Lattice QCD (INTRODUCTION)

DUBNA WINTER SCHOOL 1-2 FEBRUARY ***2005***

Main Problems

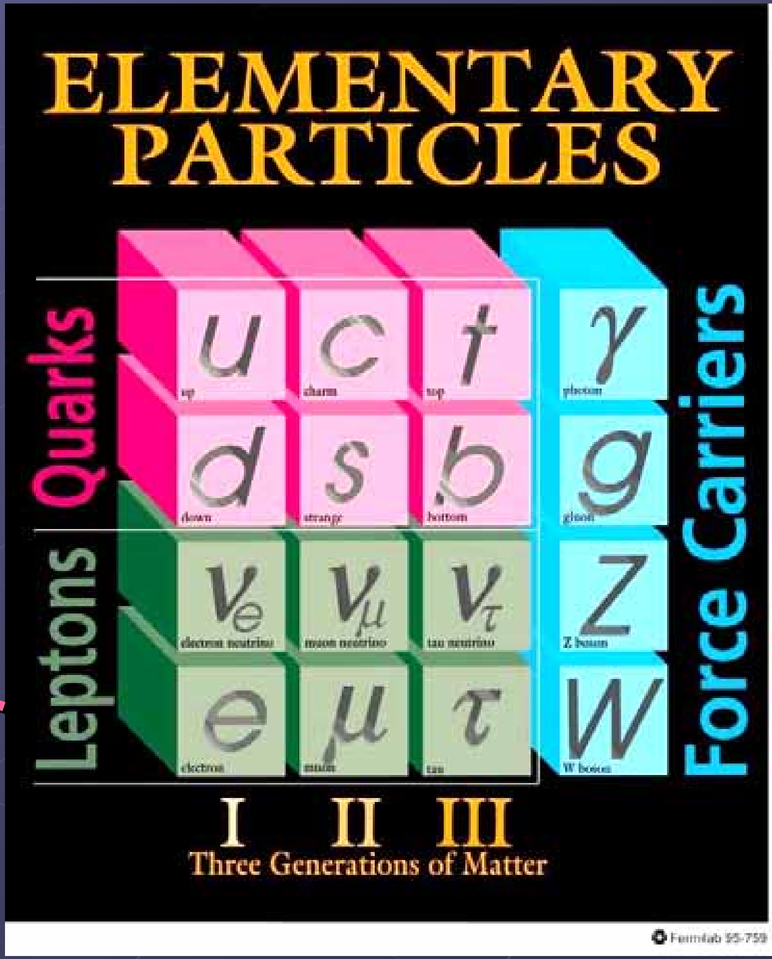
- **Starting from Lagrangian**

$$L = -\frac{1}{g^2}\,\mathrm{Tr}\,F_{\mu\nu}^2 + \sum_f \bar{\psi}_f (D+m)\psi_f$$

(1) obtain hadron spectrum,

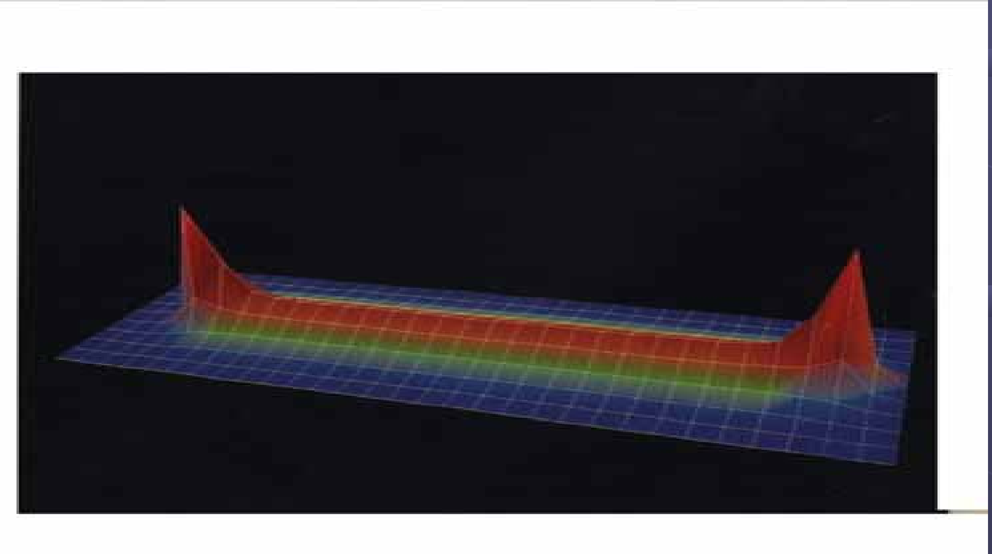
(2) describe phase transitions,

(3) explain confinement of color



http://www.claymath.org/Millennium_Prize_Problems/

The main difficulty is the absence of analytical methods, the interactions are strong and only computer simulations give results starting from the first principles.



The force between quark and antiquark is 12 tones

Methods

- Imaginary time $t \rightarrow it$

$$Z = \int D\varphi \exp\{i\, S[\varphi]\} \quad \longrightarrow \quad Z = \int D\varphi \exp\{-S[\varphi]\}$$

- Space-time discretization

$$D\varphi(x) \Rightarrow \prod_x d\,\varphi_x$$

$$Z = \int \prod_x d\varphi_x \exp\{-S[\varphi]\}$$

- Thus we get from functional integral the statistical theory in four dimensions

The statistical theory in four dimensions can be simulated by Monte-Carlo methods

- The typical multiplicities of integrals are 10^6-10^8
- We have to invert matrices $10^6 \times 10^6$
- The cost of simulation of one configuration is:

$$4 \cdot 10^{-6} \left(\frac{m_\pi}{m_\rho} \right)^{-6} \left(L[\,fm] \right)^5 \left(a[\,Gev] \right)^{-7}$$

$$Teraflops \times year$$

Three limits

$a \to 0$ **Lattice spacing**

$L \to \infty$ **Lattice size**

$m_q \to 0$ **Quark mass**

Typical values now

$a \approx 0.1\ fm$

$L \approx 2 \div 4\ fm$

$m_q \approx 100\ Mev$

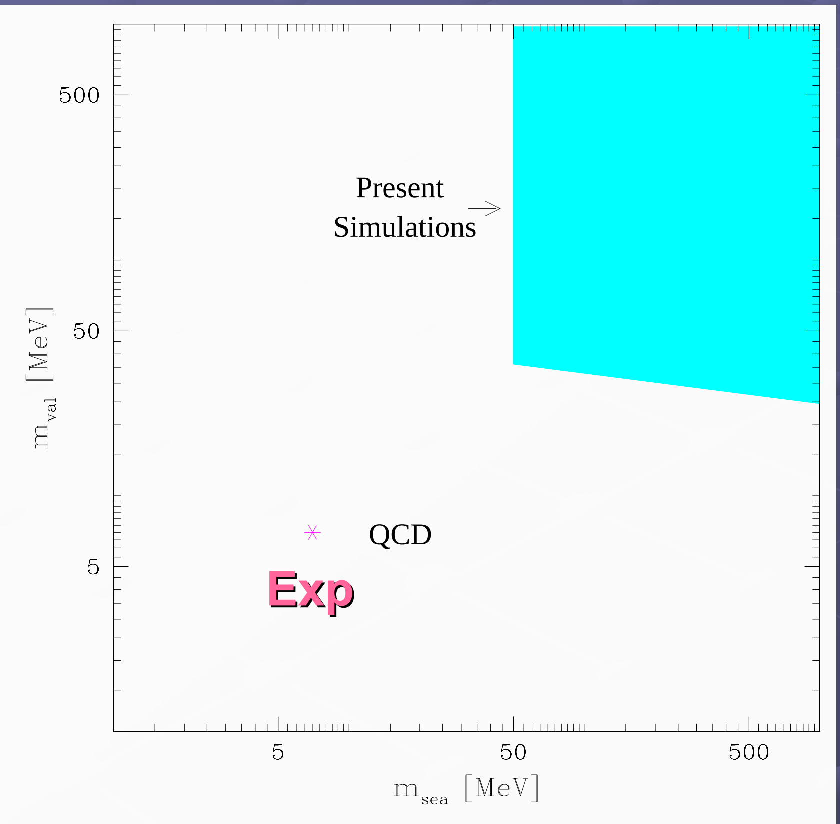
⇒ **Extrapolation**

+

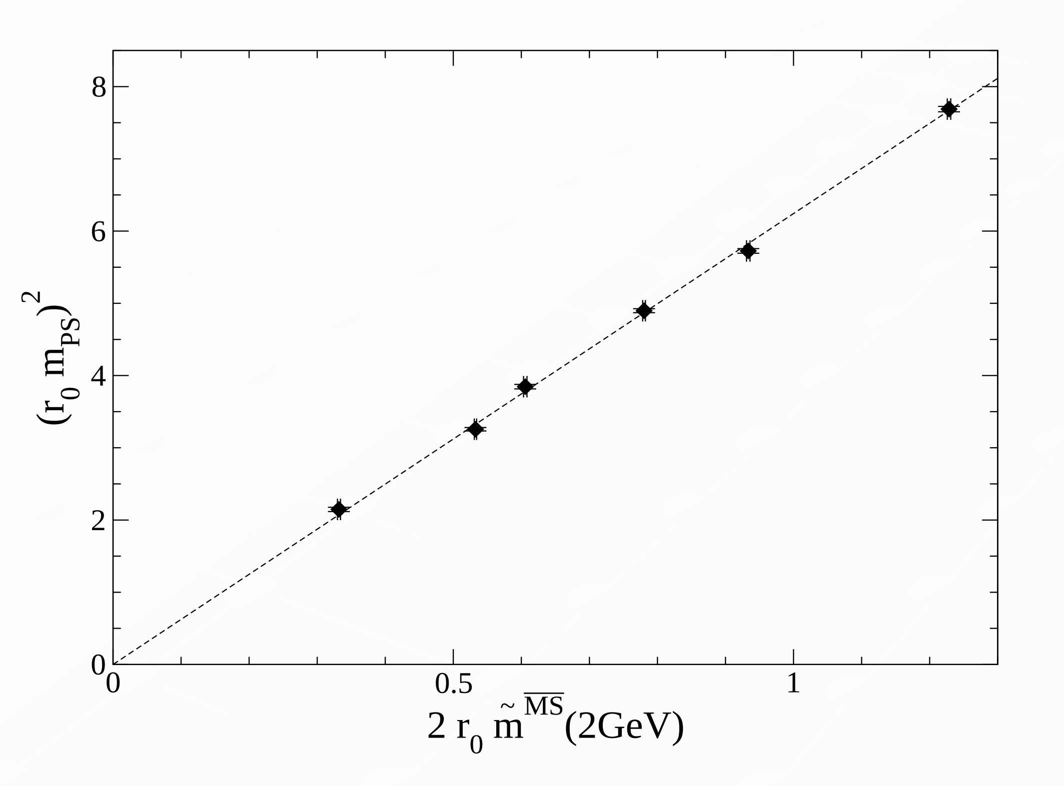
Chiral perturbation theory

Chiral limit

Quark masses



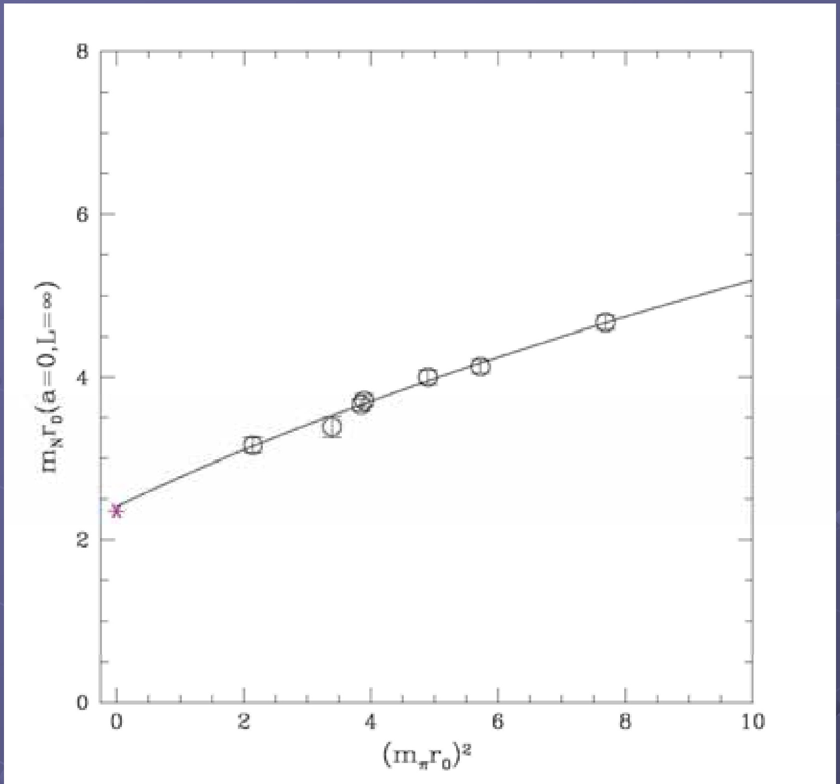
Pion mass

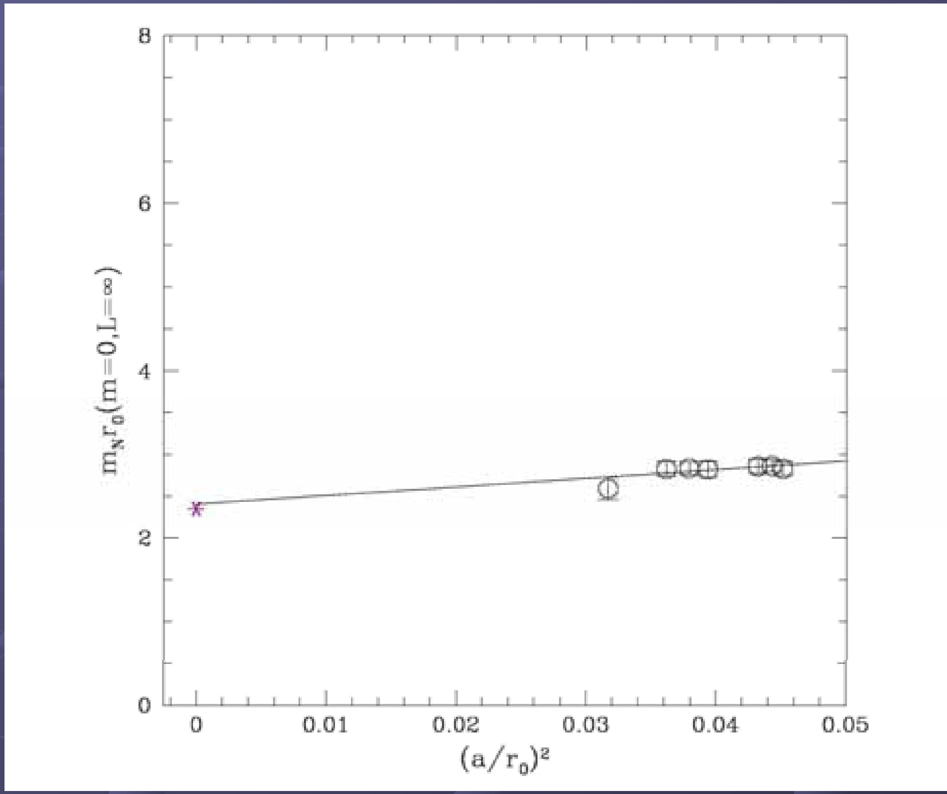


Deviation from linear

$$m_\pi^2 f_\pi^2 = m_q < \bar{\psi} \psi >$$

Nucleon mass extrapolation

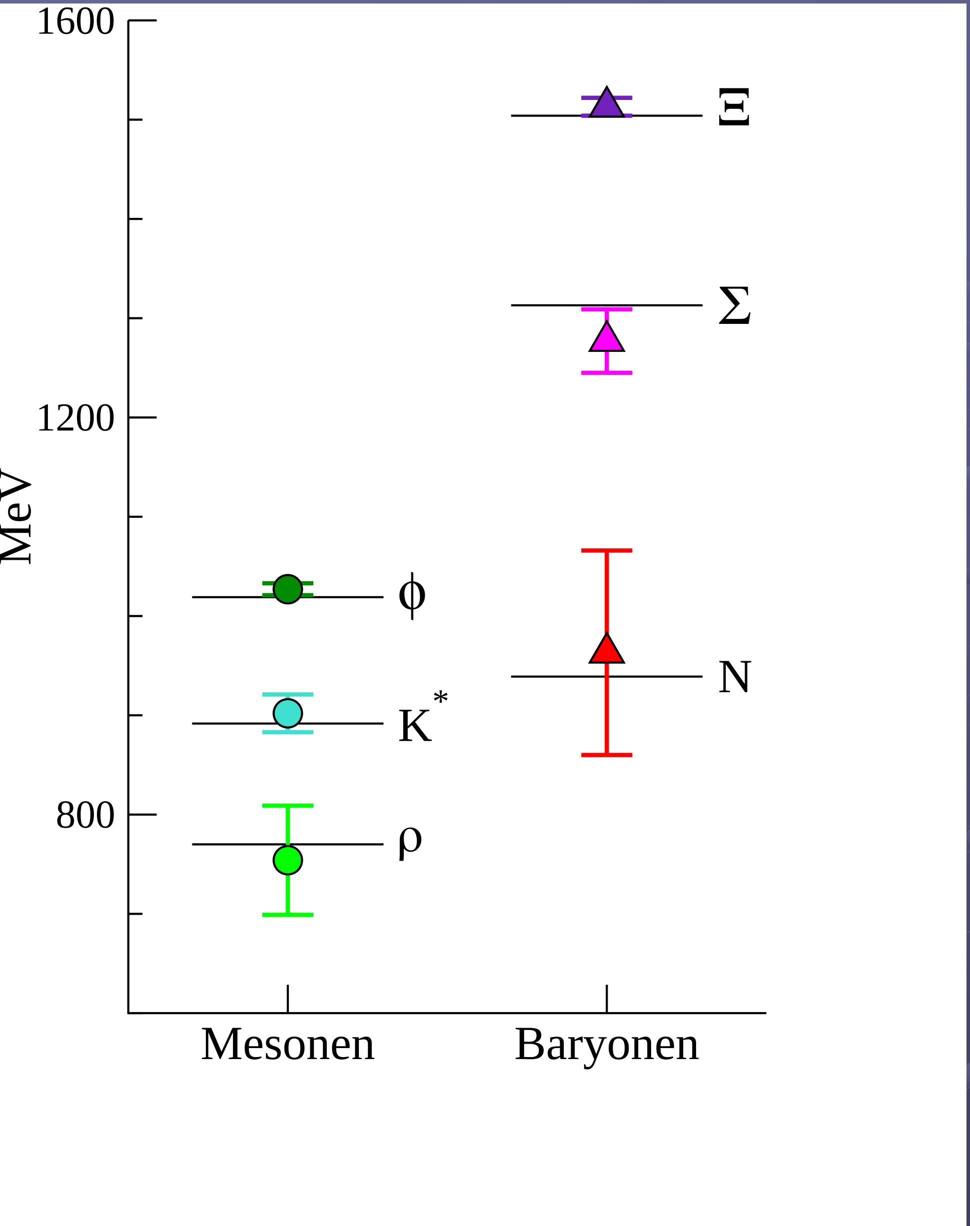




Fit on the base of the chiral perturbation theory

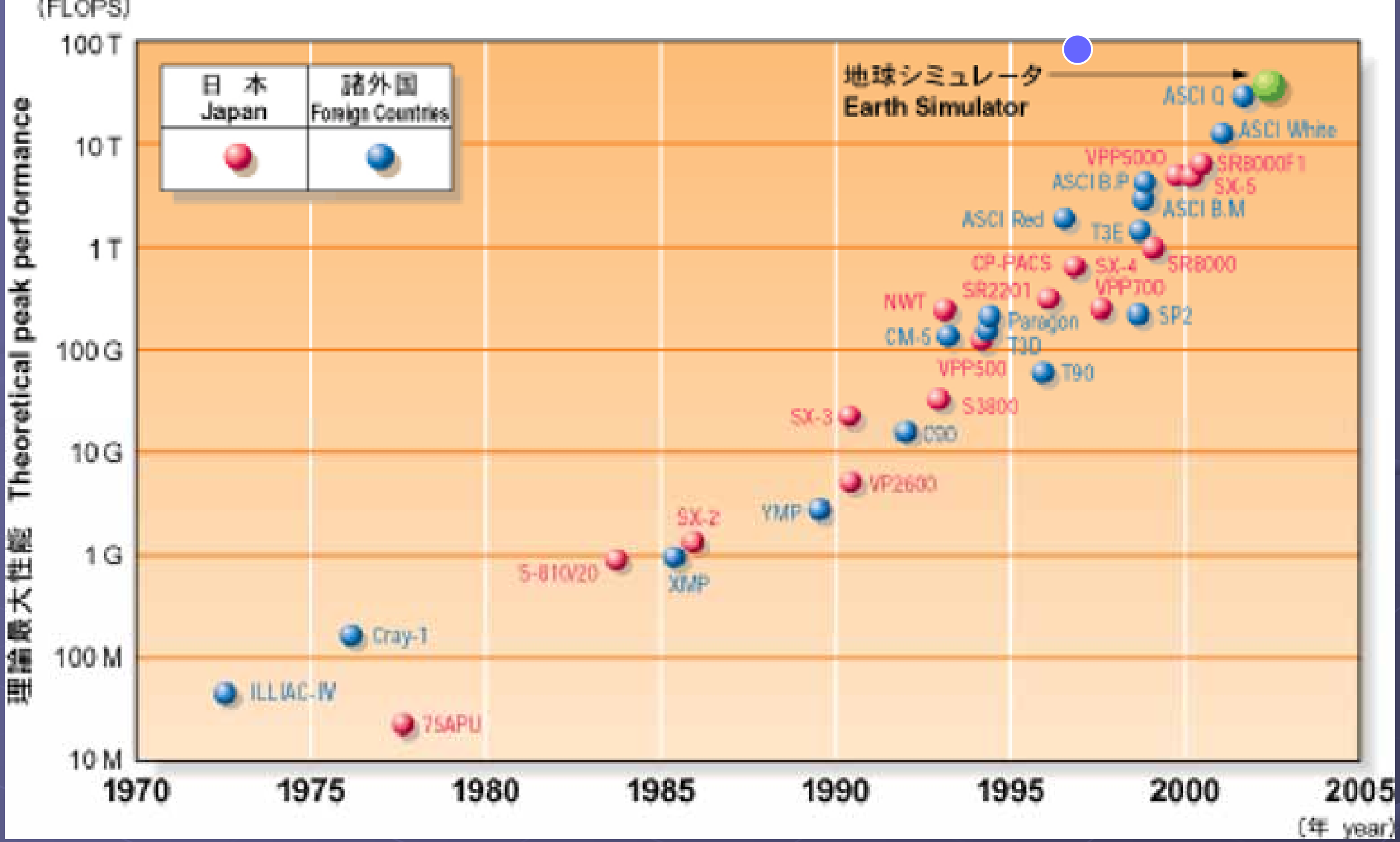
$$M_N = A + B(m_\pi r_0)^2 + C(m_\pi r_0)^3 + D(a/r_0)^2$$

Spectrum



$N_f = 2$

QCDSF-UKQCD



Earth Simulator

- Based on the NEC SX architecture, 640 nodes, each node with 8 vector processors (8 Gflop/s peak per processor), 2 ns cycle time, 16GB shared memory. – Total of 5104 total processors, 40 TFlop/s peak, and 10TB memory.
- It has a single stage crossbar (1800 miles of cable) 83,000 copper cables, 16 GB/s cross section bandwidth.
- 700 TB disk space, 1.6 PB mass store
- Area of computer = 4 tennis courts, 3 floors

Lattice QCD at finite temperature and density (INTRODUCTION)

*DUBNA WINTER SCHOOL 2-3 FEBRUARY **2006***

Finite Temperature in Field Theory

- **In imaginary time the partition function which defines the field theory is:**

$$Z = \int \prod_x d\varphi_x \exp\{-S[\varphi]\}$$

- **The action is:**

$$S[\varphi] = \int_0^{1/T} dt \iiint dxdydz\, L(\varphi, \partial_\mu \varphi)$$

QCD at Finite Temperature

- **Partition function of QCD with one flavor at temperature T is:**

$$Z = \int DA_\mu D\psi D\overline{\psi}\ \exp\{-S[A_\mu,\psi,\overline{\psi}]\}$$

$$S[A_\mu,\psi,\overline{\psi}] = \int_0^{1/T} dt \int d^3x \{(F_{\mu\nu})^2 + \overline{\psi}(i\hat{\partial} - g\hat{A}_\mu + m_q)\psi\}$$

- **In computer**

$$\int d\psi\ d\overline{\psi}\ \exp\{\overline{\psi} M \psi\} = \det M$$

Types of Fermions

Types of Fermions

$$\int d\psi \, d\bar{\psi} \, \exp\{\bar{\psi} M \psi\} = \det M$$

- **Wilson**
- **Kogut-Suskind**
- **Wilson improved**
- **Wilson nonperturbatevely improved**
- **Domain wall**
- ***Staggered***
- ***Overlap***

1. Quark mass ->0 **2. Fast algorithms**

Approximations to real QCD

- Quenched approximation (no fermion loops), gauge group SU(2), SU(3)

- Dynamical fermions, the realistic situation, heavy ***s*** quark and 5Mev ***u*** and ***d*** quarks will be available on computers in 2015(?).

Types of Algorithms

- 1. Hybrid Monte Carlo + Molecular dynamics, leap-frog
- 2. Local Boson Algorithm
- 3. Pseudofermionic Hybrid Monte Carlo
- 3. Two step multyboson
- 4. Polynomial Hybrid Monte Carlo

THE FIFTH ELEMENT

1. Earth (solid state)

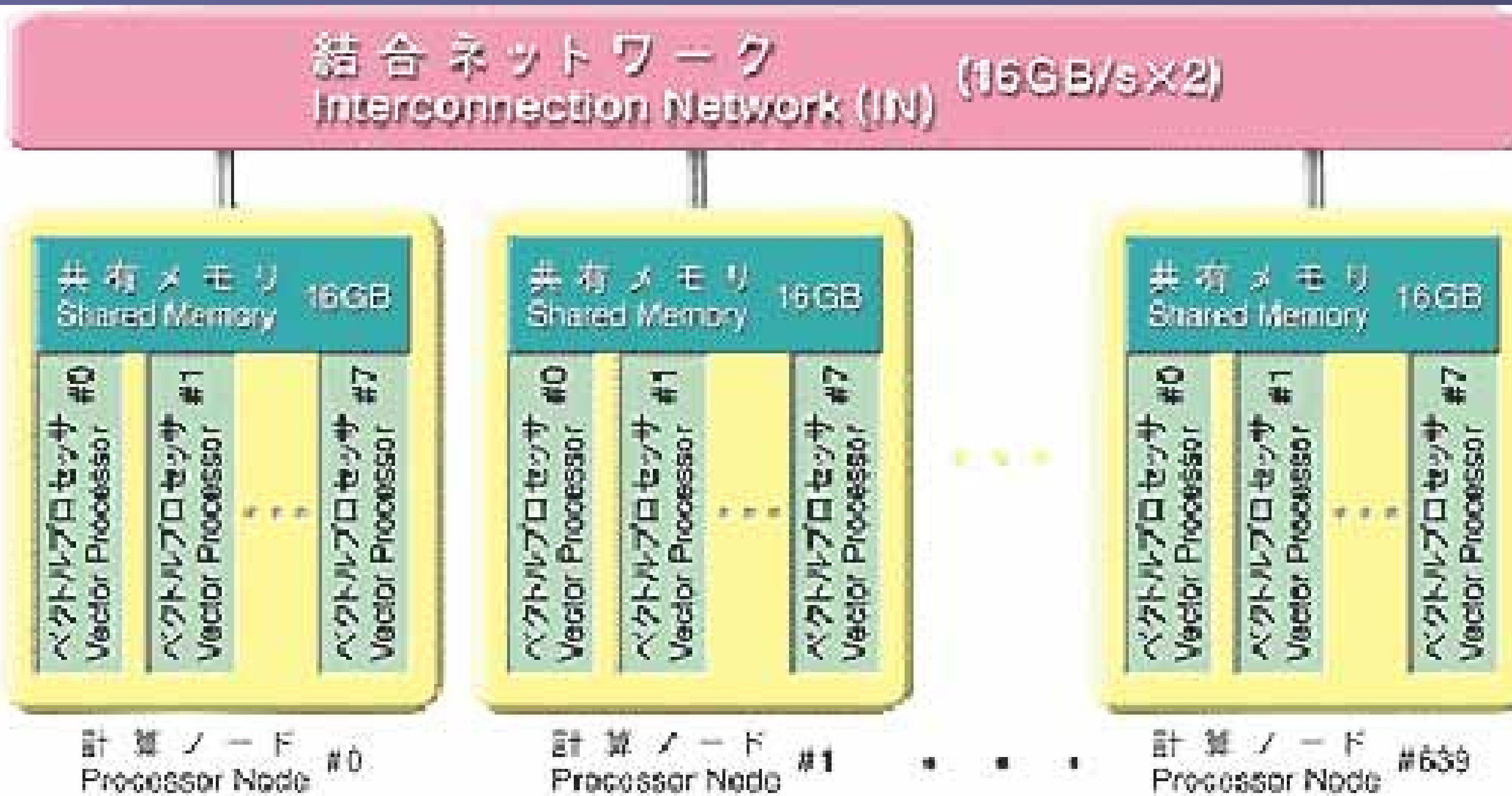
2. Water (liquid)

3. Air (gas)

4. Fire (plasma)

5.….. (quark-gluon plasma)

Earth simulator

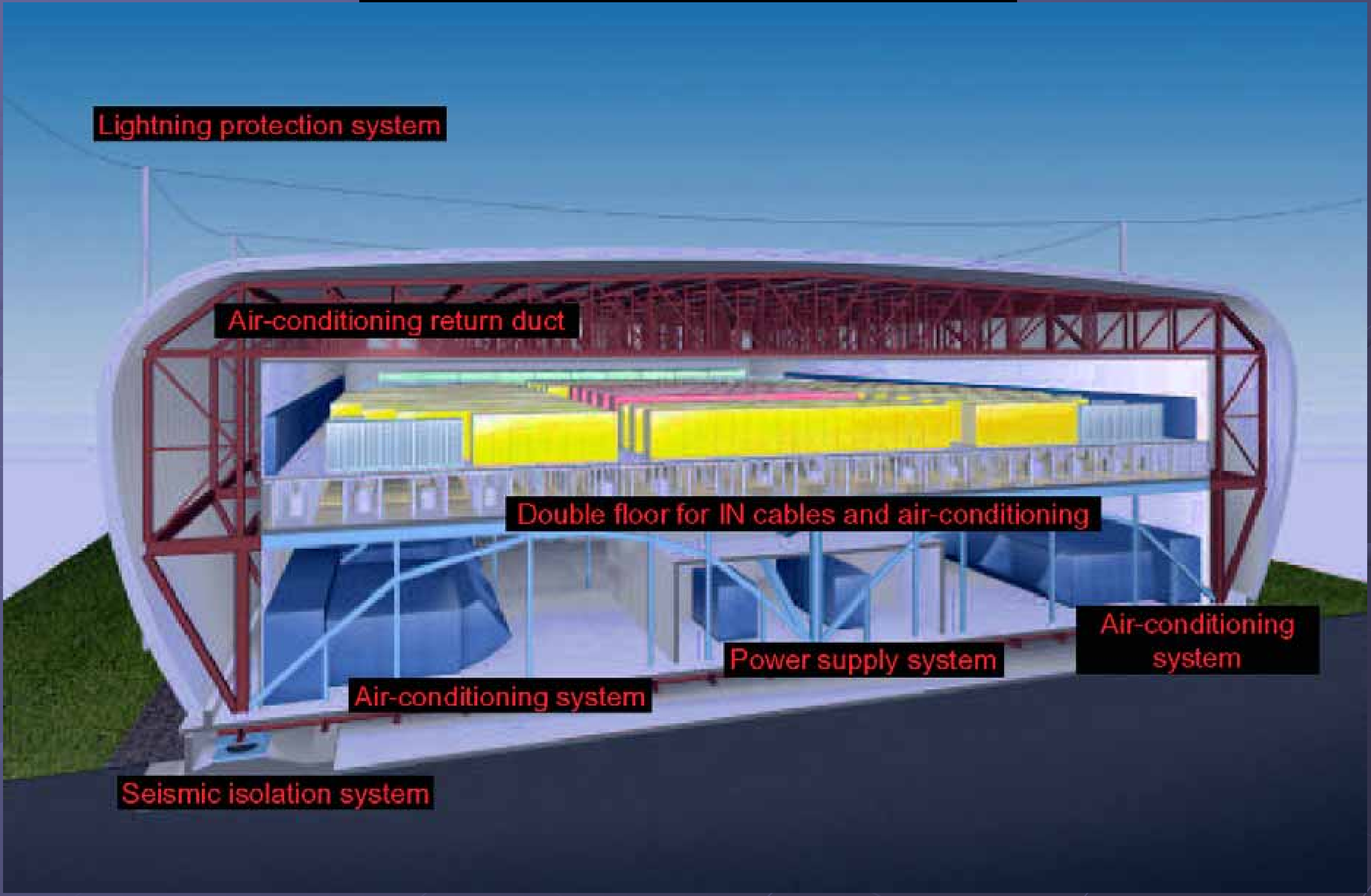


仕様
Specifications

計算プロセッサのピーク性能 Peak performance/processor	8 GFLOPS
計算ノードのピーク性能 Peak performance/node	64 GFLOPS
計算ノードの主記憶容量 Shared memory	16 GB

総プロセッサ数 Total number of processors	5,120
総計算ノード数 Total number of nodes	640
ピーク性能 Total peak performance	40 TFLOPS
主記憶容量 Total main memory	10 TB

Earth simulator



Earth simulator

Earth simulator

Earth simulator

How to find quark gluon plasma?

ORDER PARAMETERS

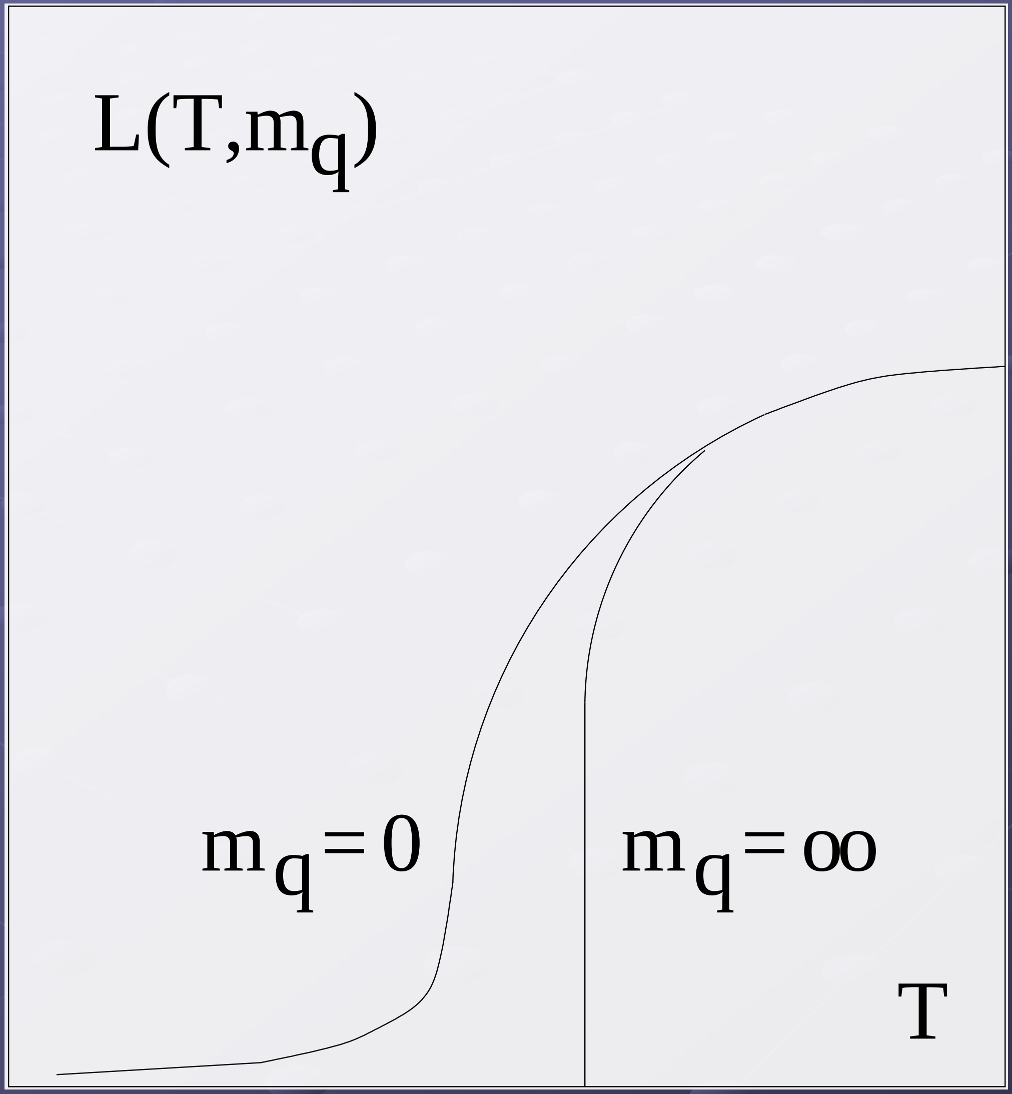
$$m_q = \infty$$

$$\text{Polyakov line} = \langle P \exp\{ i \int_0^{1/T} A_0 \, dx_0 \} \rangle$$

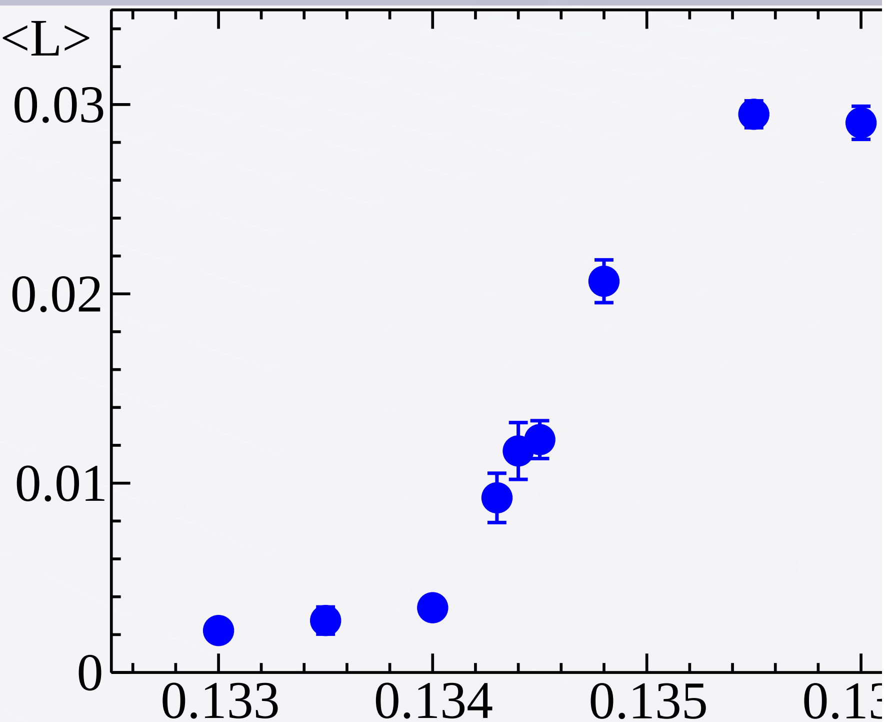
$$m_q = 0$$

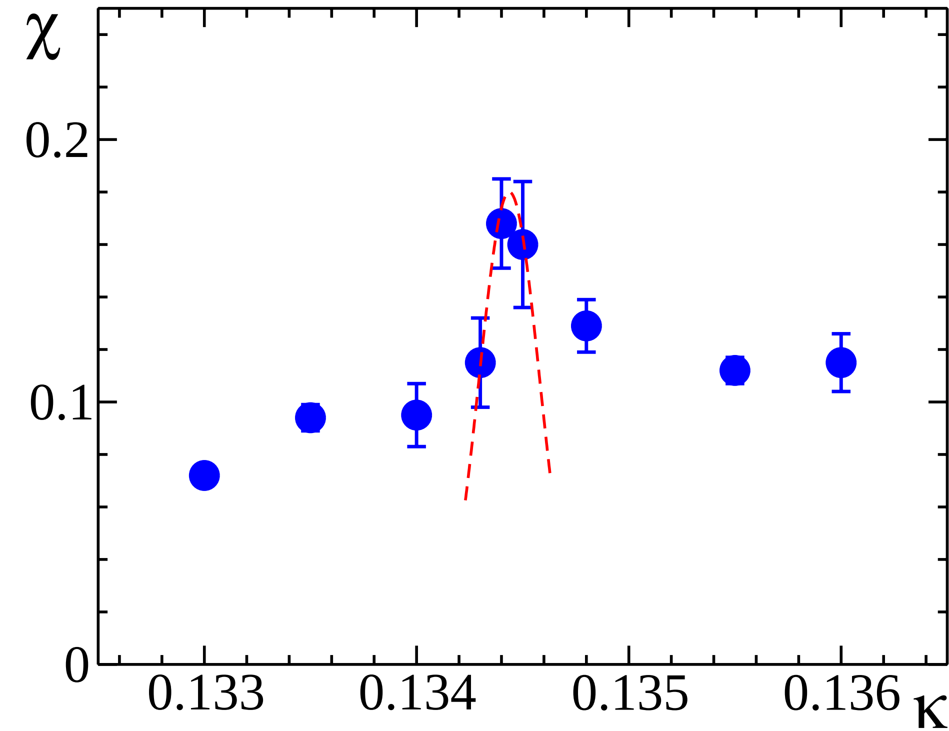
$$\text{Quark condensate} = \langle \bar{\psi} \psi \rangle$$

Example



Lattice calcula



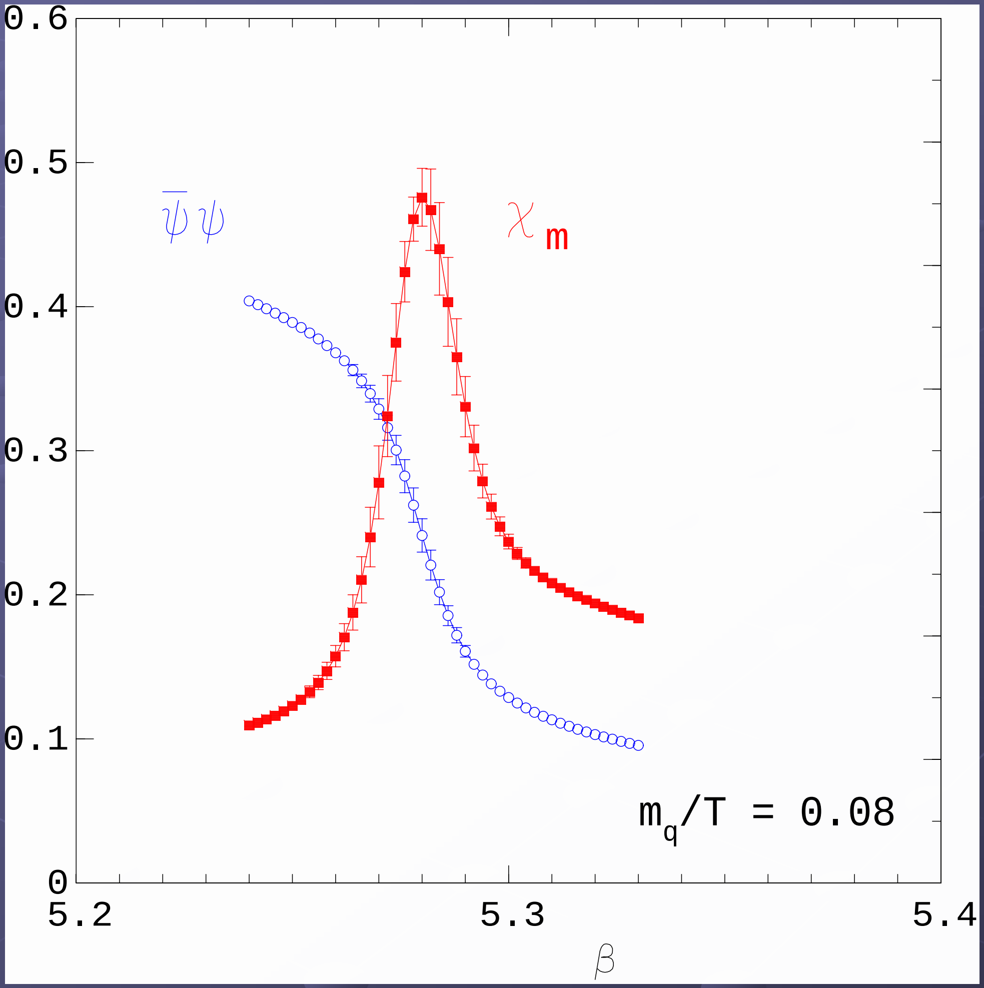


Polyakov loop **susceptibility**

clower improved Wilson fermions

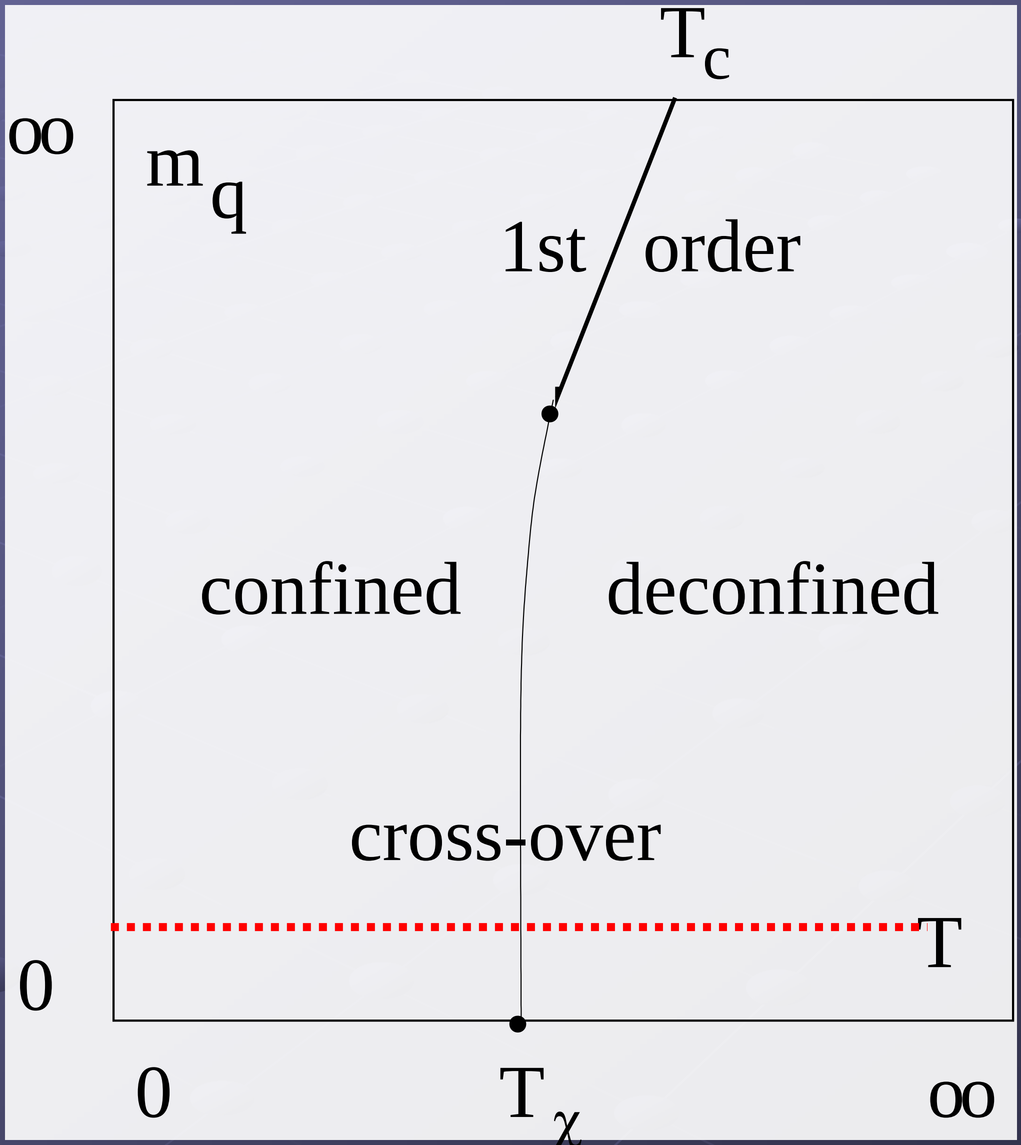
163*8 lattice**

Quark condensate

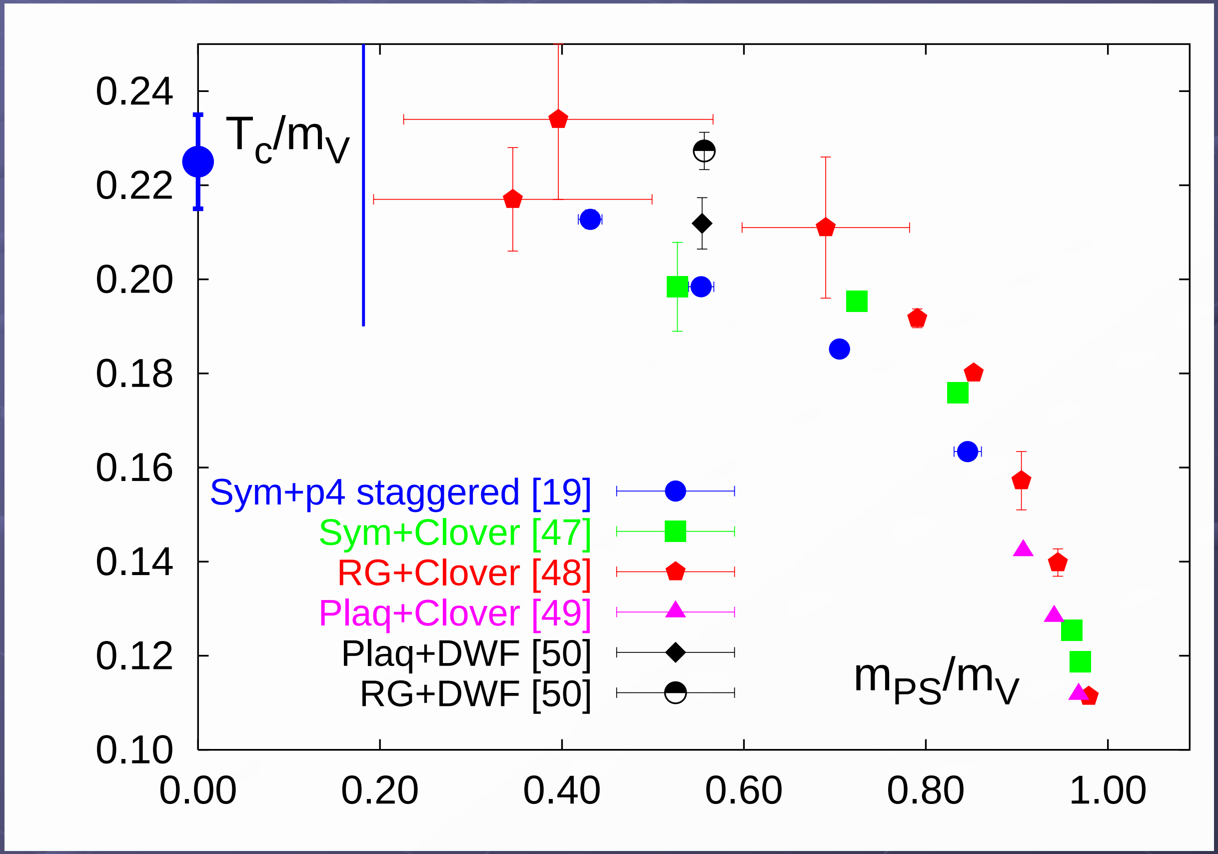


Fermion condensate vs. T, F.Karsch et al.

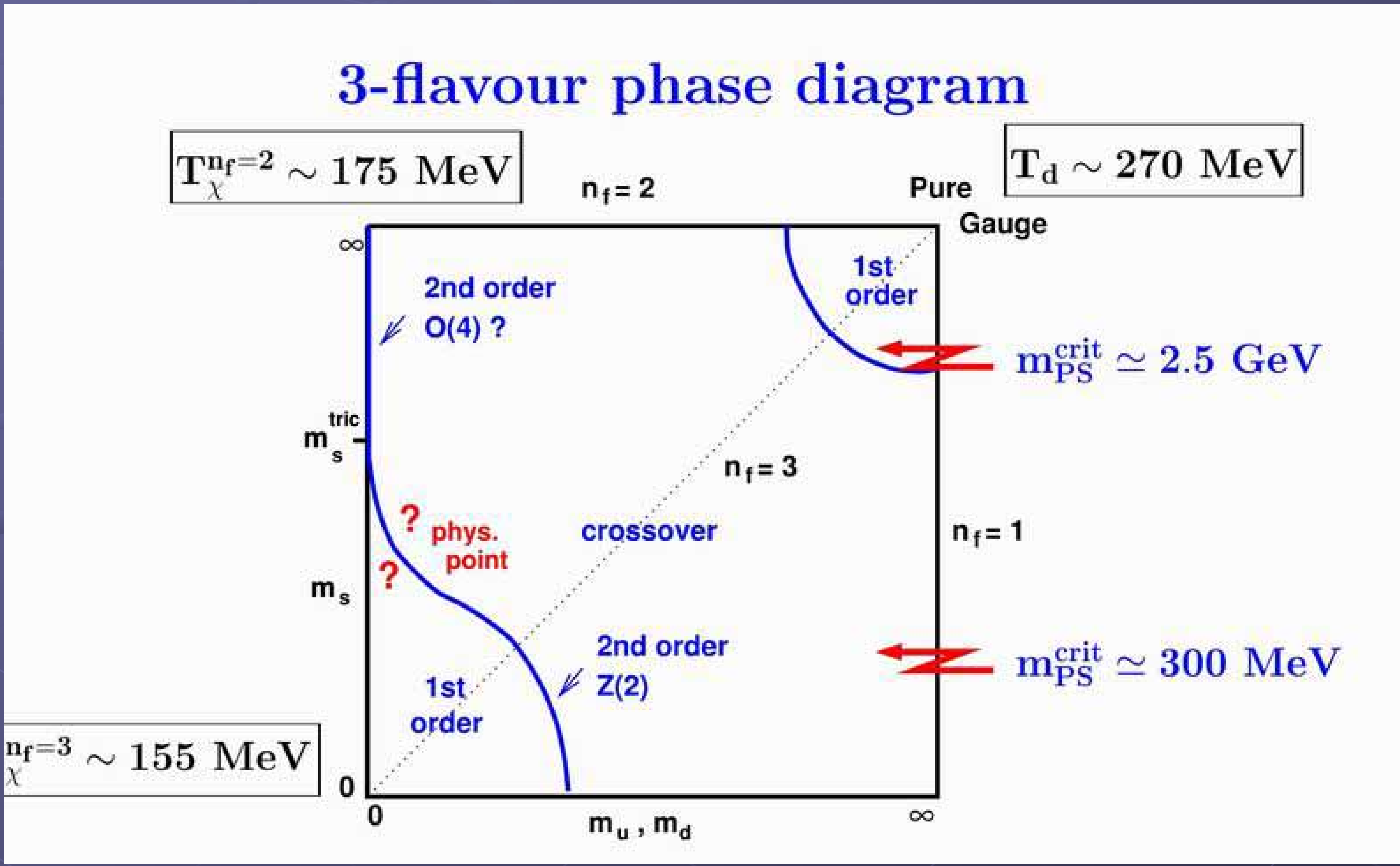
Phase diagram m_q-T, one flavor



Quark mass dependence of Tc



Three quarks



Critical temperature for pure glue and for various dynamical quarks

Pure SU(3) glue (simplest case)

Temperature of the phase transition

- **Pure glue SU(3)** $$T_c = (271 \pm 2)\ Mev$$ ***F. Karsch***

Temperature of the phase transition

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- **Two flavor QCD, clover improved Wilson fermions**

$$T_c = (171 \pm 4)\ Mev$$

C.Bernard (2005)

$$T_c = (173 \pm 3); \quad (166 \pm 3)\ Mev$$

DIK collaboration (2005)

Temperature of the phase transition

- Pure glue SU(3) $T_c = (271 \pm 2)\ Mev$ *F. Karsch*

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F.Karsch (2000)


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Temperature of the phase transition

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DIK collaboration (2005)

- Two flavor QCD, improved staggered fermions

$$T_c = (173 \pm 8)\ Mev$$

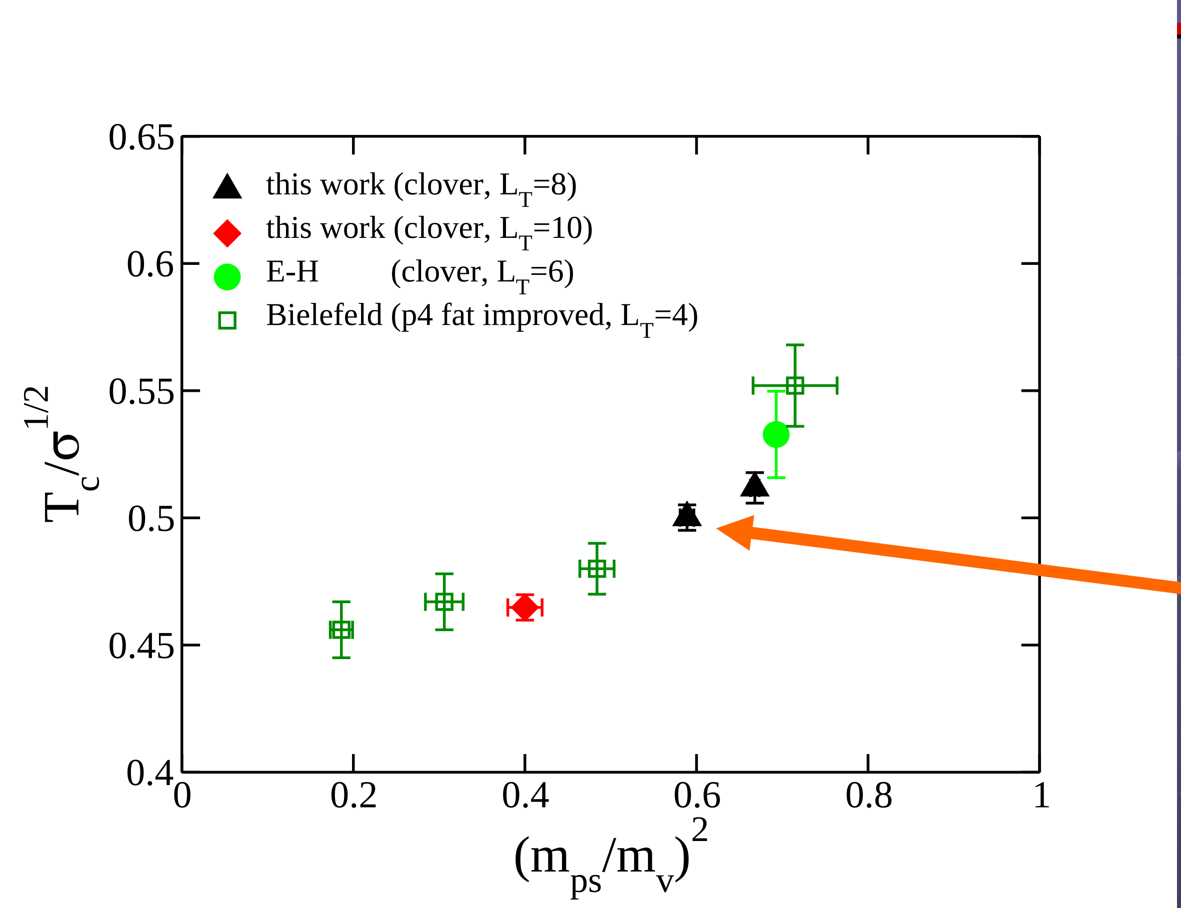
F.Karsch (2000)

- Three flavor QCD, improved staggered fermions!

$$T_c = (154 \pm 8)\ Mev$$

F.Karsch (2000)

Example of extrapolation (DIK 2005)



Russian (JSCC) supercomputer M1000

$$T_c(m_\pi, a) = T_c + C_1\left(\frac{a}{r_0}\right) + C_2(m_\pi a)^\alpha$$

Plasma thermodynamics

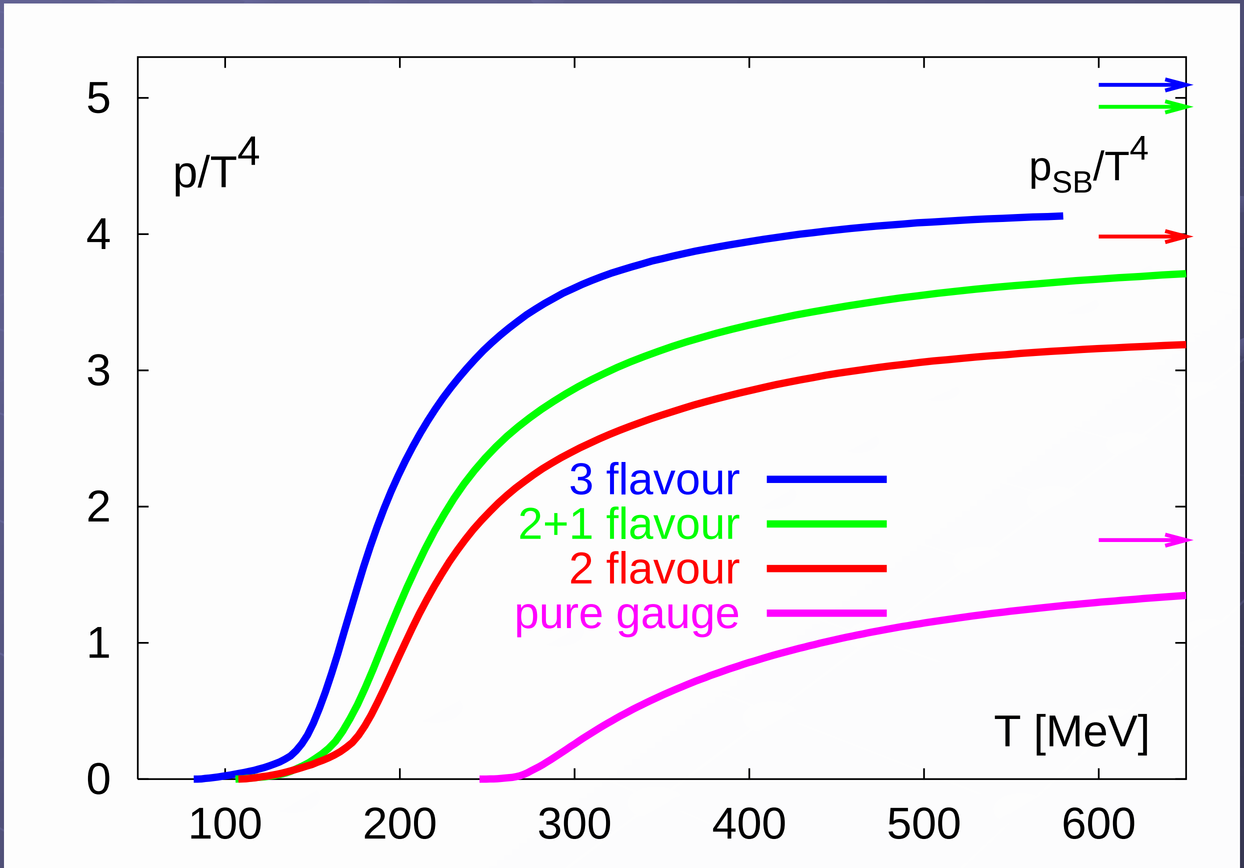
- Free energy density $$f = -\frac{T}{V} Z(T,V)$$

- ε energy, s entropy, c_s velosity of sound, p pressure

$$p = -f;$$

$$\frac{\varepsilon - p}{T^4} = T \frac{d}{dT}\left(\frac{P}{T^4}\right); \frac{s}{T^3} = \frac{\varepsilon + p}{T^4}; c_s^2 = \frac{dp}{d\varepsilon}$$

Example: pressure



F. Karsch (2001-2005)

FULL PROBLEM

Non zero chemical potential

- QCD partition function at finite temperature and chemical potential

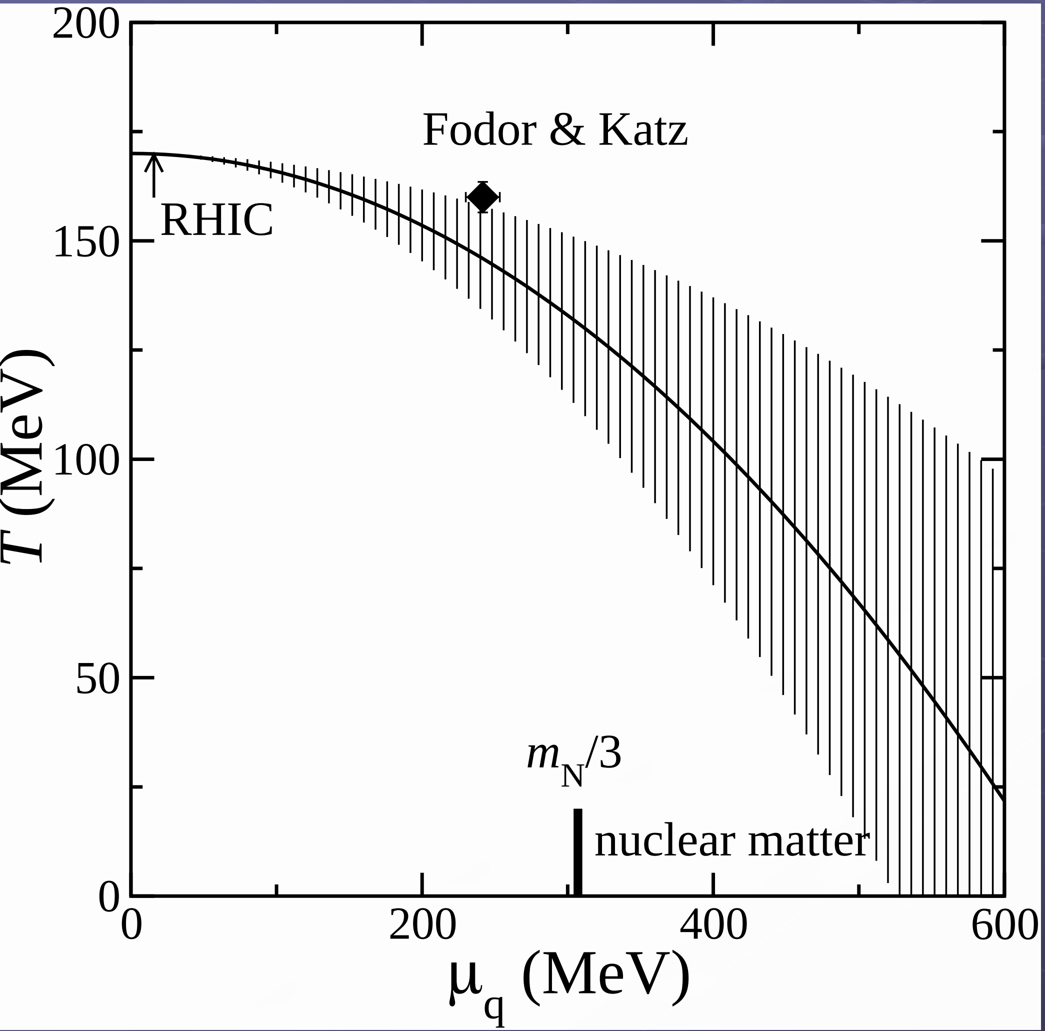
$$Z = \int DA_\mu D\psi D\bar{\psi}\ \exp\{-S[A_\mu,\psi,\bar{\psi}] + \mu \int_0^{1/T} dt \int d^3x \bar{\psi}\gamma_0\psi\}$$

- At finite chemical potential the fermionic determinant is not positively defined! Thus we have no interpretation of the partition function as probability wait (big difficulties with MC).

$$\int d\psi\ d\bar{\psi}\ \exp\{\bar{\psi} M \psi\} = \det M$$

Анекдот

Mu-T diagramme, example of calculation (weighted expantion on mu)



C.R. Aallton et al. (2005)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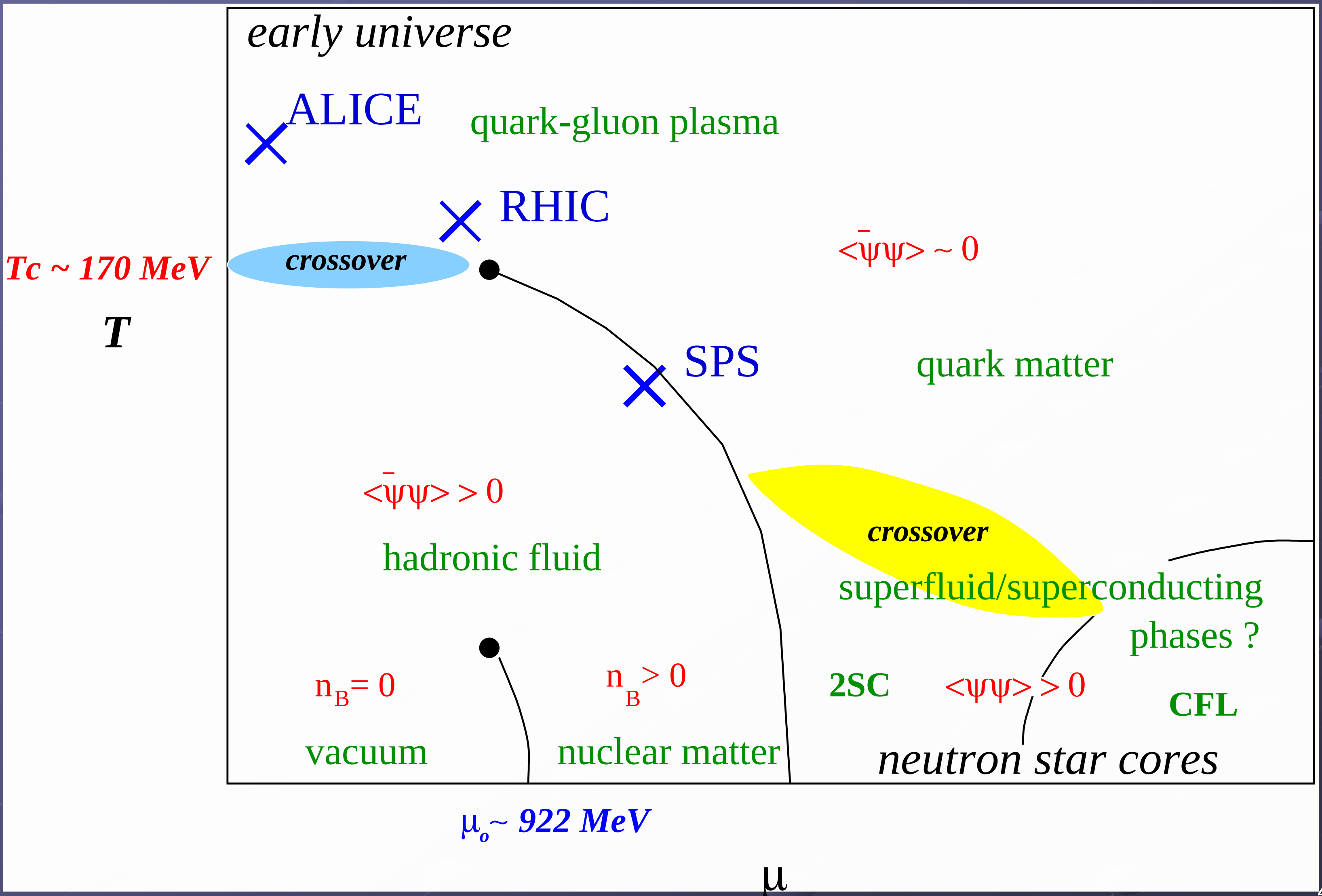
Full diagram (theory) C.R. Aallton et al. (2005)



Literature

- H. Satz hep-ph/0007209
- F. Karsch hep-lat/0106019
- C.R. Allton et al. hep-lat/0504011
- F. Karsch hep-lat/0601013
- DIK (DESY-ITEP-Kanazawa) collaboration hep-lat/0509122, hep-lat/0401014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VALERY SCHEKOLDIN
(photo)