho_i = \frac{d_i}{2\pi^2} \int_0^\infty \kappa^2 d\kappa f_i$$

$$\delta_i = \frac{d_i}{2\pi^2} \int_0^\infty \kappa^2 d\kappa \sqrt{\kappa^2 + m_i^2} f_i$$

$$P_i = \frac{d_i}{2\pi^2} \int_0^\infty \kappa^2 d\kappa \frac{\kappa^2}{3\sqrt{\kappa^2 + m_i^2}} \cdot f_i$$

$$P = \sum_i P_i, \quad \rho_B = \sum_i b_i \rho_i$$

$$P(T, \mu_B)$$

$$S = \frac{\partial P}{\partial T}, \quad \rho_B = \frac{\partial P}{\partial \mu_B}$$

$$\varepsilon = TS + \mu_B \rho_B - P$$

$$\psi_m(T) = g \frac{m^2 T}{2\pi^2} \mathcal{K}_2\left(\frac{m}{T}\right)$$

$g$  degeneracy factor

$$g = (2j+1)$$

$$p(T) = T \psi_m(T)$$

$$h(T) = \psi_m(T)$$

$$\varepsilon(T) = T^2 \frac{d\psi_m}{dT}$$

$$\psi_m = \frac{g}{2\pi^2} \int_0^{\infty} k^2 dk \exp\left[-\frac{(k^2+m^2)^{1/2}}{T}\right]$$

$$\psi_m = \frac{g}{2\pi^2} \int_0^{\infty} k^2 dk \frac{1}{\exp\left[\pm\frac{(k^2+m^2)^{1/2}}{T}\right] \pm 1}$$

$$\underline{m/\tau \rightarrow 0}$$

$$\begin{aligned} \psi_0(\tau) &= \frac{2}{2\pi^2} \int_0^\infty k^2 dk \exp\left[-\frac{k}{\tau}\right] \equiv \\ &\equiv \frac{2}{2\pi^2} \tau^3 \int_0^\infty x^2 dx e^{-x} = \frac{2}{\pi^2} \tau^3 \end{aligned}$$

$$p^{id}(\tau, m=0) = \frac{2}{\pi^2} \tau^4$$

$$\varepsilon^{id}(\tau, m=0) = \frac{3}{\pi^2} \tau^4$$

$$\varepsilon = \sigma T^4$$

$\sigma$  - SB const

	Boltzmann	Bose	Fermi
	$e^{-x}$	$\frac{1}{e^x - 1}$	$\frac{1}{e^x + 1}$
S-B constant	$\frac{3}{\pi^2} \cong 0.304$	$\frac{\pi^2}{30} \cong 0.329$	$\frac{7}{8} \cdot \frac{\pi^2}{30} \cong 0.288$

$$\underline{m/\tau \gg 1}$$

$$\psi_m(\tau) \cong \left(\frac{m\tau}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{\tau}\right)$$

5<sup>11</sup>

$$\psi_m(T) = \frac{1}{2\pi^2} \int_0^{\infty} p^2 dp e^{-\frac{\sqrt{p^2 + m^2}}{T}}$$

$$m/T \rightarrow \infty$$

$$\sqrt{p^2 + m^2} \approx m + \frac{p^2}{2m}$$

$$\psi_m(T) = \frac{e^{-m/T}}{2\pi^2} \int_0^{\infty} p^2 dp e^{-\frac{p^2}{2mT}}$$

$$= \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$Z(T, V)$  g.c. partition function

$$P(T) = T \lim_{V \rightarrow \infty} \frac{\ln Z(T, V)}{V}, \text{ pressure}$$

$$\varepsilon(T) = T \frac{dP}{dT} - P(T), \text{ energy density}$$

$$S(T) = \frac{dP}{dT}, \text{ entropy density}$$

$$\varepsilon + P = TS, \text{ thermod. identity}$$

$P(T)$  is a continuous function

$$T = T_c$$

A discontinuity of  $\frac{dP}{dT}$  1st order PT

A discontinuity of  $\frac{d^2P}{dT^2}$  2nd order PT

....

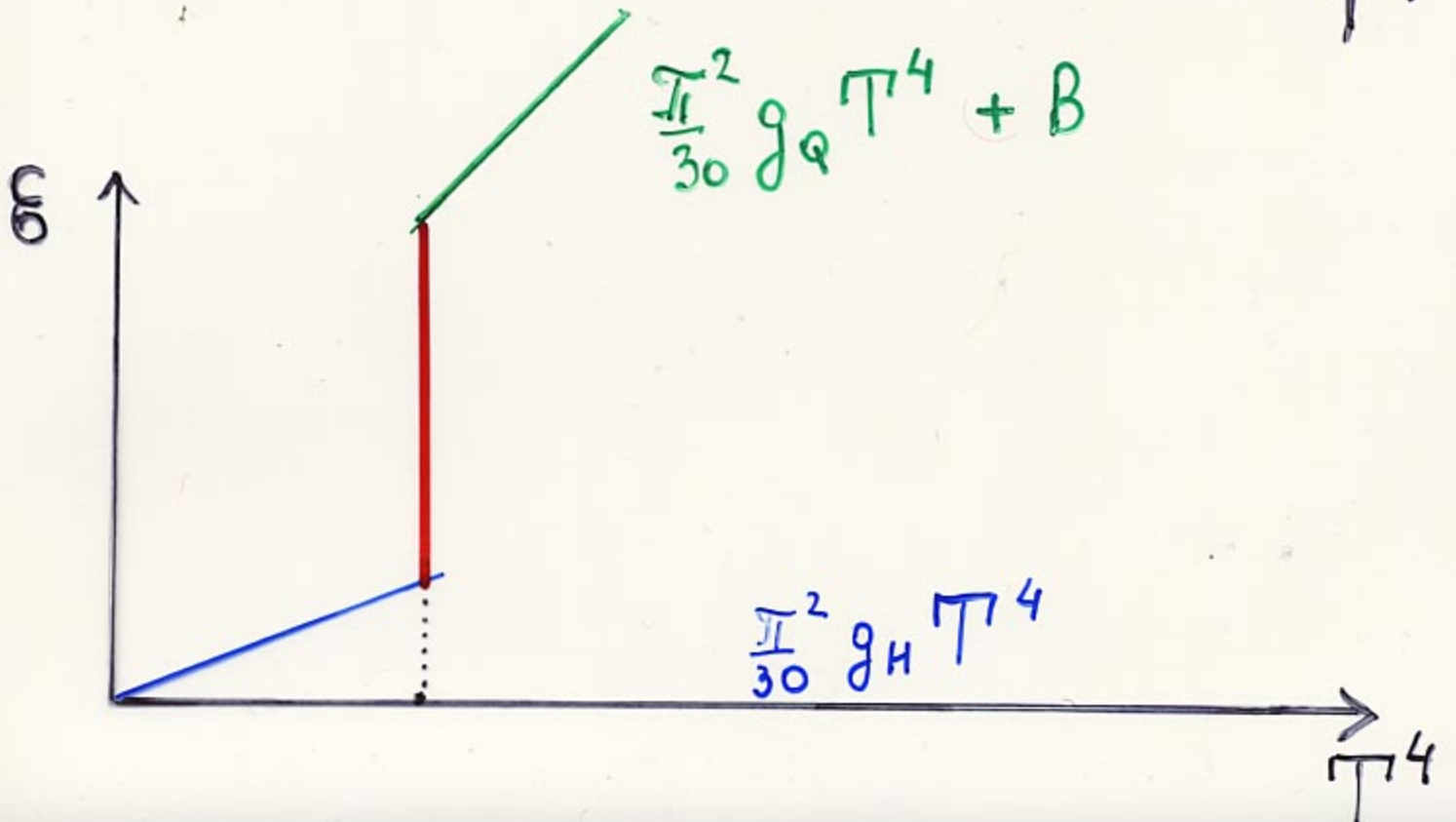
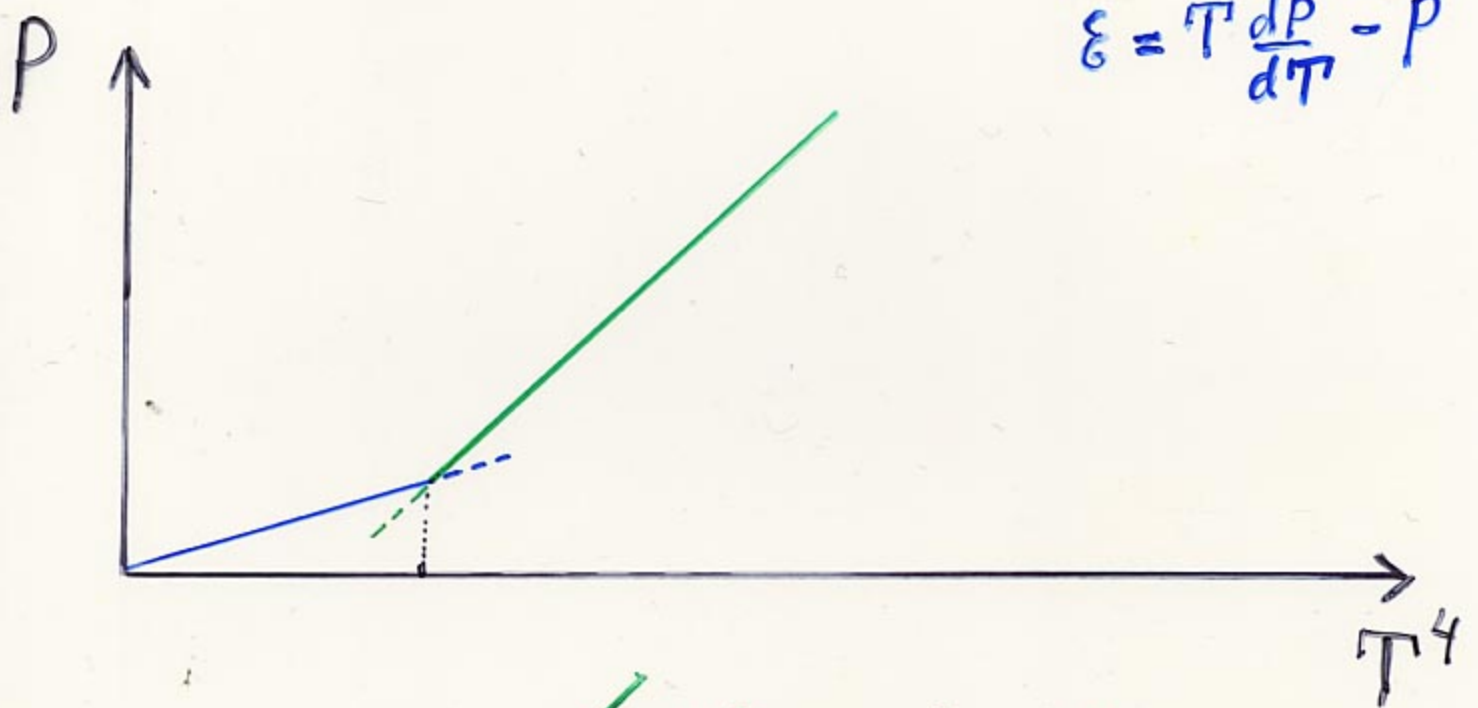
QGP  $P_Q = \frac{\pi^2}{90} g_Q T^4 - B$

$g_Q = 8 \cdot 2 + \frac{7}{8} \cdot 2 \cdot 3 \cdot 3 \cdot 2 = 47.5$

HG  $P_H = \frac{\pi^2}{90} g_H T^4$

$g_H < g_Q$

$\epsilon = T \frac{dP}{dT} - P$

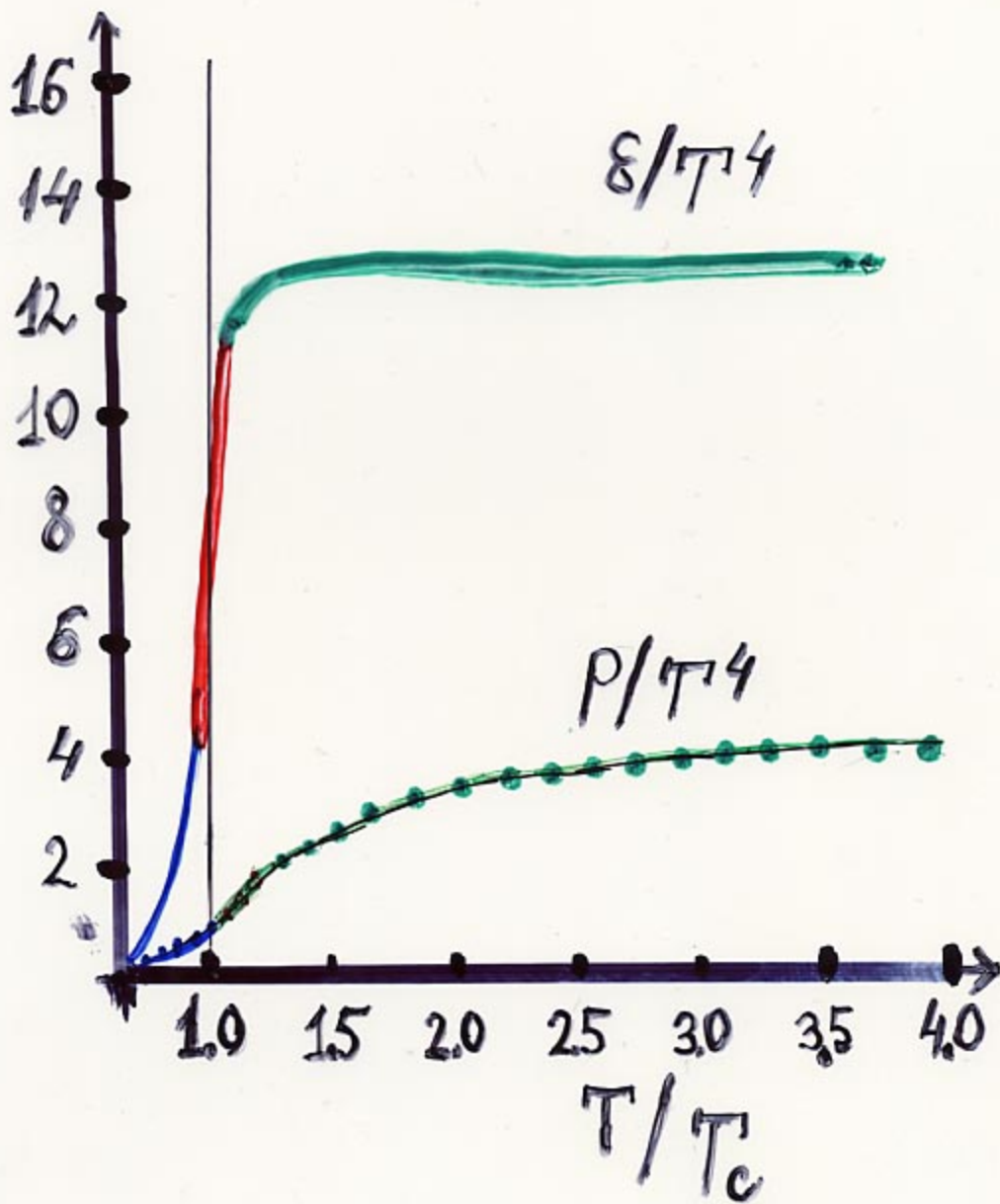


# Lattice QCD

$$T \gg T_c$$

$$\varepsilon \approx \sigma T^4, \quad \rho \approx \frac{1}{3} \varepsilon$$

$$\sigma = \frac{\pi^2}{30} \left( \underbrace{8 \cdot 2}_{N_c^2 - 1} + \frac{7}{8} \cdot \underbrace{2}_{(2j+1)} \cdot \underbrace{3}_{N_c} \cdot \underbrace{3}_{n_f} \cdot \underbrace{2}_{\bar{q}} \right)$$



$$T_c \approx 170 \text{ MeV}$$

"generalized"  
mixed phase



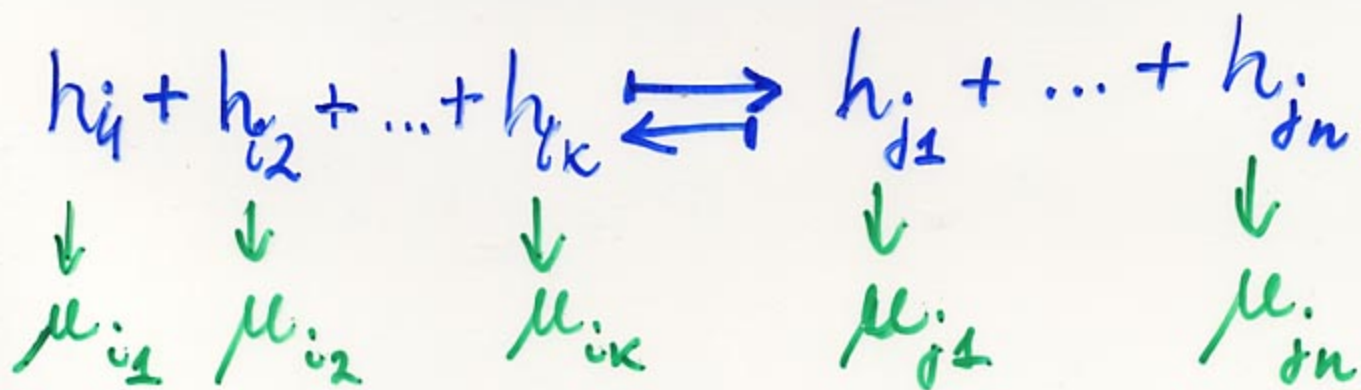
$$P(T, \mu) = T \lim_{V \rightarrow \infty} \frac{1}{V} \ln Z$$

$$S(T, \mu) = \frac{\partial P}{\partial T}$$

$$n(T, \mu) = \frac{\partial P}{\partial \mu}$$

$$\varepsilon(T, \mu) = T \frac{\partial P}{\partial T} + \mu \frac{\partial P}{\partial \mu} - P$$

$\mu_i$



$$\mu_{i_1} + \mu_{i_2} + \dots + \mu_{i_k} = \mu_{j_1} + \dots + \mu_{j_n}$$

$$\mu_i = v_i \mu_B + s_i \mu_S + q_i \mu_e$$



1).  $\mu_S - ? \quad \mu_B > 0 \rightleftharpoons N_{\lambda} > N_{\bar{\lambda}}$

$p_S = 0 \quad \mu_S = \mu_S(T, \mu_B) > 0, \rightleftharpoons N_{K^+} > N_{K^-}$   
( $\bar{s}u$ ) ( $s\bar{u}$ )

2).  $\mu_e - ?$

$p_e \approx p_B/2 \quad \mu_e \approx 0, \quad p_e \approx 0.4 p_B$

$\mu_e < 0 \rightleftharpoons N_{\pi^-} > N_{\pi^+}$

B	S	Q	C
$\mu_B$	$\mu_S$	$\mu_e$	$\mu_c$

$$\mu_i = b_i \mu_B + S_i \mu_S + q_i \mu_e + c_i \mu_c$$

$$\mu_{\pi^+} = \mu_e$$

$$\mu_n = \mu_B$$

$$\mu_{\pi^0} = 0$$

$$\mu_p = \mu_B + \mu_e$$

$$\mu_{\pi^-} = -\mu_e$$

$$\mu_{\bar{n}} = -\mu_B$$

$$\mu_{\bar{p}} = -\mu_B - \mu_e$$

$$\mu_{K^+} = \mu_S + \mu_e$$

$$\mu_{\Lambda} = \mu_B - \mu_S$$

$$\begin{array}{|c|c|} \hline \rho_B & \rho_S \\ \hline \rho_e & \rho_c \\ \hline \end{array}$$

$\pi$



$$\{T, \mu_B, \mu_S, \mu_e, \mu_c\}$$

$$\rho_S = 0$$

$$\rho_c = 0$$

$$A + A$$

$$\pi \begin{array}{|c|c|} \hline n & p \\ \hline \pi^+ & \pi^0 \pi^- \\ \hline \end{array}$$

$$\rho_B \rho_e$$

$$1). N_n = N_p$$

$$\rho_e = \frac{1}{2} \rho_B$$

$$\mu_B \quad \mu_e = 0$$

$$2). N_n > N_p$$

$$\rho_e \approx 0.4 \rho_B$$

$$\mu_B, \mu_e < 0$$

$$\mu_{\pi^+} = \mu_e$$

$$\mu_{\pi^0} = 0$$

$$\mu_{\pi^-} = -\mu_e$$

$$N_{\pi^-} > N_{\pi^0} > N_{\pi^+}$$



$$Q = \frac{1}{2} B$$

$$\mu_e = 0$$

$$\mu_B \quad \mu_S$$

$$\mu_N = \mu_B$$

$$\mu_{K^+} = \mu_S$$

$$\mu_\Lambda = \mu_B - \mu_S$$

$$\left. \begin{array}{l} \mathcal{P}_B > 0 \\ \mathcal{P}_S = 0 \end{array} \right\} \rightarrow \mu_B, \mu_S ?$$

$$\mu_S = \mu_S(\bar{T}, \mu_B)$$

$$\mu_B > \mu_S$$

$$\mu_\Lambda > \mu_{\bar{\Lambda}}$$

$$N_\Lambda > N_{\bar{\Lambda}}$$

$$\mu_S > 0 \leftarrow N_{K^+} > N_{K^-}$$

QGP

$$m_u, m_d, m_s \ll T$$

$$\mu_s = 0, \quad \frac{1}{3} \mu_B - \mu_s = 0,$$

$$P_{id} = \frac{\pi^2}{90} (8 \cdot 2 + \frac{7}{8} \cdot 2 \cdot 3 \cdot 3 \cdot 2) T^4 +$$
$$+ \left( \frac{\mu_B}{3} \right)^2 T^2 + \frac{1}{2\pi^2} \left( \frac{\mu_B}{3} \right)^4$$

$$P_{QGP} = P_{id}(T, \mu_B) - B$$

MIT bag model

$$\epsilon_{QGP} = \epsilon_{id}(T, \mu_B) + B$$

$$f_i = \frac{1}{\exp\left[\frac{\sqrt{k^2 + m_i^2} - \mu_i}{T}\right] + 1}$$

u, d

$$\mu_{u,d} = \frac{1}{3} \mu_B$$

s

$$\mu_s = \frac{1}{3} \mu_B - \mu_s$$

$$\mu_s = 0$$

$$\underline{\underline{\mu_s = \frac{1}{3} \mu_B}}$$

$f_g =$

$$\frac{1}{\exp\left(\frac{k}{T}\right) - 1}$$

# "Discovery" of the QGP in Pb+Pb at 160 A GeV

2000

## 1. $J/\psi$ Suppression

$$R = \frac{N_{J/\psi}}{N_{\psi}}$$

T. Matsui, H. Satz  
PLB 178 (1986) 416

$$\left(\frac{N_{J/\psi}}{N_{\psi}}\right)_{AA} < \left(\frac{N_{J/\psi}}{N_{\psi}}\right)_{pp}$$

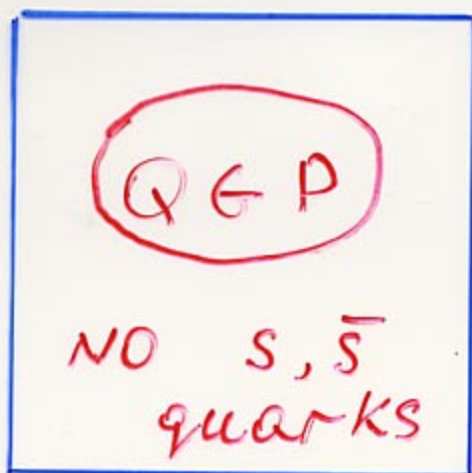
## 2. Strangeness Enhancement

$$K/\pi, \dots, \rho/\pi, \rho/N_p$$

$$\frac{(K/\pi)_{AA}}{(K/\pi)_{pp}} > 1, \quad \gg 1 \quad \text{for } \rho$$

J. Rafelski, B. Müller  
PR 48 (1982) 1066  
56 (1986) 2334E  
.....





T = 150 MeV

$t_{eq} \approx 100 \text{ fm}/c$

$t_{eq} \approx 10 \text{ fm}/c$

$$\frac{\left(\frac{K^+}{\bar{\pi}^+}\right)_{AA}}{\left(\frac{K^+}{\bar{\pi}^+}\right)_{PP}}$$

should strongly increase  
if the QGP is formed  
! strangeness enhancement

\* Real A+A gives different initial  
condit. (RQM, UQM)

\*\* multiple hadron collisions

\*

$$\left(\frac{\text{strangenes}}{\text{entropy}}\right)_{HG}^{eq} > \left(\frac{\text{strangeness}}{\text{entropy}}\right)_{QGP}^{eq}$$

$$N_s + N_{\bar{s}} \quad - ?$$

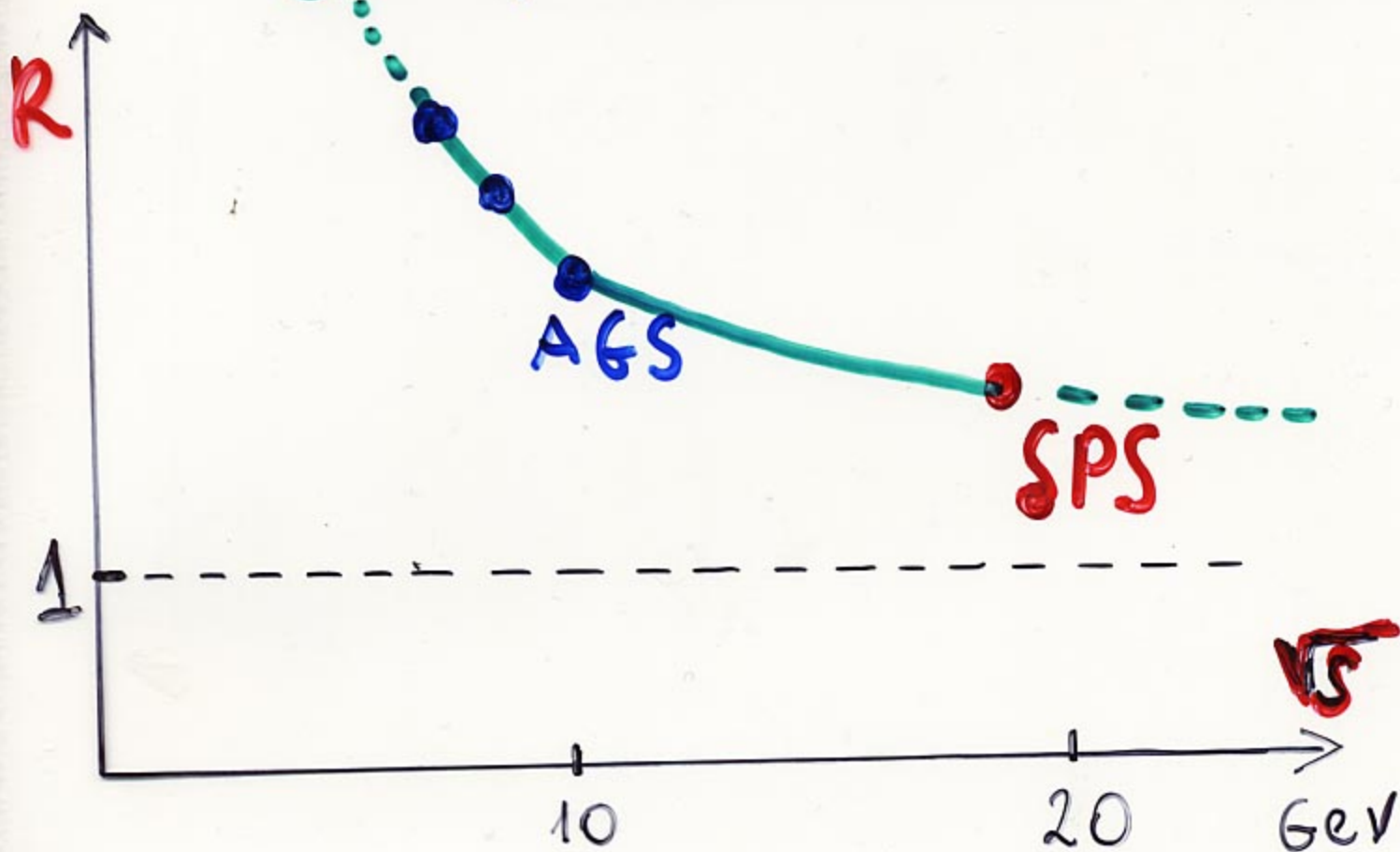
$$f = \frac{N_s + N_{\bar{s}}}{\langle \pi \rangle}, \quad \frac{N_s + N_{\bar{s}}}{N_p}, \quad \frac{K^+}{\pi^+}, \dots$$

...,  $R/\pi$

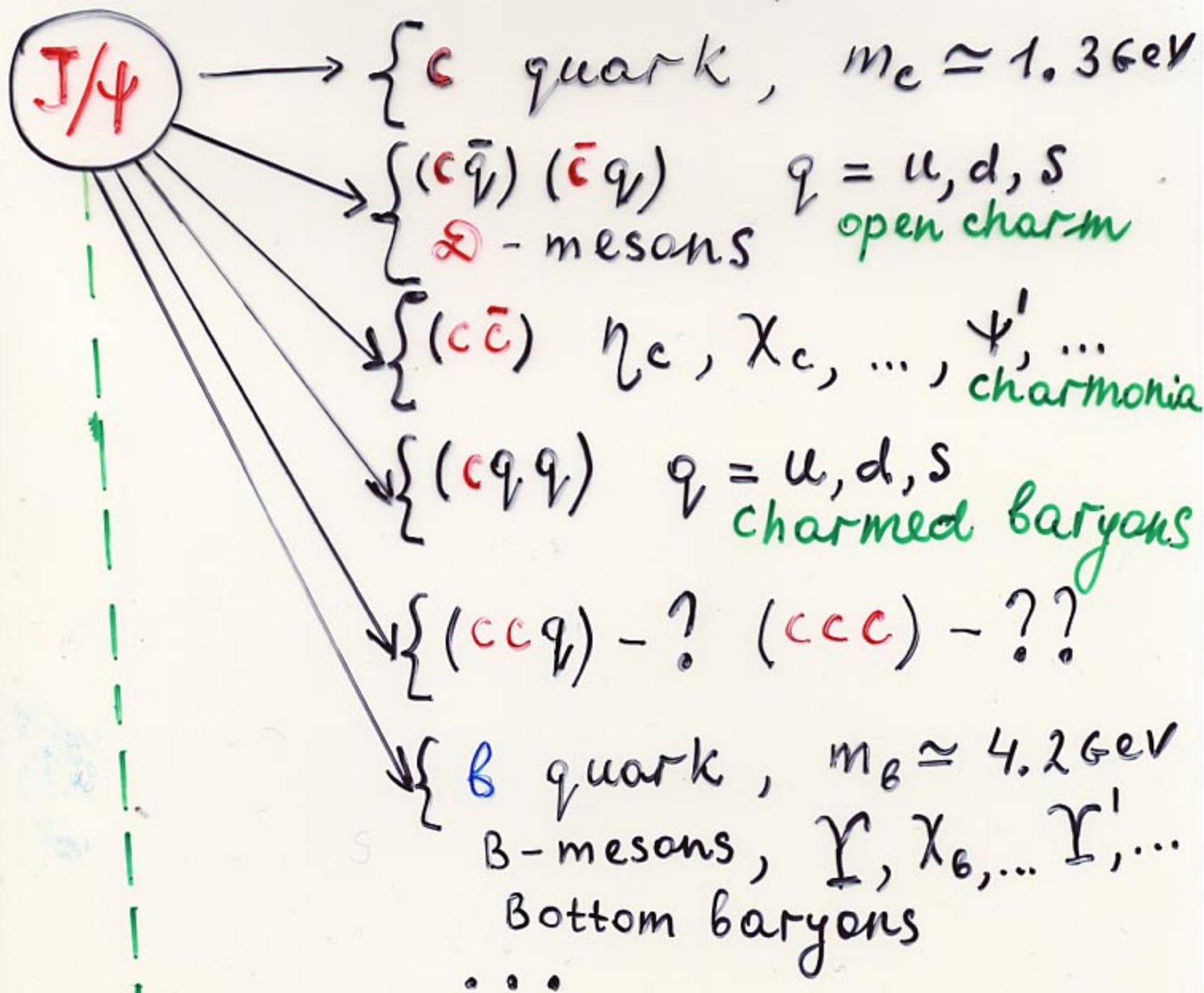
$f_{AA}(\sqrt{s})$  **strong** increase if QGP is formed

$$R(\sqrt{s}) = \frac{f_{AA}}{f_{PP}}$$

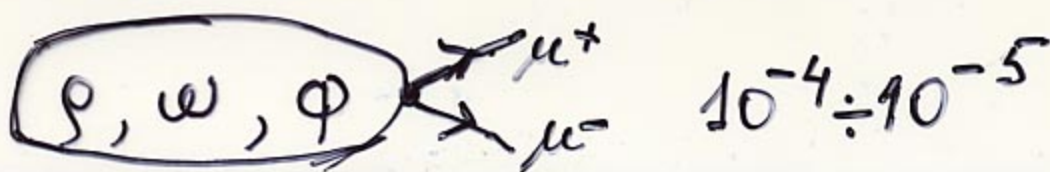
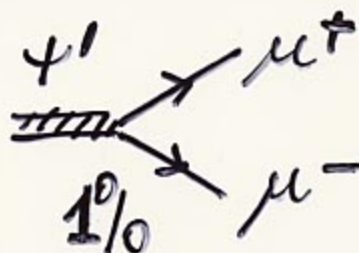
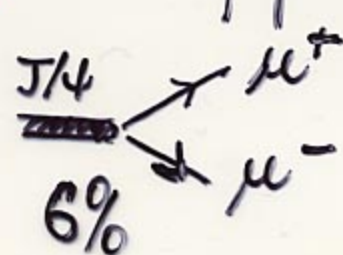
strangeness enhancement

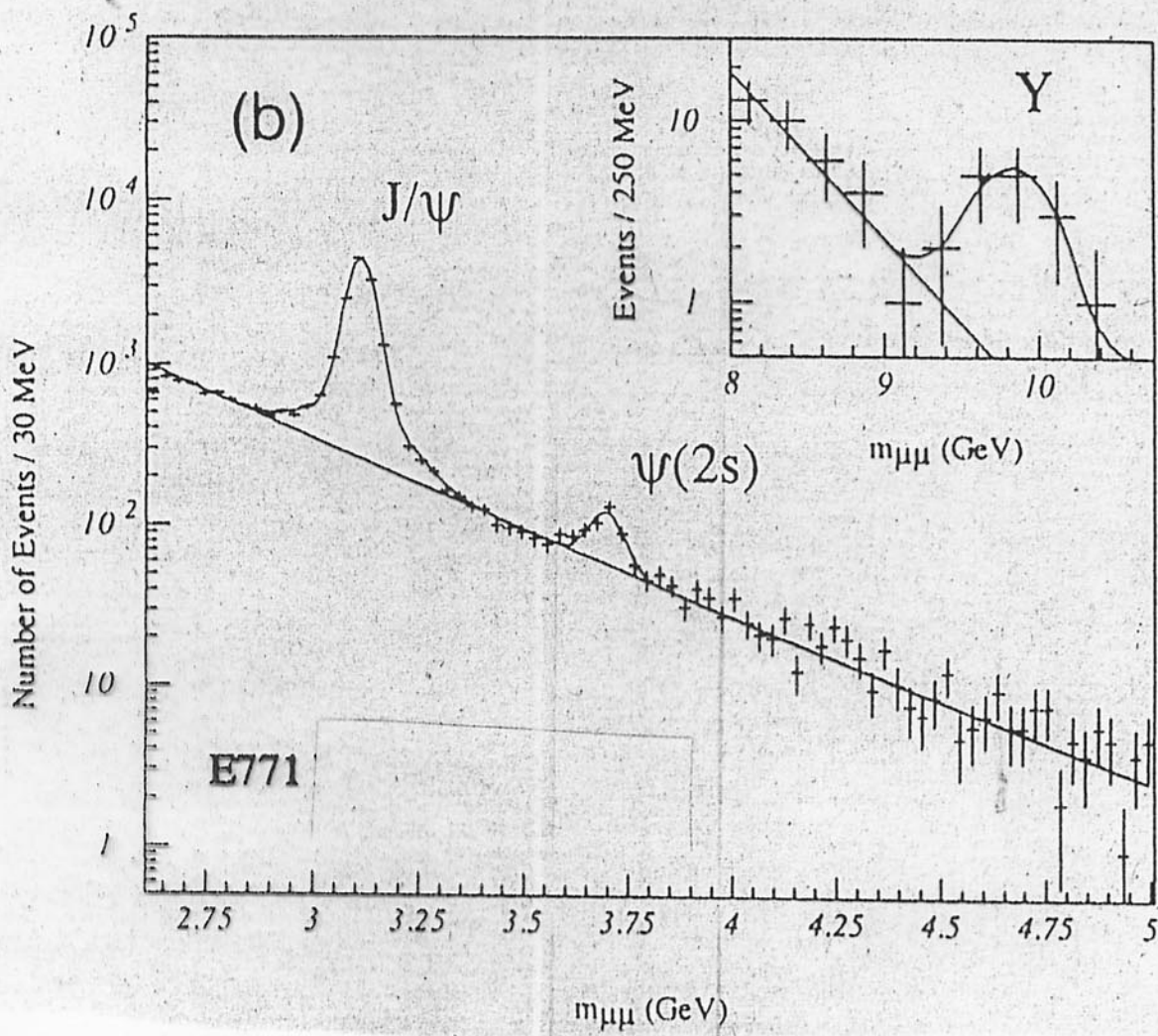
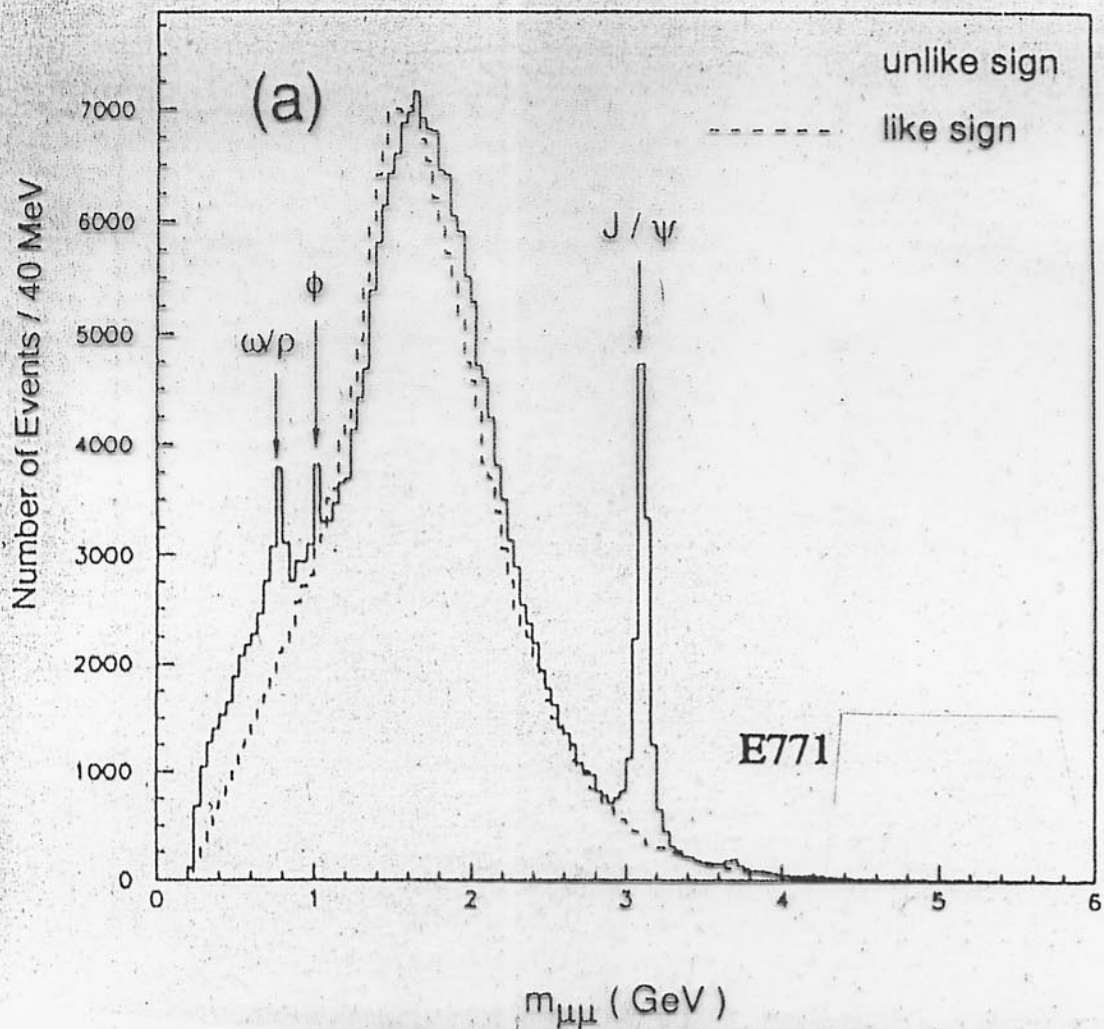


(1974)  $J/\psi$   $j=1$   $m_{J/\psi} \approx 3.1 \text{ GeV}$



$J/\psi$  suppression in QGP





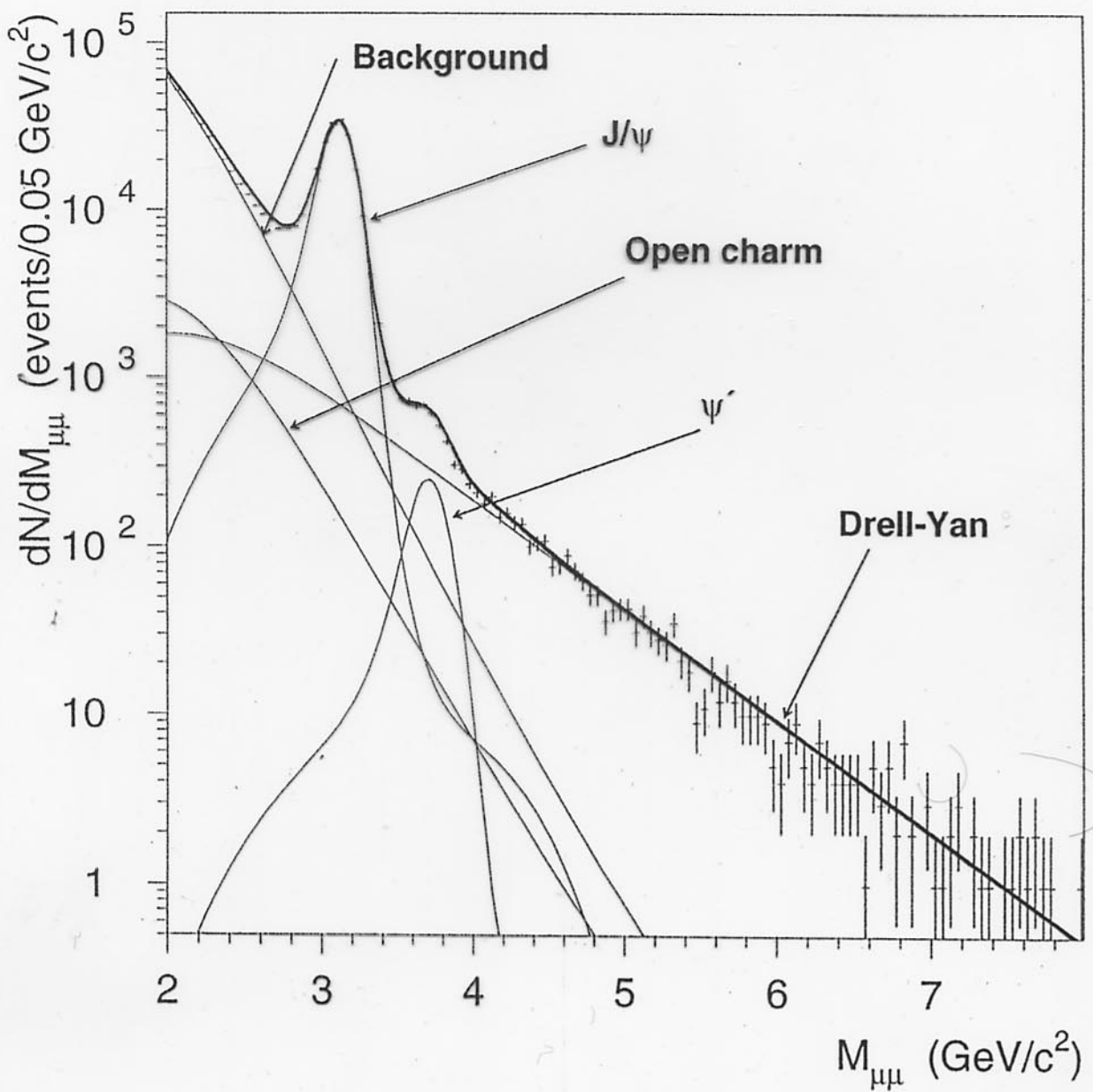
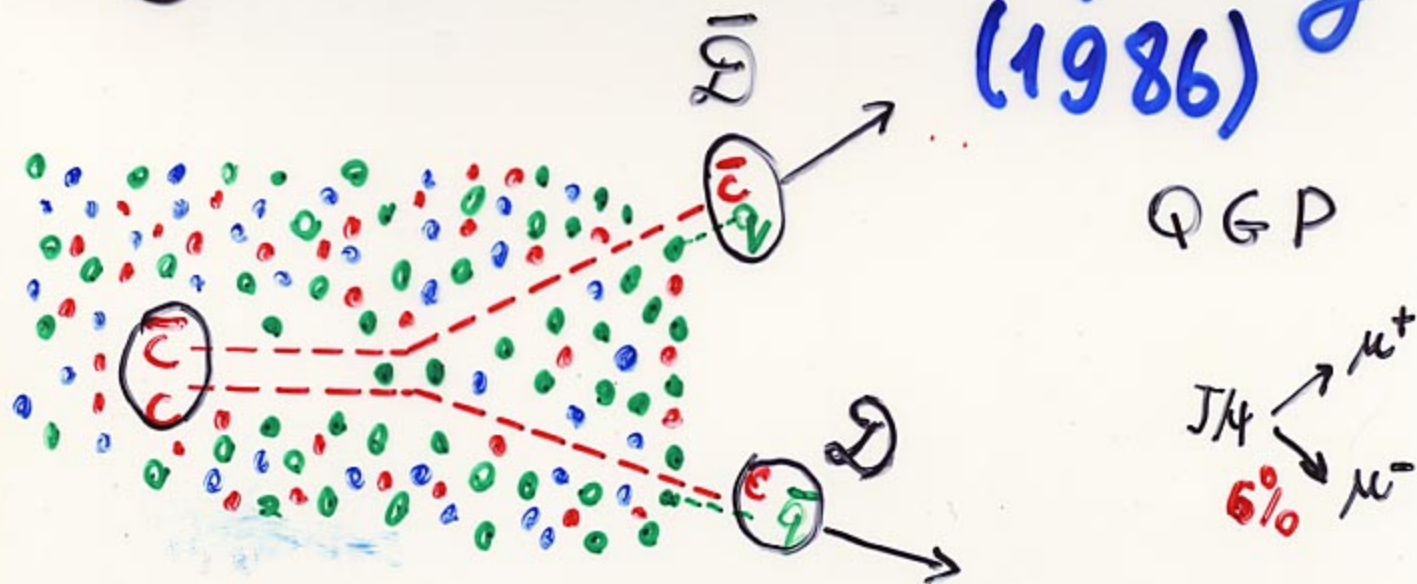
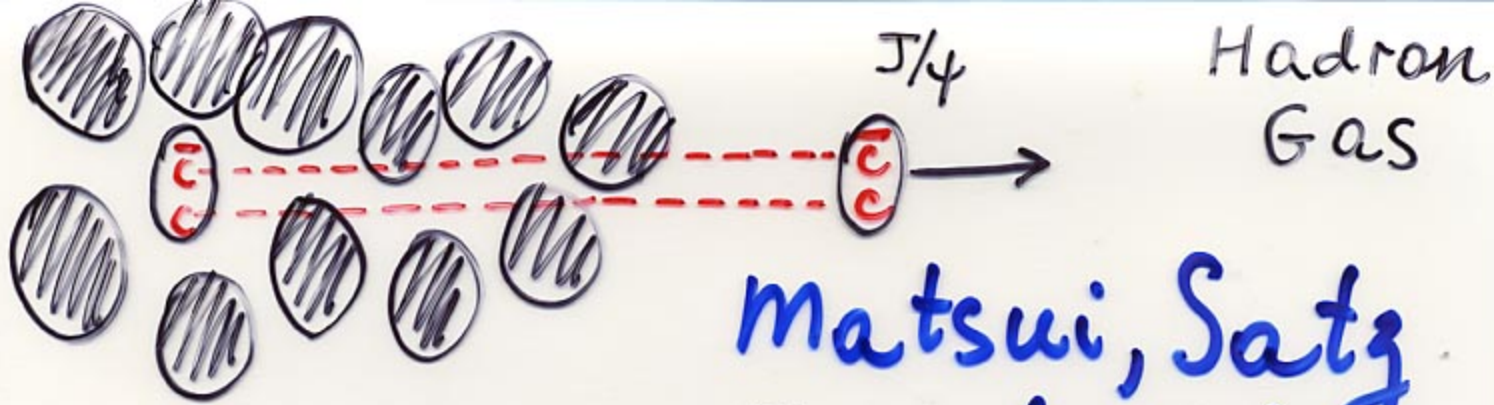


Figure 1: Opposite-sign muon pair invariant mass spectrum for Pb-Pb collisions at 158 GeV/c incident momentum. Data collected in 1996.



$$\langle J/\psi \rangle_{\text{primary}} \sim \langle \psi \bar{\psi} \rangle \sim N_p^{4/3}$$

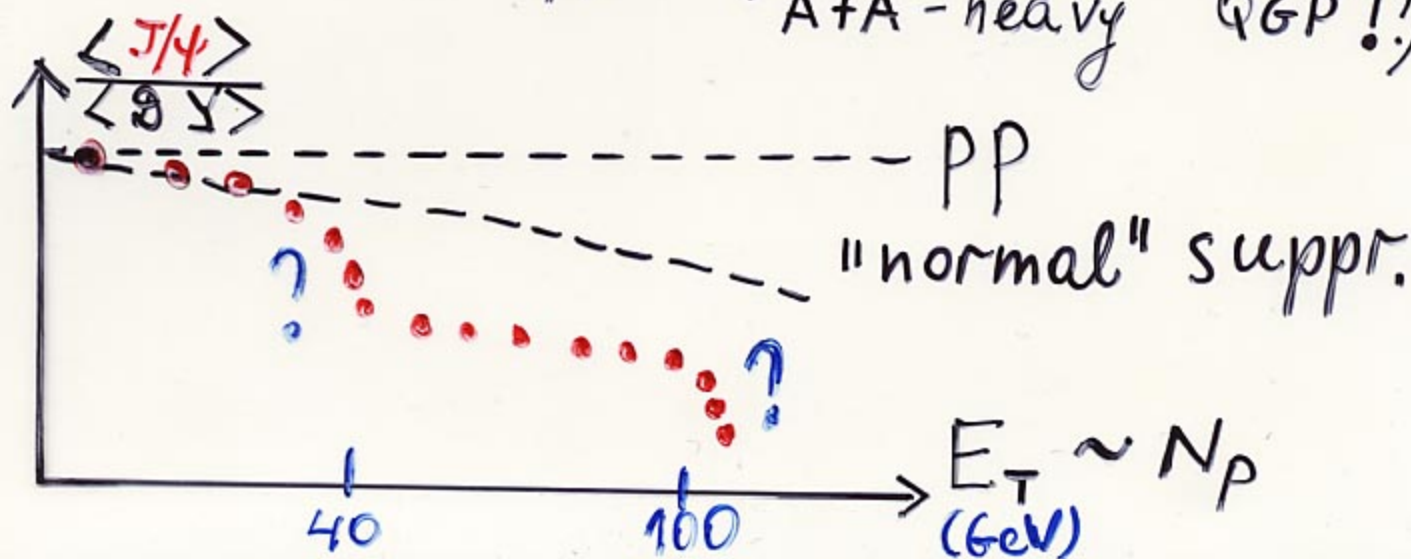
$$\langle J/\psi \rangle \approx \langle J/\psi \rangle_{\text{prim.}} \quad (\text{no suppr. } p+p)$$

$$\langle J/\psi \rangle \lesssim \langle J/\psi \rangle_{\text{prim.}} \quad (\text{"normal" suppr.})$$

$p+A, A+A$  - light

$$\langle J/\psi \rangle \ll \langle J/\psi \rangle_{\text{prim.}} \quad (\text{"anomalous" suppr.})$$

$A+A$  - heavy QGP !?



$$\textcircled{2} \langle J/\psi \rangle = \frac{(2j+1)}{2\pi^2} V \int_0^\infty \frac{k^2 dk}{\exp\left[\frac{(k^2+m_\psi^2)^{1/2}}{T}\right]-1}$$

$$\cong (2j+1) V \left(\frac{m_\psi T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\psi}{T}\right)$$

$$j=1, \quad m_\psi = 3097 \text{ MeV}$$

$$\frac{\langle J/\psi \rangle}{\langle h^- \rangle} \cong \text{const}(A, \sqrt{s})$$

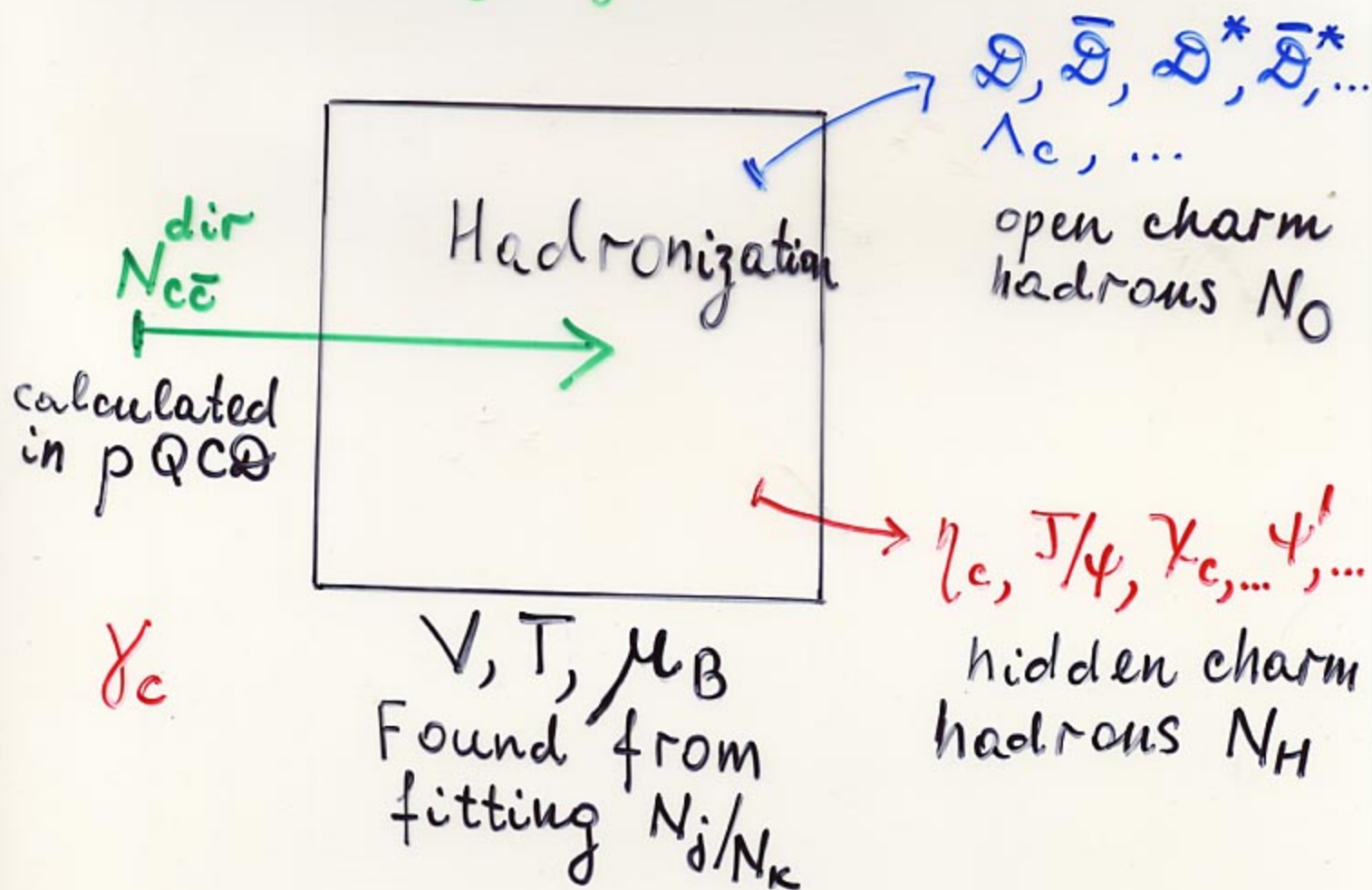
m. Gaździcki, M. Gorenstein

Phys. Rev. Lett. 83 (1999) 4009

$$T \approx 176 \text{ MeV}$$

$$\frac{\langle J/\psi \rangle}{\langle h^- \rangle}$$

# Braun-Munzinger, Stachel (2000)



$$N_{c\bar{c}}^{dir} = \gamma_c N_0 + \gamma_c^2 N_H$$

$N_0 (V, T, \mu_B)$

g.c.e.

1).  $N_0$

c.e.

2).

$P(N_{c\bar{c}}^{dir})$

- Poisson distr.

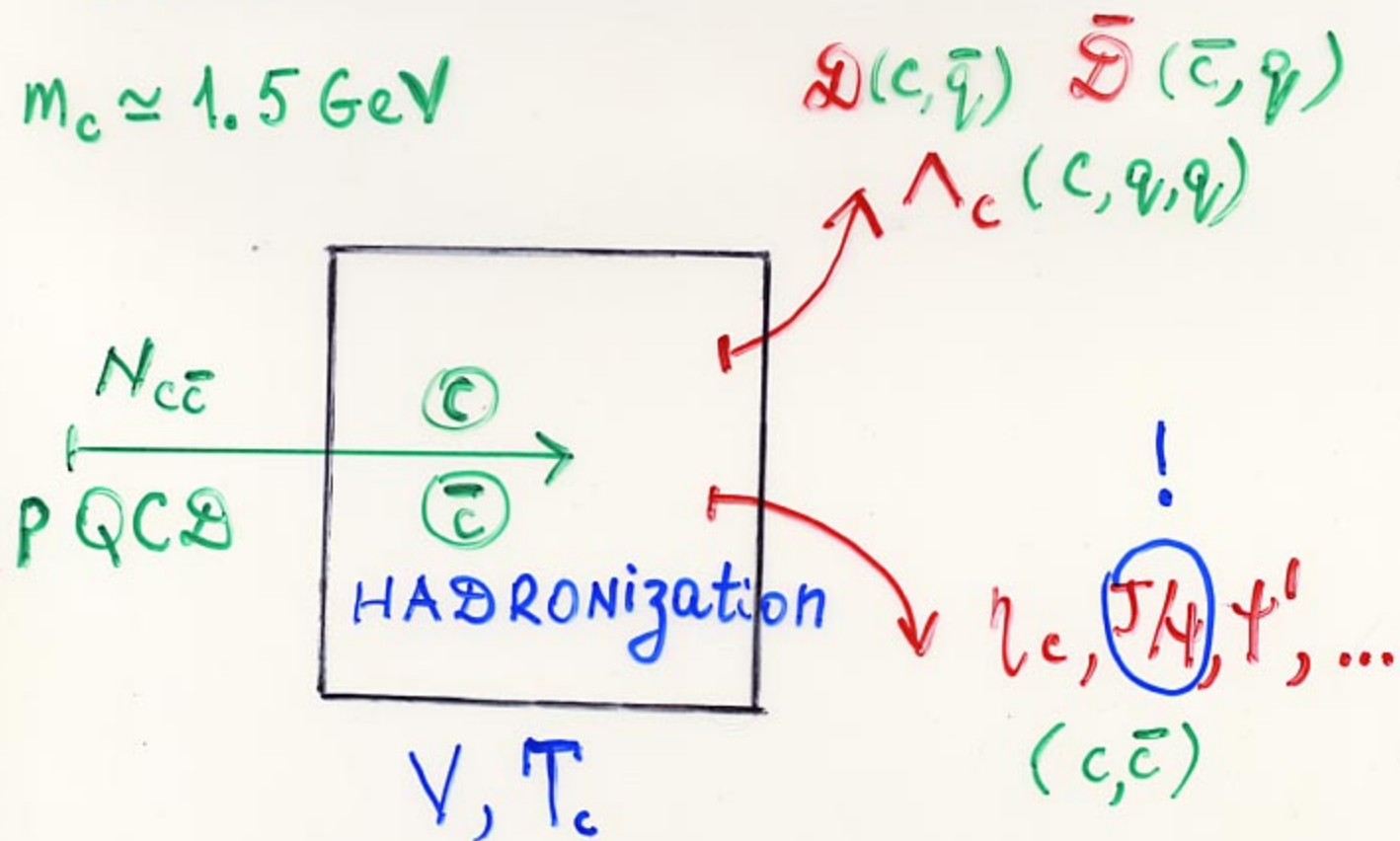
$$\langle J/\psi \rangle \cong \bar{N}_{c\bar{c}} (1 + \bar{N}_{c\bar{c}}) \frac{N_{J/\psi}^{tot}}{N_0^2}$$

m.G., Kostyuk, Stöcker, Greiner (2001)  
Phys. Lett. B (2001, 2002)



$A + A$

$m_c \approx 1.5 \text{ GeV}$



$Pb + Pb \quad 160 A \cdot \text{GeV}$

$\langle N_{c\bar{c}} \rangle < 1$

Exact Charm Conservation

~~! g.e.e.~~

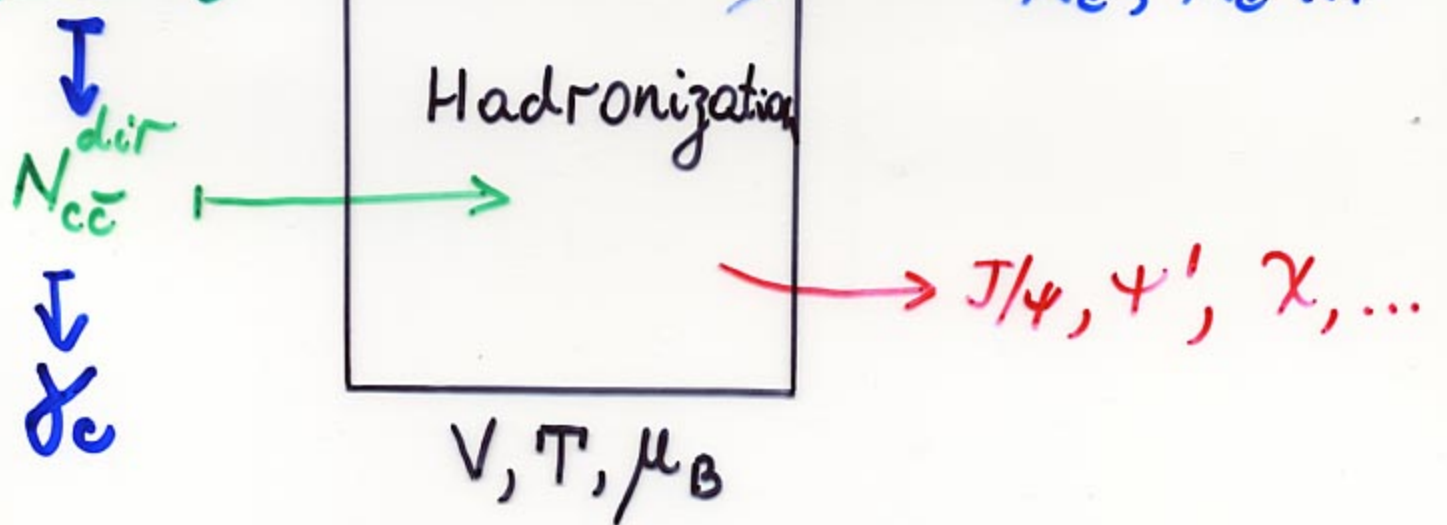
M. Gorenstein et al

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Phys. Lett B 509 (2001) 277

B523 (2001) 60, B524 (2002) 265

# Braun-Munzinger Stachel (2000)



$$N_j = d_j V e^{\mu_j/T} \left( \frac{m_j T}{2\pi} \right)^{3/2} \exp\left(-\frac{m_j}{T}\right)$$

$$\mu_j = b_j \mu_B + s_j \mu_s + c_j \mu_c$$

$$\mu_s = \mu_s(T, \mu_B) \quad \mu_c = \mu_c(T, \mu_B)$$

$$N_H^{c.e.} = N_H$$

$$N_0^{c.e.} = N_0 \frac{I_1(N_0)}{I_0(N_0)}$$

$$N_H = \sum_{i \in \text{hidden}} N_i$$

$$N_0 = \sum_{j \in \text{open}} N_j$$

$$\frac{I_1(N_0)}{I_0(N_0)}$$

$$\rightarrow 1, \quad N_0 \gg 1$$

$$\rightarrow \frac{N_0}{2}, \quad N_0 \ll 1$$

$$\frac{\langle \psi' \rangle_{th}}{\langle J/\psi \rangle_{th}} = \left( \frac{m_{\psi'}}{m_{J/\psi}} \right)^{3/2} \exp \left[ - \frac{(m_{\psi'} - m_{J/\psi})}{T} \right]$$

$$\frac{\langle \psi' \rangle_{th}}{\langle J/\psi \rangle_{th}} = \begin{array}{l} \rightarrow 0.04 \quad (T = 170 \text{ MeV}) \\ \rightarrow 0.05 \quad (T = 180 \text{ MeV}) \end{array}$$

$$\frac{\langle \psi' \rangle}{\langle J/\psi \rangle} = 0.04 \div 0.05$$

Pb + Pb (158 A GeV)

$$N_{part} = 100 \div 400$$

$$R(N_p, \sqrt{s}) = \frac{\langle J/\psi \rangle}{N_{c\bar{c}}^{\text{dir}}}$$

$$N_{c\bar{c}}^{\text{dir}} = \frac{1}{2} \gamma_c N_0 \frac{I_1(\gamma_c N_0)}{I_0(\gamma_c N_0)} + \gamma_c^2 N_H \sim 0$$

$$\langle J/\psi \rangle = \gamma_c^2 N_{J/\psi}^{\text{tot}}$$

$$N_{J/\psi}^{\text{tot}} \sim N_0 \sim \sqrt{V} \sim \langle \pi \rangle \sim N_p \sqrt{s}^{1/2}$$

$$N_{c\bar{c}}^{\text{dir}} \sim N_p^{4/3} \sqrt{s}^\beta, \quad \beta \approx 1.13$$

m. G., A. Kostyuk, et. al.

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hep-ph/0012292 (J. Phys. G) 2001

I.  $N_{c\bar{c}}^{dir} \ll 1$

$$N_{c\bar{c}}^{dir} \approx \frac{1}{4} \gamma_c^2 N_0^2$$

$$R(N_p, \sqrt{s}) \approx \frac{N_{J/4}^{tot}}{N_0^2/4} \sim \frac{1}{V} \sim N_p^{-1} \sqrt{s}^{-\frac{1}{2}}$$

J/4 suppression

II.  $N_{c\bar{c}}^{dir} \gg 1$

$$N_{c\bar{c}}^{dir} \approx \frac{1}{2} \gamma_c N_0$$

$$R(N_p, \sqrt{s}) \approx \frac{\gamma_c N_{J/4}^{tot}}{N_0/2} \sim \frac{N_{c\bar{c}}^{dir}}{V}$$

$$\sim N_p^{1/3} (\sqrt{s})^{\beta - 1/2}$$

J/4 enhancement

