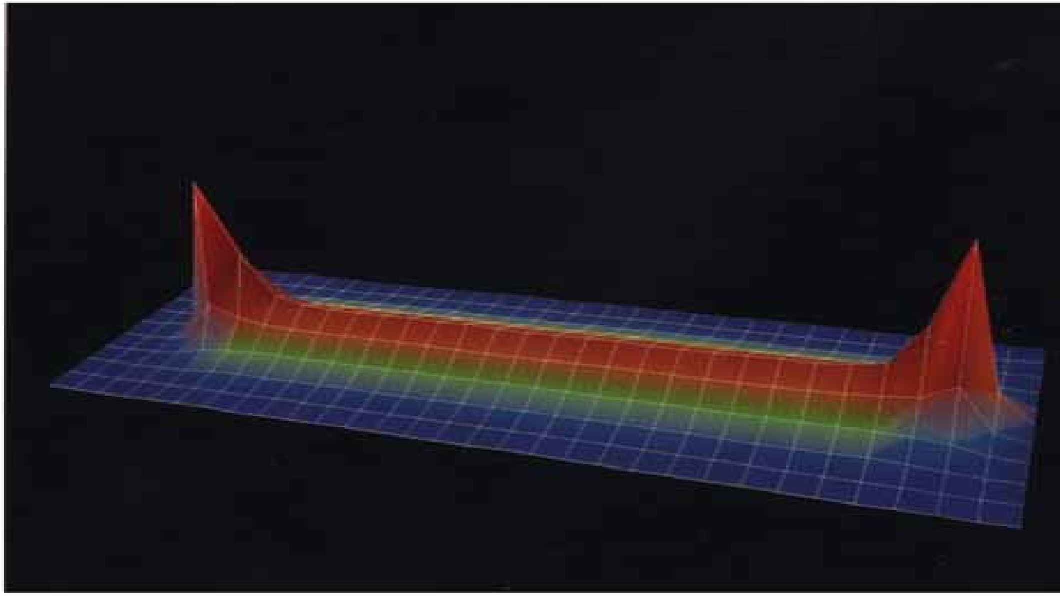

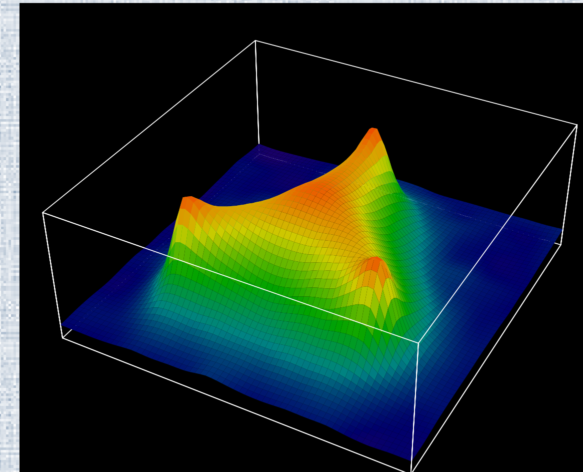

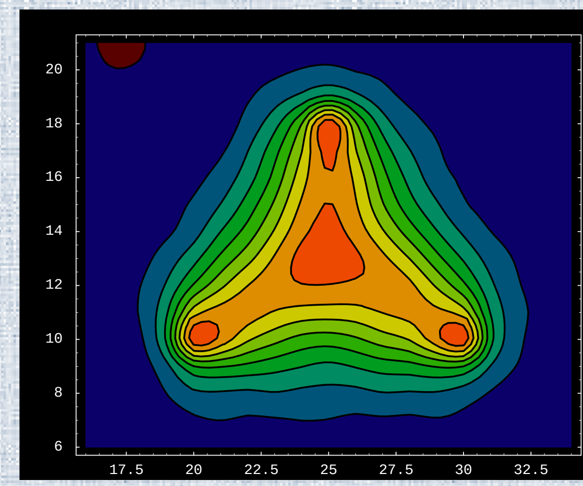
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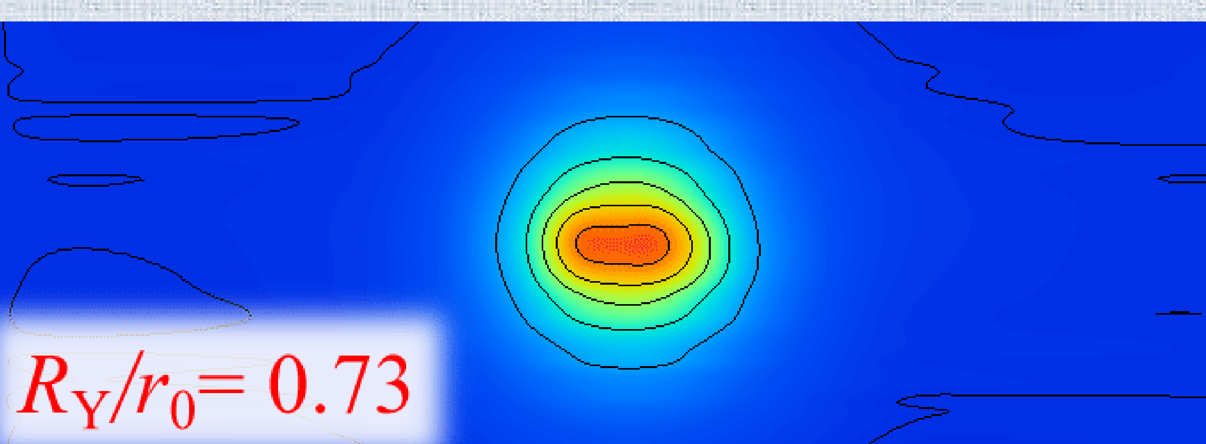
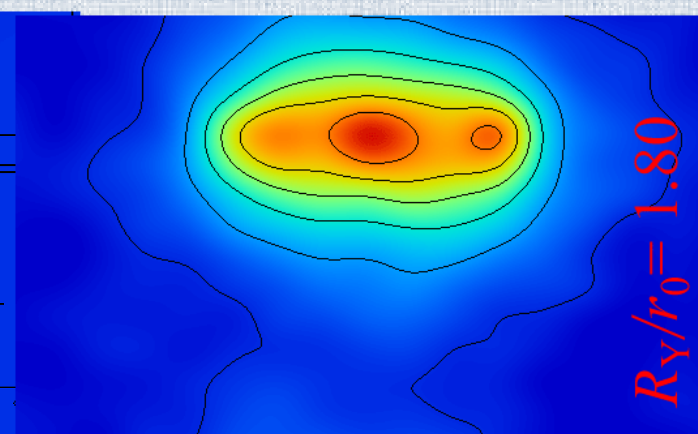
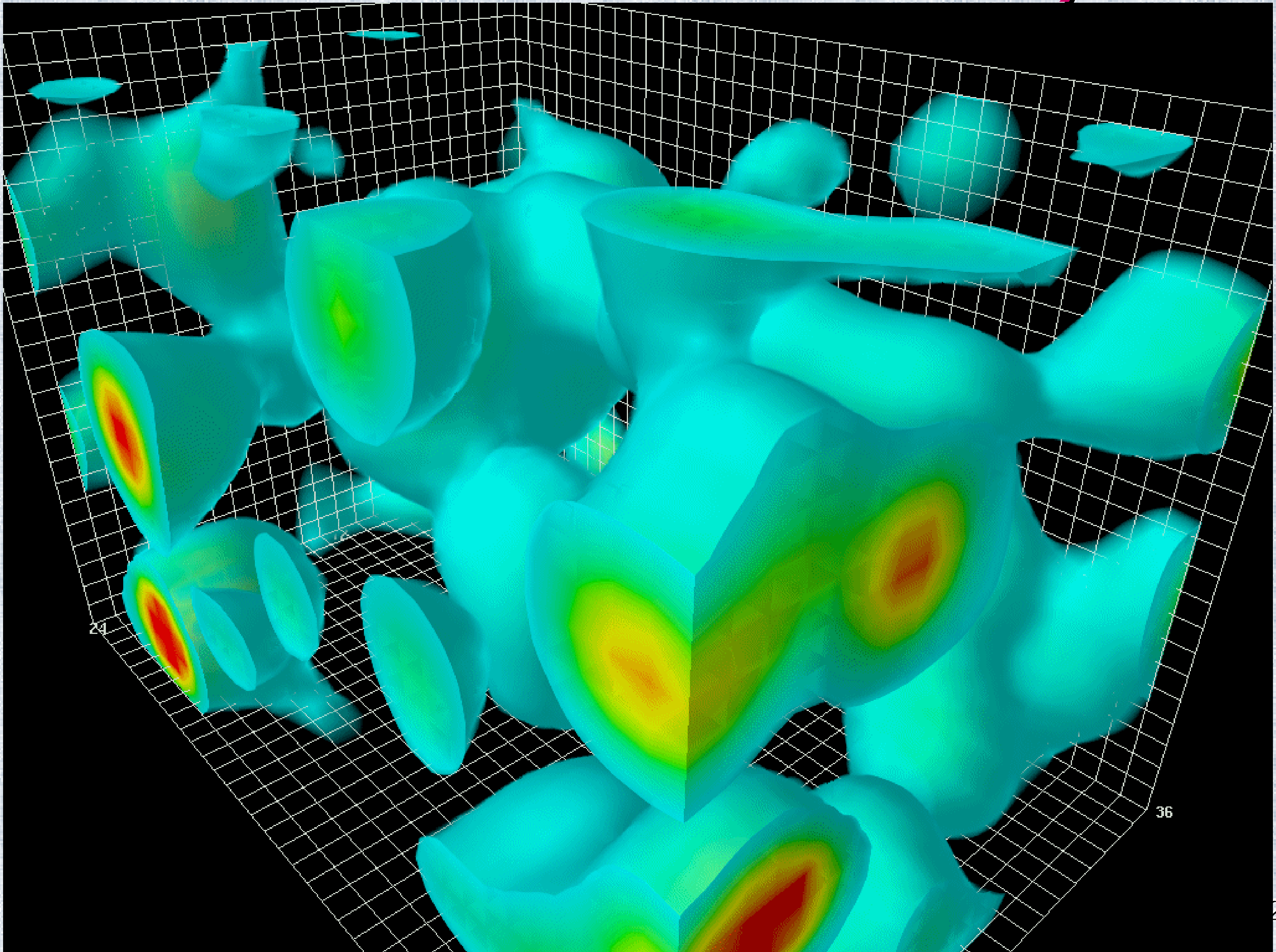

$R_Y/r_0 = 0.73$


$R_Y/r_0 = 1.80$

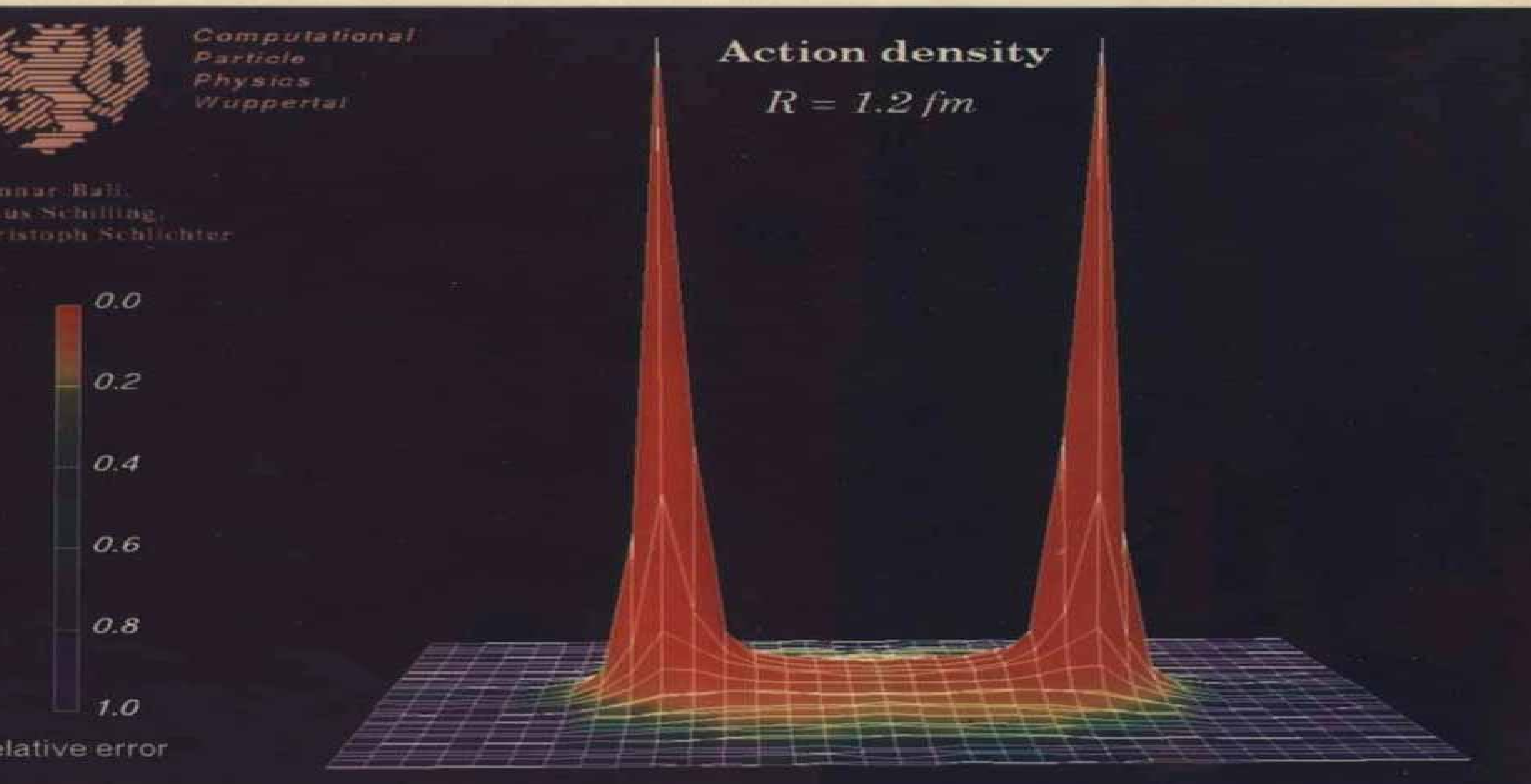
QCD vacuum: 3d time slices, action



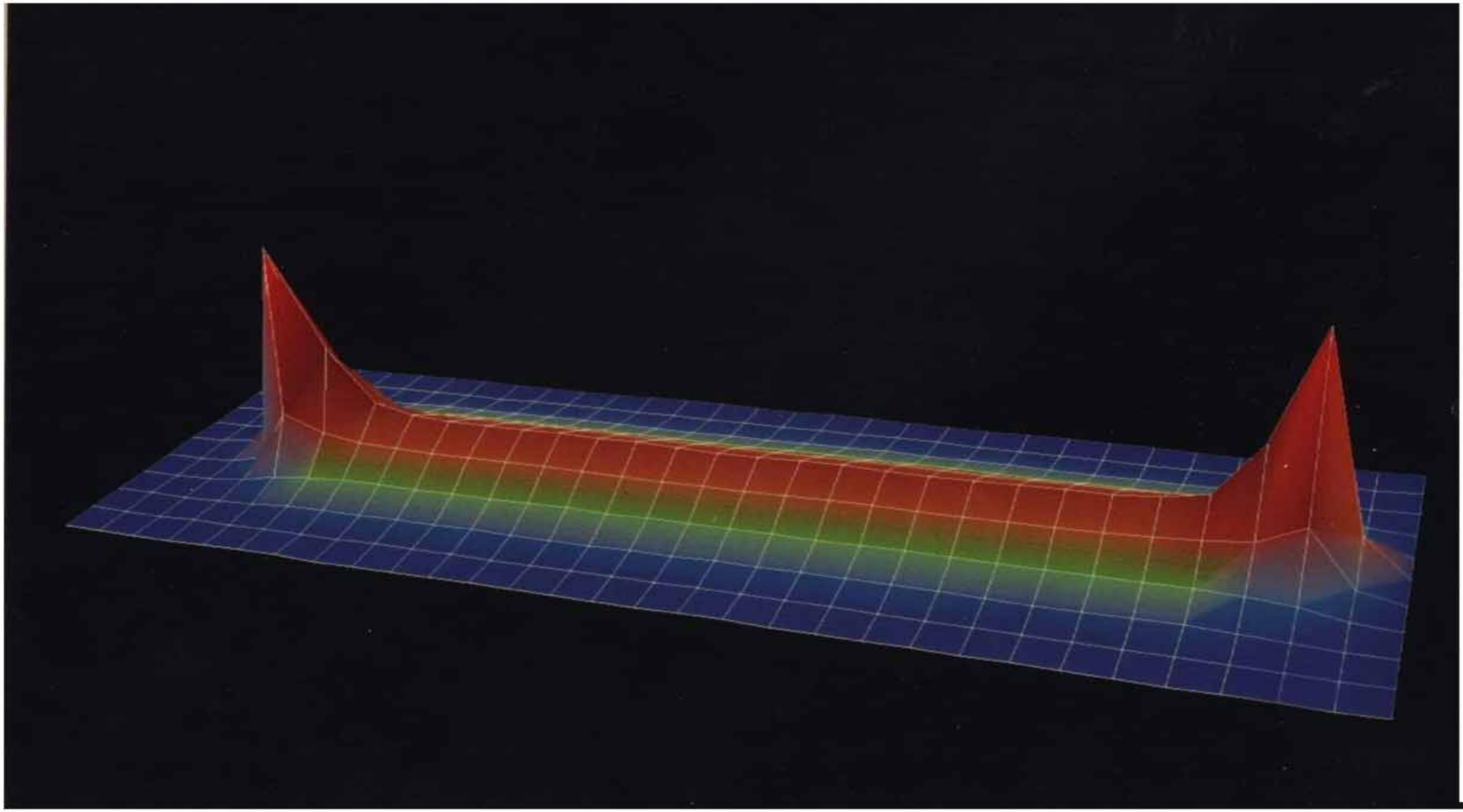
Lattice QCD
(INTRODUCTION)

*DUBNA WINTER SCHOOL 1-2 FEBRUARY **2005***

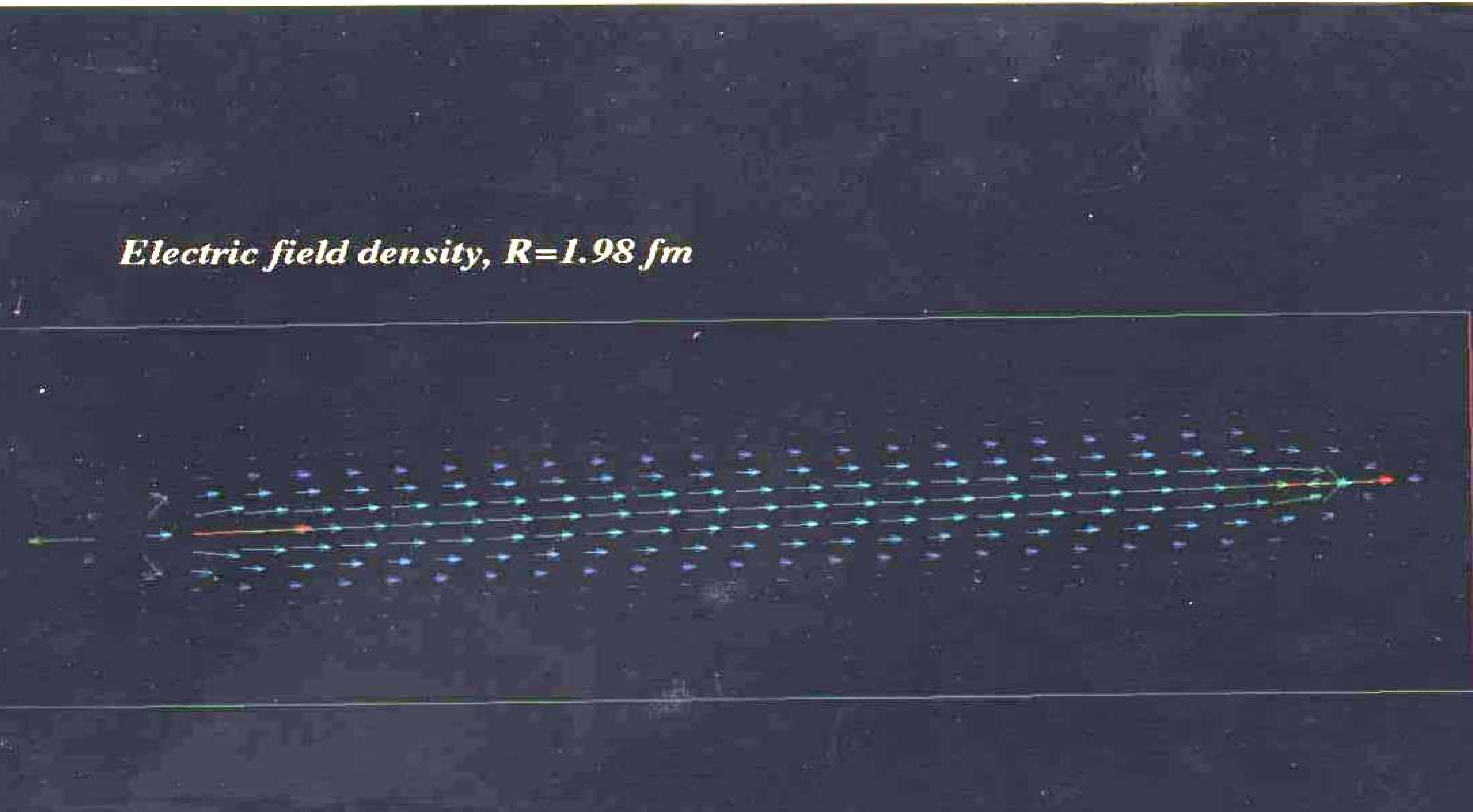
Confining String (*Bali, Schlichter, Schilling*)



Confining String



Electric field of confining String



$$L = \frac{1}{4g^2} F_{\mu\nu}^2 + \frac{1}{2} |(\partial_\mu - iB_\mu)\Phi|^2$$

$$+ \lambda(|\Phi|^2 - \eta^2)^2$$

Confinement in Abelian theories

- **Compact electrodynamics**

  Confinement is due to monopoles, which are condensed, and vacuum is a dual superconductor

- **Z(2) gauge theory**

  Confinement is due to Z(2) vortices.

Monopole confinement
Compact Electrodynamics

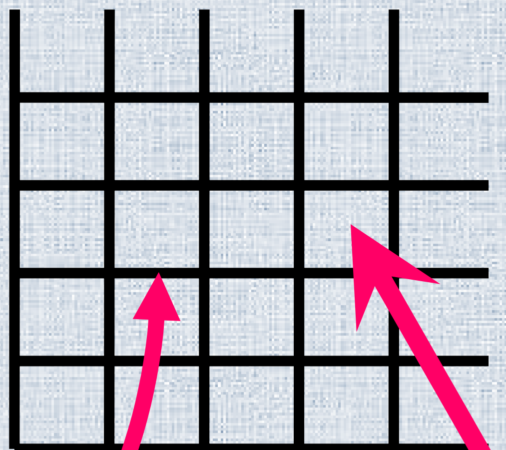
Vortex Confinement
Z(2) gauge theory

Confinement in compact electrodynamics

Partition function:

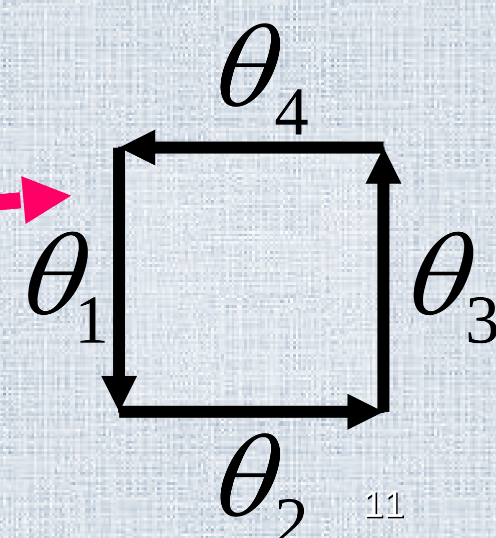
$$Z = \int_{-\pi}^{\pi} D\theta_l \, e^{\beta \sum_P \cos\theta_P}$$

$$Z = \int_{-\pi}^{\pi} D\theta_l \, e^{\beta \sum_P \cos\theta_P}$$



θ_l

$$\boxed{\theta_P = \theta_1 + \theta_2 - \theta_3 - \theta_4}$$



Confinement in compact electrodynamics

Various representations of partition function

$$Z = \int_{-\pi}^{\pi} D\theta_l \, e^{\beta \sum_P \cos\theta_P} = const. \sum_{\delta j = 0} \exp\{-4\pi^2 \beta (j, \Delta^{-1} j)\} =$$

$$= const. \lim_{\substack{\lambda\to\infty \\ \kappa\to\infty}} \int DB_\mu D\varphi \exp\{-S^{AHM}_{dual}[B_\mu, \varphi]\}$$

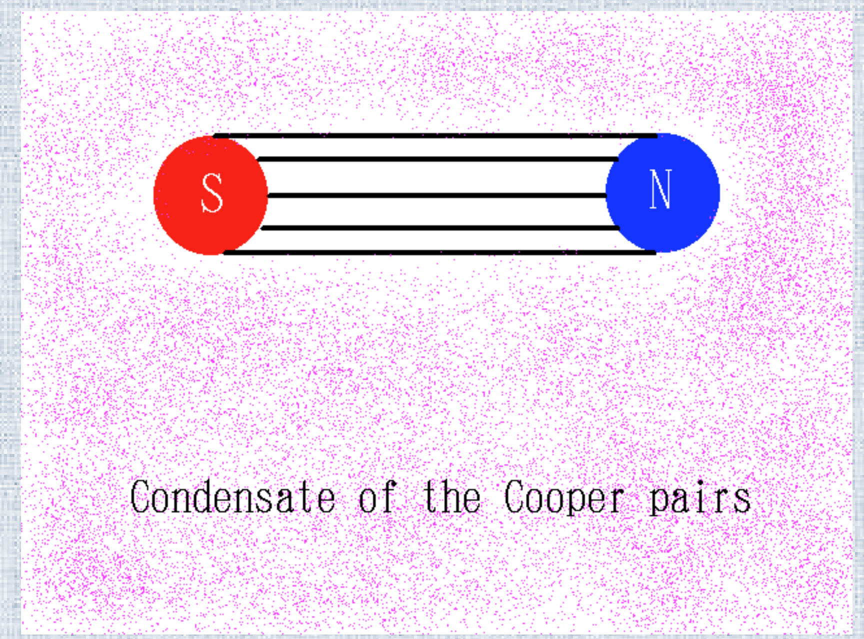
Dual superconductor Action

$$S^{AHM}_{dual} = \int d^4x [\frac{1}{g^2} G^2_{\mu\nu} + \kappa |(B_\mu + ig\partial_\mu)\Phi|^2 + \lambda(|\Phi|^2 - 1)^2]$$

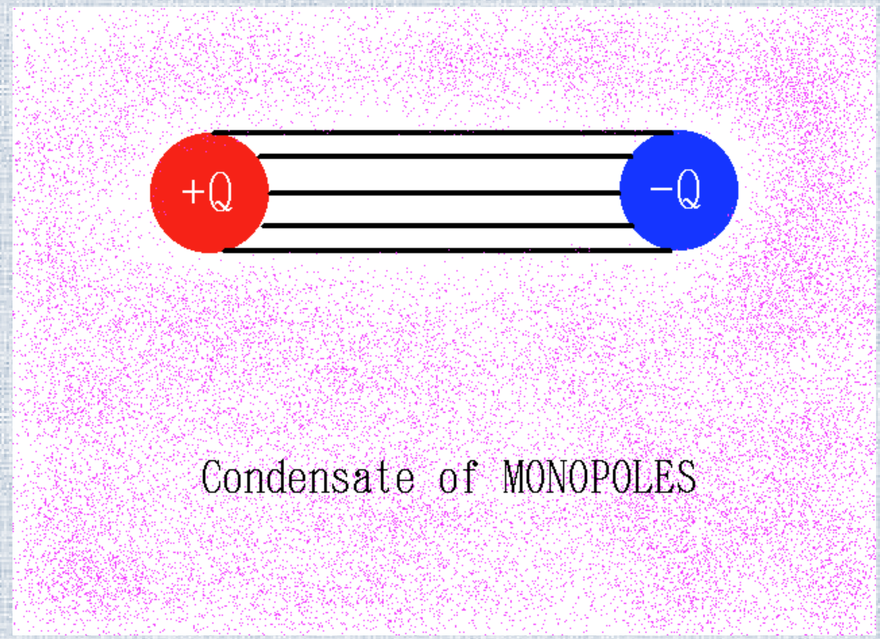
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Vacuum of compact QED is dual to supercoductor

Monopole-antimonopole in superconductor

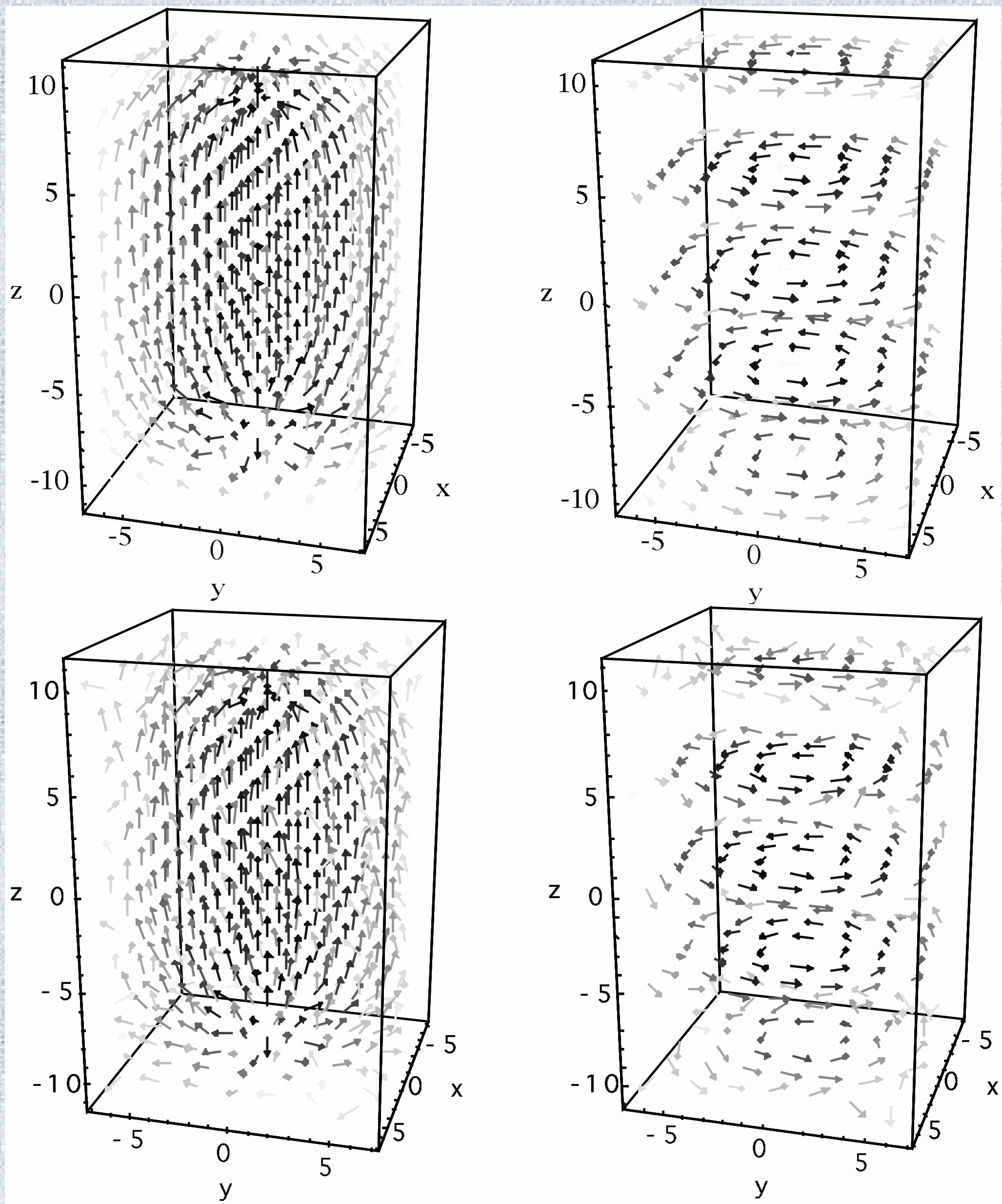


Charge-anticharge in cQED vacuum



MONOPOLE CONFINEMENT

hep-lat/0302006, Y. Koma, M. Koma, E.M. Ilgenfritz, T. Suzuki, M.I.P.



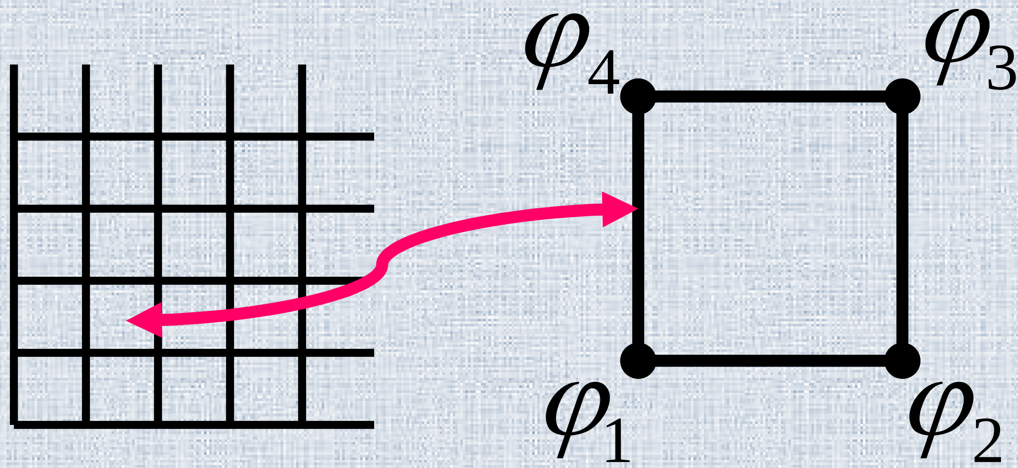
Dual Abelian Higgs Model

Abelian Projection of SU(2) gluodynamics

$$\sigma_{mon} \approx 90\% \, \sigma_{SU(2)}$$

Monopole is a topological defect

EXAMPLE: 2D topological defect



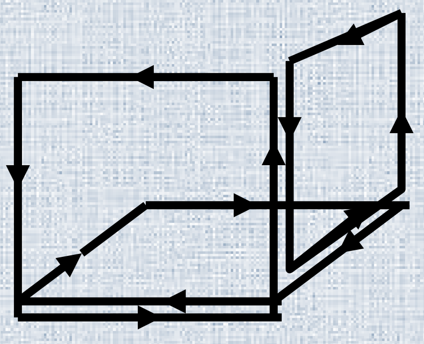
$$m = \frac{1}{2\pi}\{[\varphi_1 - \varphi_2]_{2\pi} + [\varphi_2 - \varphi_3]_{2\pi} + [\varphi_3 - \varphi_4]_{2\pi} + [\varphi_4 - \varphi_5]_{2\pi}\}$$

$$-\pi \le \varphi_i < \pi$$

$$m = 0, \pm 1$$

Topology: from real numbers we get integer, thus small, but finite variation of angles does not change topological number m !!!

Monopole is a topological defect

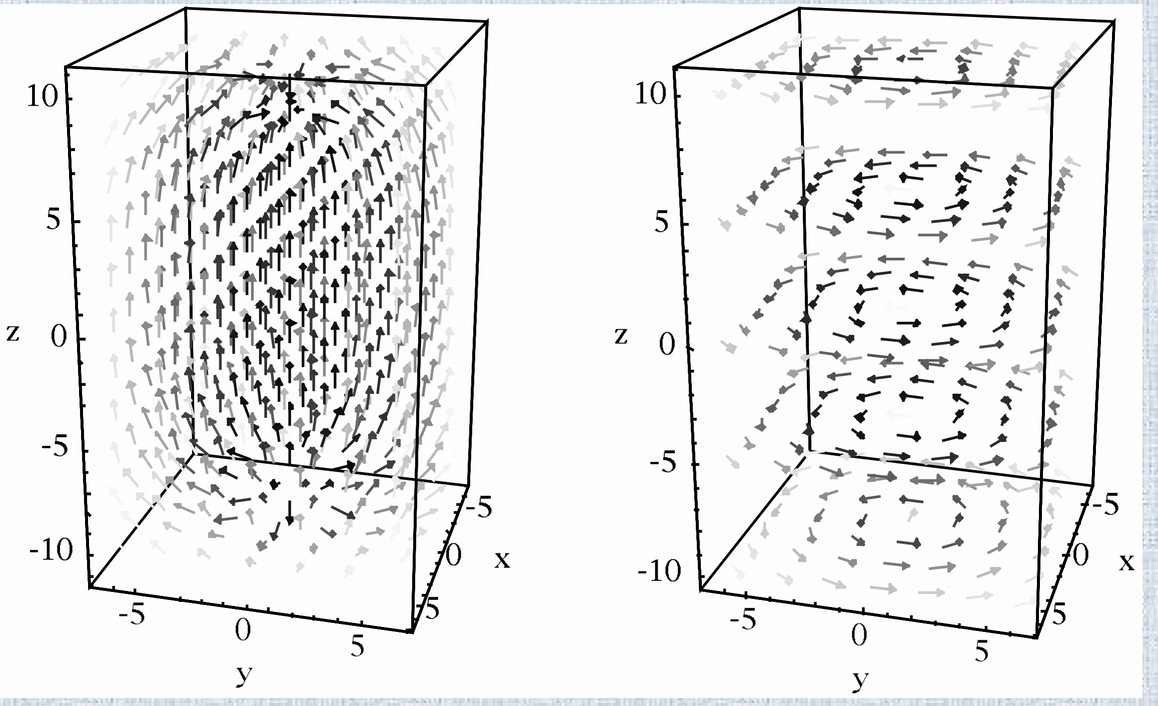


$$\theta_p = \theta_a + \theta_d - \theta_c - \theta_d$$

For each cube we have integer valued current *j*

$$2\pi\, j_{mon} = [\theta_{P1}] + [\theta_{P2}] + [\theta_{P3}] + \\ + [\theta_{P4}] + [\theta_{P5}] + [\theta_{P6}]$$

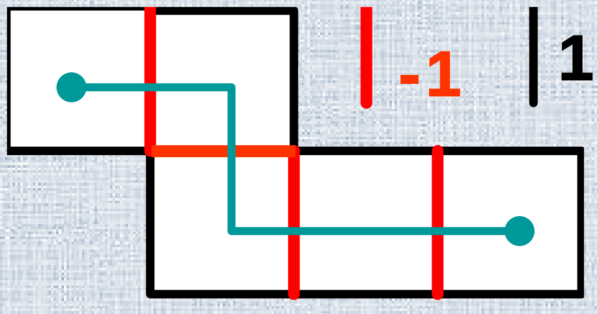
These monopoles are responsible for the formation of electric string



Confinement in Z(2) Gauge Theory

$$z = \sum_{\{s=\pm 1\}} e^{-\beta S} ; S = \sum_P s_p ; s_p = s_1 s_2 s_3 s_4$$

2D example of "vortices" (they are points connected by "Dirac line")



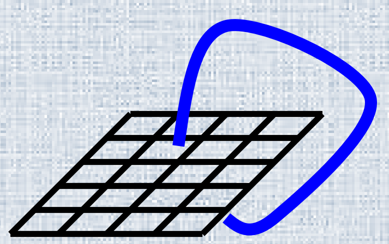
In 3D vortices are closed lines, Dirac strings are 2d surfaces spanned on vortices

In 4D vortices are closed surfaces, Dirac strings are 3d volumes spanned on vortices

Confinement in 3D Z(2) Gauge Theory

If ***p*** is the probability that the plaquette is pierced by vortex then the expectation value of the plaquette is:

$$< P >= (1 - p)(+1) + p(-1) = 1 - 2p$$

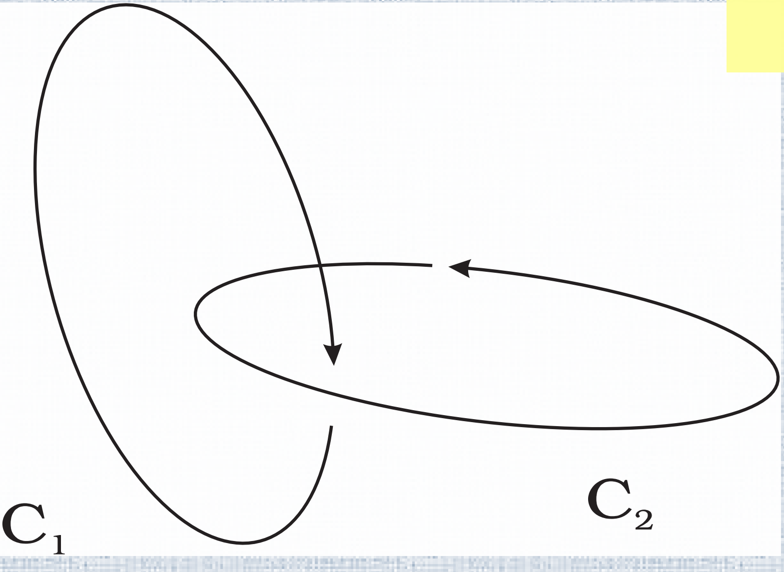


If vortices are uncorrelated (random) then expectation value of the Wilson loop is:

$$< W >= (1 - 2p)^{Area} = e^{-\sigma \cdot Area} ; \sigma = -\ln(1 - 2p)$$

Linking number

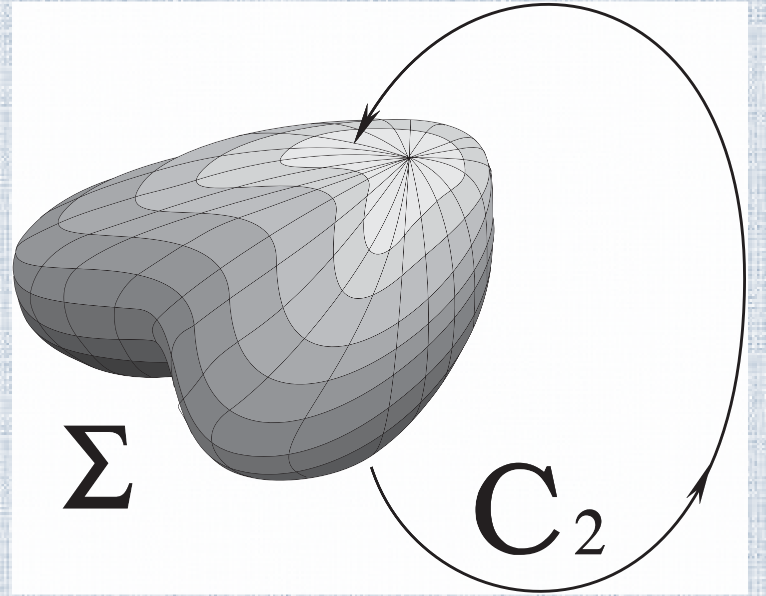
3D



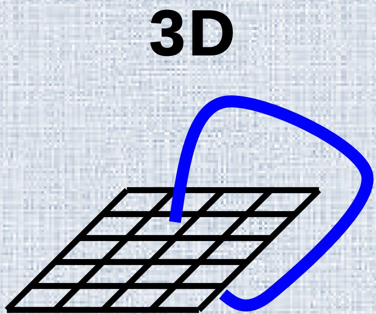
$$L = \frac{1}{4\pi} \oint_{C_1} dx_i \oint_{C_2} dy_k \, \varepsilon_{ikl} \partial_l \frac{1}{|x-y|}$$

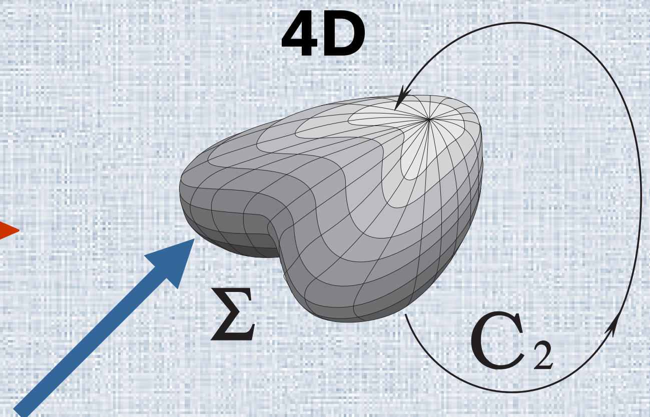
4D

$$L = \frac{1}{8\pi^2} \oint_{C_1} d\Sigma_{\alpha\beta}(x) \oint_{C_2} dy_\gamma \, \varepsilon_{\alpha\beta\gamma\delta} \partial_\delta \frac{1}{|x-y|}$$



Confinement in 4D Z(2) Gauge Theory





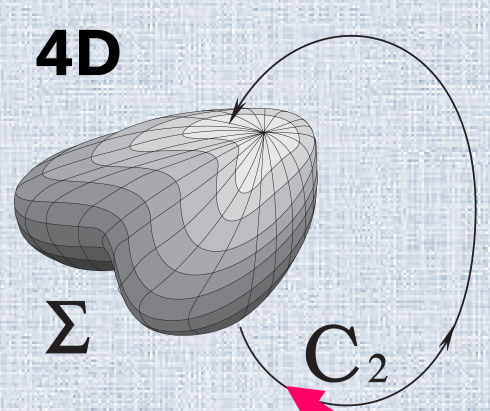
Vortex in 4D is a closed surface, 3d Dirac volume is enclosed by the vortex. If vortices are random then the string tension is the same as in 3D:

$$\sigma = -\ln(1-2p)$$

Confinement in 4D SU(2) Gauge Theory by Center Vortices

Center projection: partial gauge fixing

$$U_l \Rightarrow Z_l$$



Each piercing of the surface spanned on the Wilson loop by center vortex give (-1) to W.

$$\sigma_{vortex} \approx (60\% \div 70\%)\, \sigma_{SU(2)}$$

Monopole confinement
Compact Electrodynamics

Vortex Confinement
Z(2) gauge theory

Anderson localization in lattice QCD

Dubna 2006, JINR Winter School

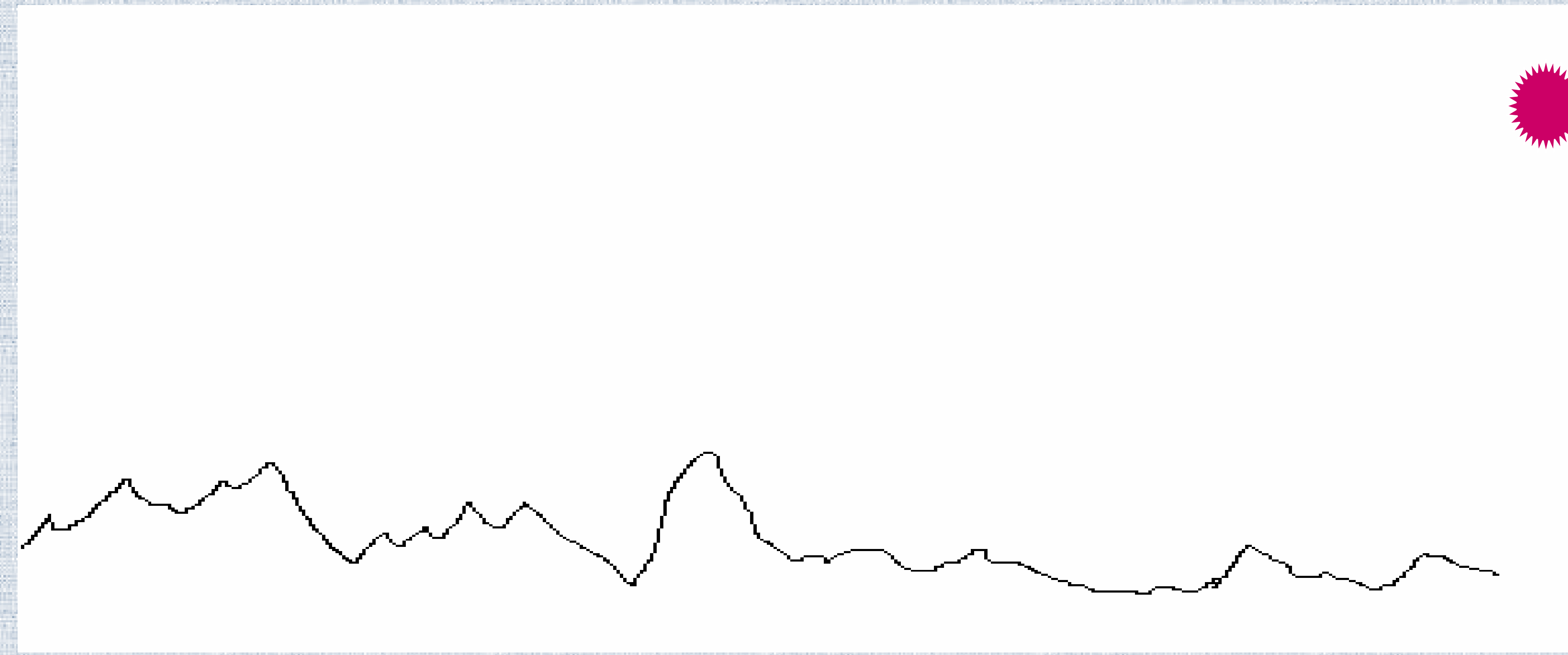
LECTURE II

J. Greensite, F.V. Gubarev, A.V.Kovalenko, S.M. Morozov, S. Olejnik, MIP, S.V. Syritsyn, V.I. Zakharov

hep-lat/0505016, hep-lat/0504008

Anderson localization in 3D

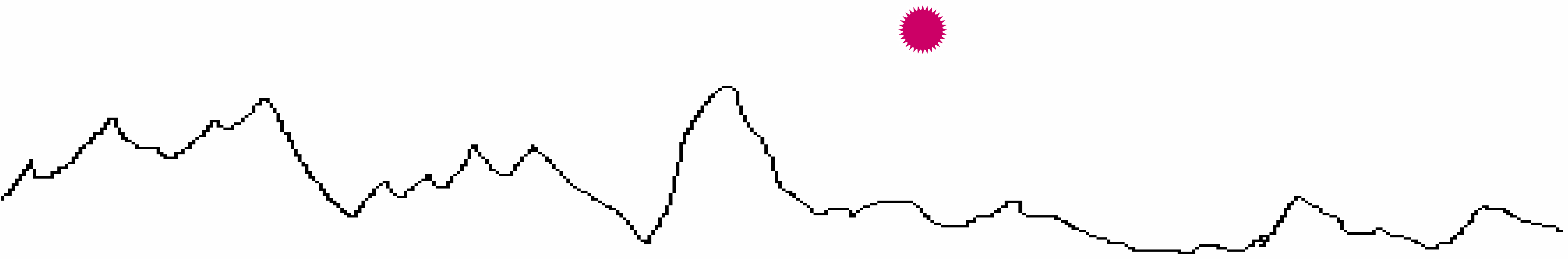
$E > E_{cr}$ (mobility edge)



IPR is small

Anderson localization in 3D

$E<E_{cr}$ (mobility edge)



IPR is large

Eigenmodes localization is characterized by IPR

B.Kramer, A.MacKinnon (1993); C.Gattringer et al. (2001); T.Kovacs (2003); C. Aubin et al. [MILC Collaboration] (2004), J.Greensite et al. (2005) F. Bruckmann, E.-M. Ilgenfritz (2005) (Solid state physics, lattice fermions and bosons). Talks at Lattice 2005: N.Cundy, T.De Grand, C.Gattringer, J.Greensite, J.Hetrick, I.Horvath, Y.Koma, S.Solbrig,B.Svetitsky, S.Syritsyn, M.I.P.

By definition the inverse participation ratio (IPR) is:

$$I_\lambda = V \sum_x \rho_\lambda^2(x), \text{ where } \rho_\lambda(x) = \left|\varphi_\lambda(x)\right|^2,$$

$$\sum_x \rho_\lambda(x) = 1, \quad \hat{H}\varphi_\lambda(x) = \lambda\varphi_\lambda(x).$$

$$I_\lambda \approx 1 \quad \text{for delocalized states}$$

$$I_\lambda \approx V \quad \text{for extremely localized states}$$

$$\boxed{I_\lambda = \frac{V}{b},} \quad \text{where } b \text{ is the localization volume}$$

IPR EXAMPLES

$$I_\lambda = V\sum_x \rho_\lambda^2(x); \quad \sum_x \rho_\lambda(x) = 1 \quad \Longleftrightarrow \quad I_\lambda = \frac{V\sum_x \rho_\lambda^2(x)}{\left(\sum_x \rho_\lambda(x)\right)^2}$$

$$\rho_\lambda(x) = \left|\varphi_\lambda(x)\right|^2$$

$$1)\ \varphi_\lambda(x) = e^{ikx} \Rightarrow \rho_\lambda(x) = 1 \Rightarrow I_\lambda = \frac{V^2}{V^2} = 1$$

$$2)\ \varphi_\lambda(x) = \delta_{x,y} \Rightarrow I_\lambda = \frac{V\cdot 1}{1^2} = V$$

Scalars

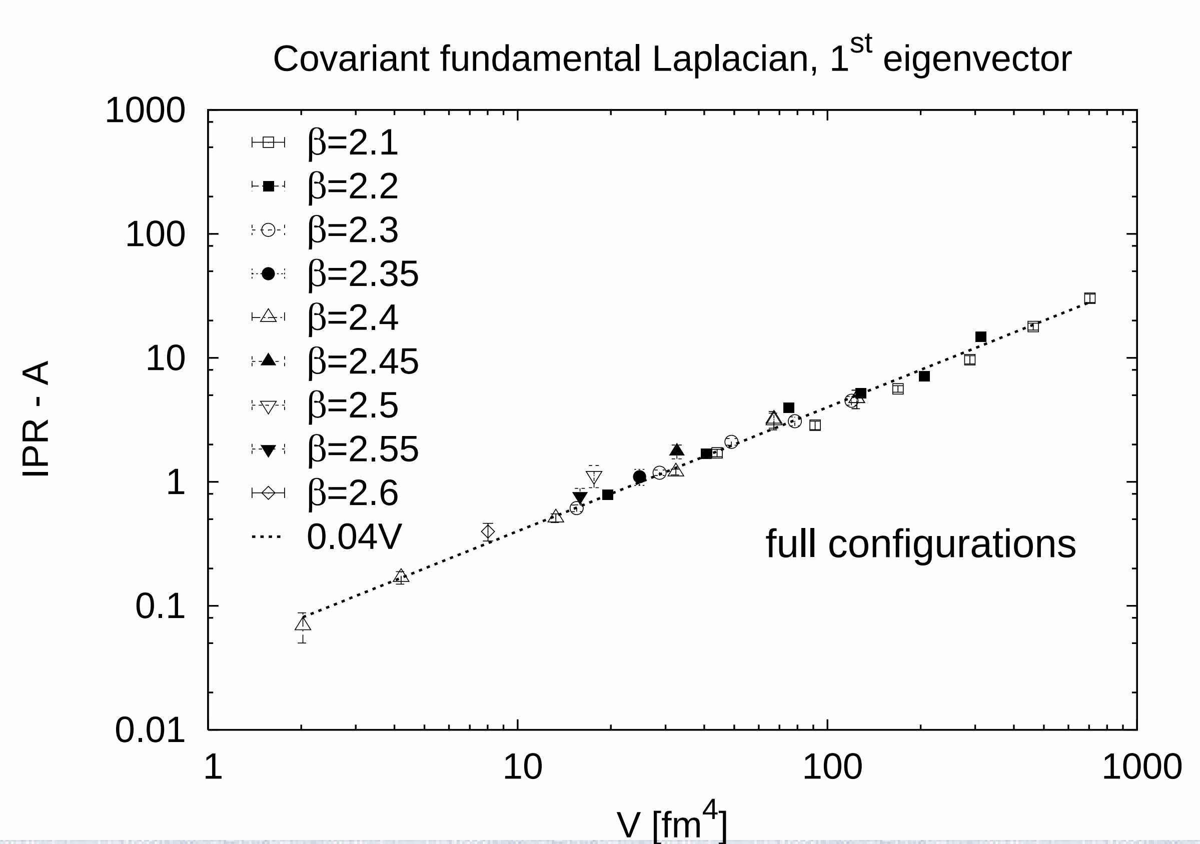
fundamental representation (j=1/2) SU(2) lattice gluodynamics

$$\boxed{\Delta(A)\varphi_\lambda(x) = \lambda\varphi_\lambda(x)}$$

gauge field A_μ is generated by MC on the lattice

Scalars (j=1/2)

$$V_{localization} \approx$$
$$\approx (2\, fm)^4$$



SCALING

$V_{localization}$ **is constant in physical units**

$$< I_\lambda >= C_0 + \frac{L^4 a^4}{b a^4} = C_0 + \frac{V_{phys}}{V_{localization}} =$$

$$= C_0 + C_1 a^{d-4}; \text{ from fit } d = 4$$

Removing center vortices we get zero string tension and zero quark condensate

P. de Forcrand and M. D'Elia, Phys.Rev.Lett. 82 (1999) 4582

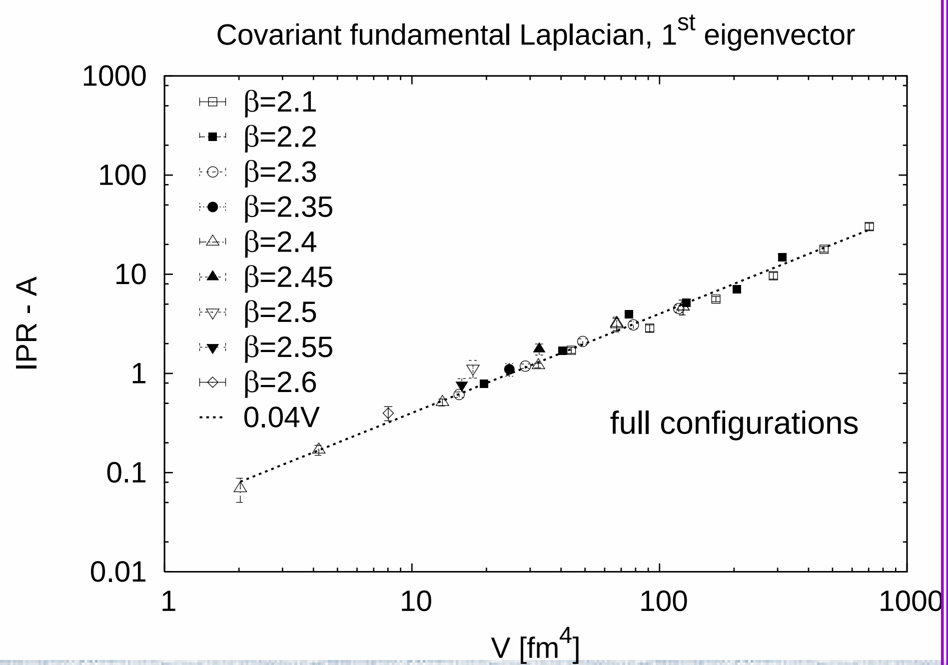
Z(2) gauge fixing

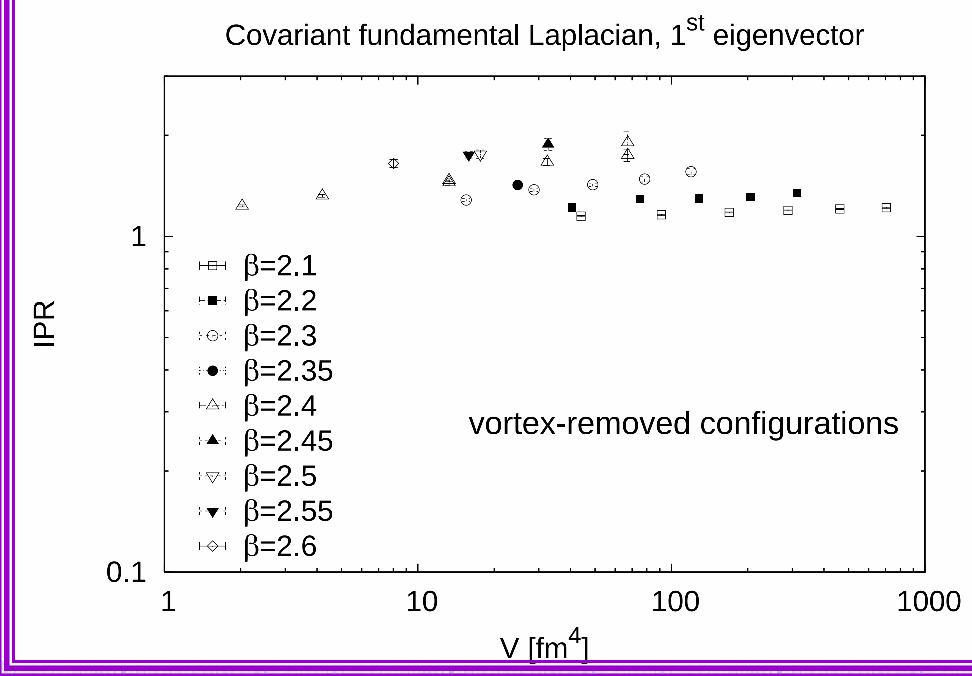
$$Z_{x\mu} = sign\, TrU_{x\mu}$$

$$\tilde{U}_{x\mu} = U_{x\mu} Z_{x\mu}$$

$$\sigma = 0; < \bar{\psi}\psi >= 0$$

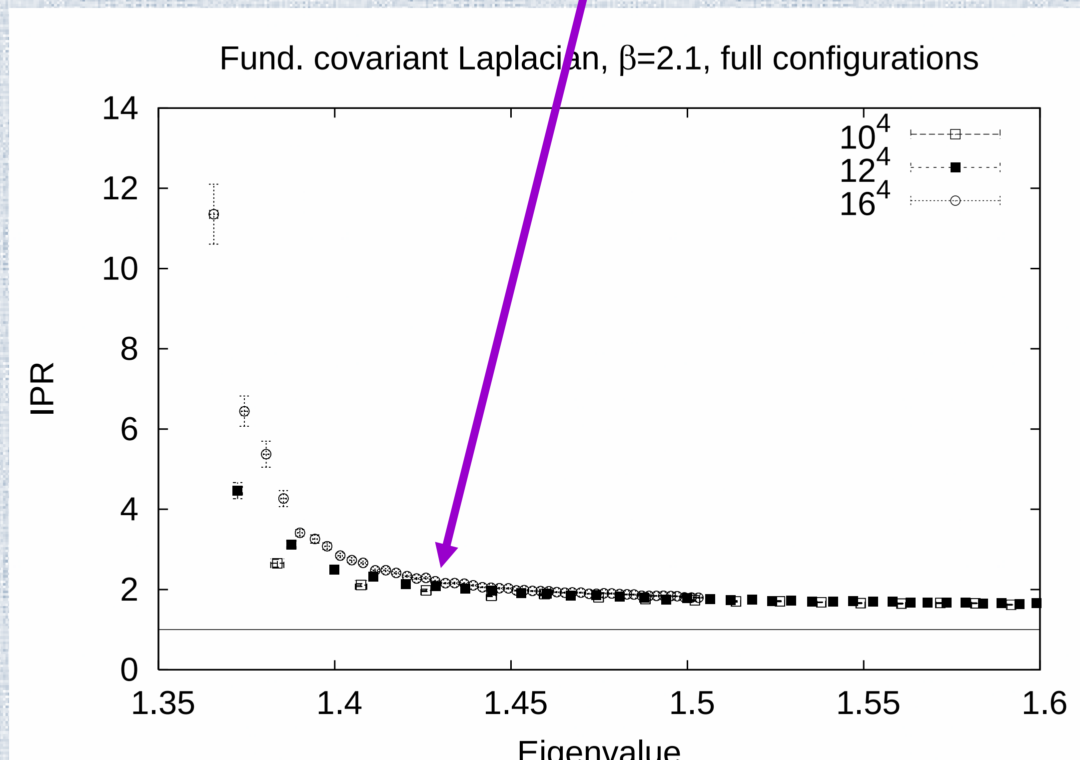
IPR for full and vortex removed gauge field configurations for SU(2) lattice gluodynamics (fundamental Laplasian, j=1/2)





Confinement is similar to Anderson localization?

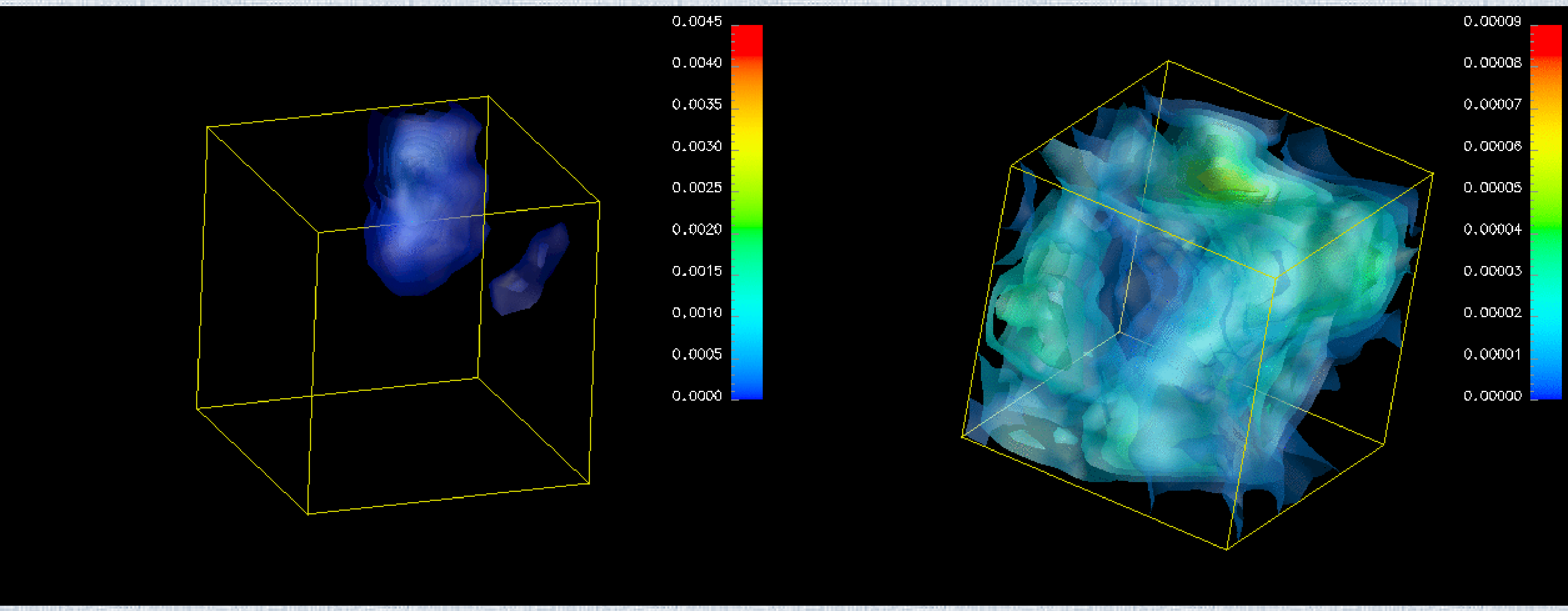
IPR for large enough ● is small
?mobility edge?



$$IPR \propto (\lambda_n - \lambda_{mobility\ edge})^{-\alpha}$$

Visualization of localization

Time slices, intensity of color is proportional to $\left|\varphi_\lambda(x)\right|^2$



IPR=52 **IPR=1.9**

Fermions

overlap Dirac operator, SU(2) gluodynamics

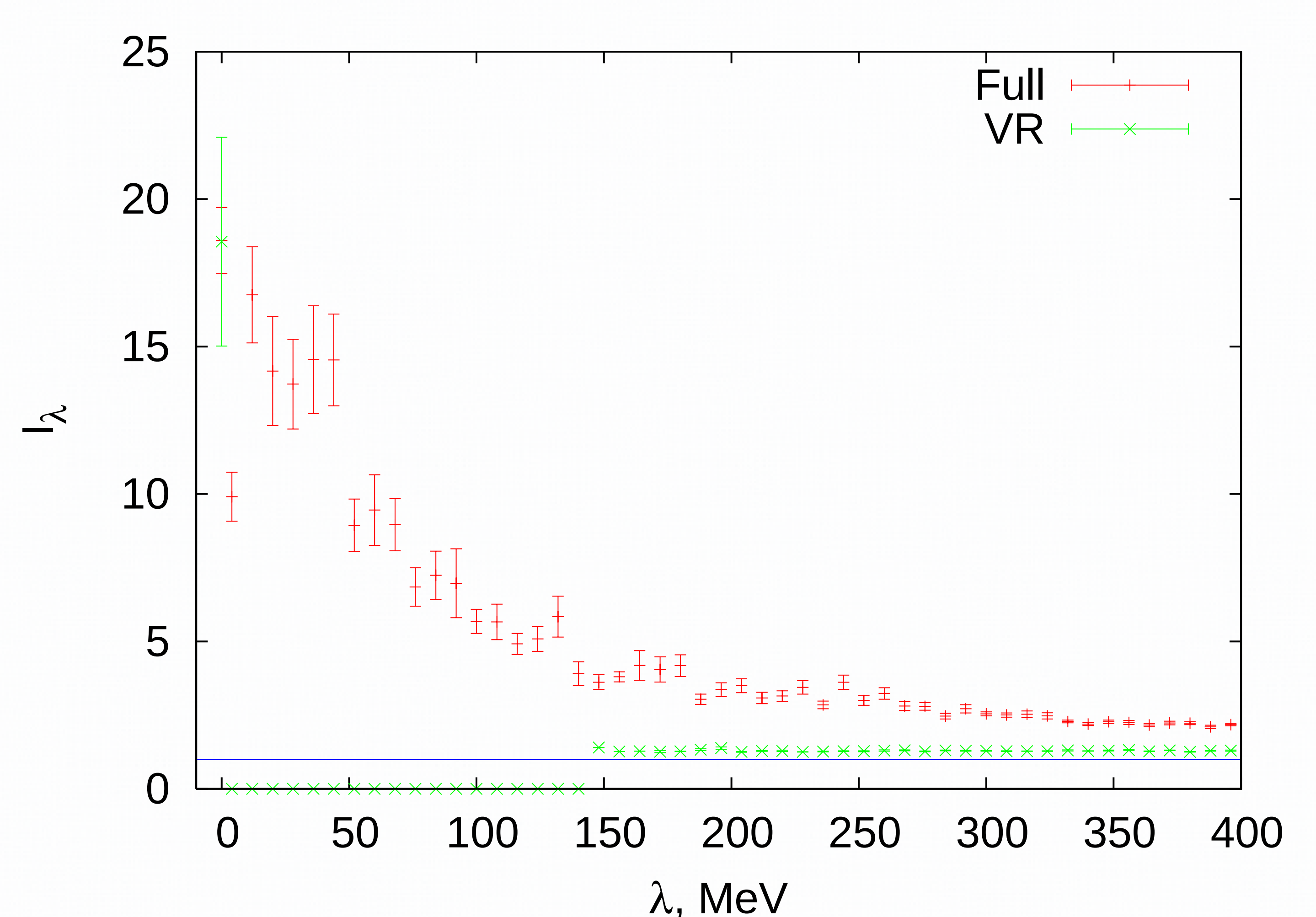
Overlap computer code was given to ITEP group by G. Schierholz and T. Streuer

$$\boxed{D(A)\psi_{\lambda}(x) = \lambda\psi_{\lambda}(x)}$$

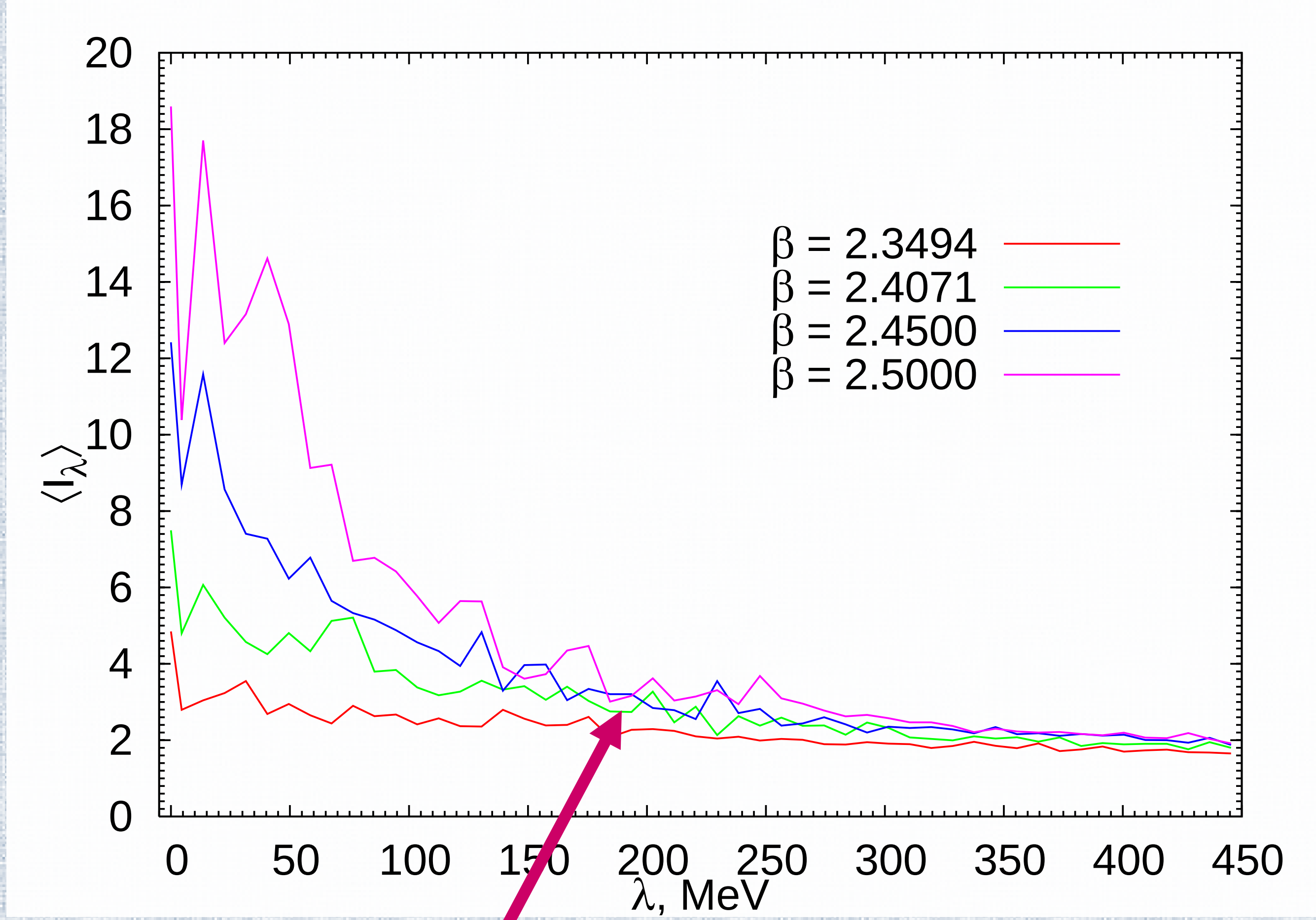
gauge field A_{μ} is generated by MC on the lattice

IPR for overlap lattice fermions before and after removing center vortices

Confinement and chiral condensate disappears after removing center vortices (P. de Forcrand and M. d'Ellia (1999); J. Gattnar et al. (2005), what happens with localization?



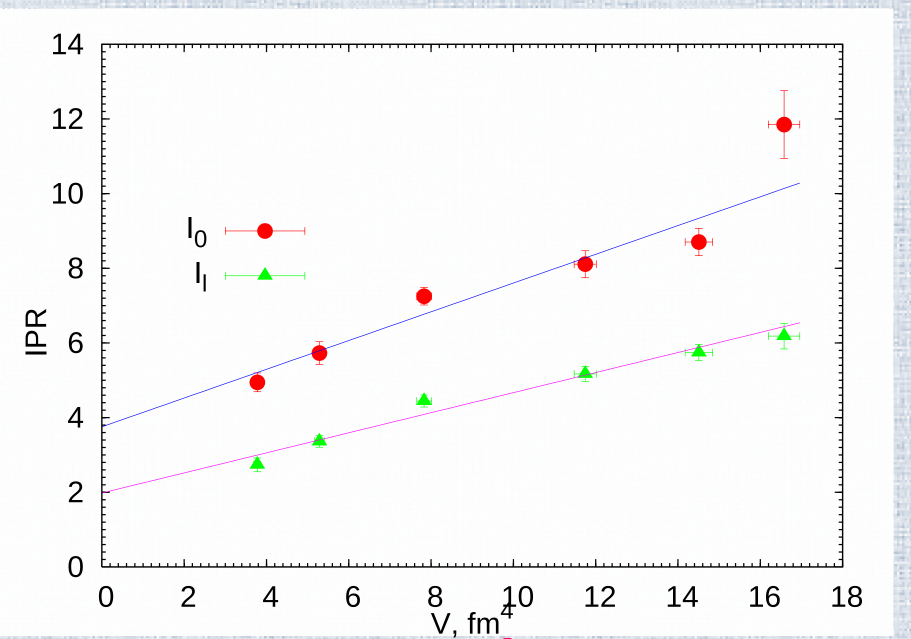
IPR for various lattice spacings



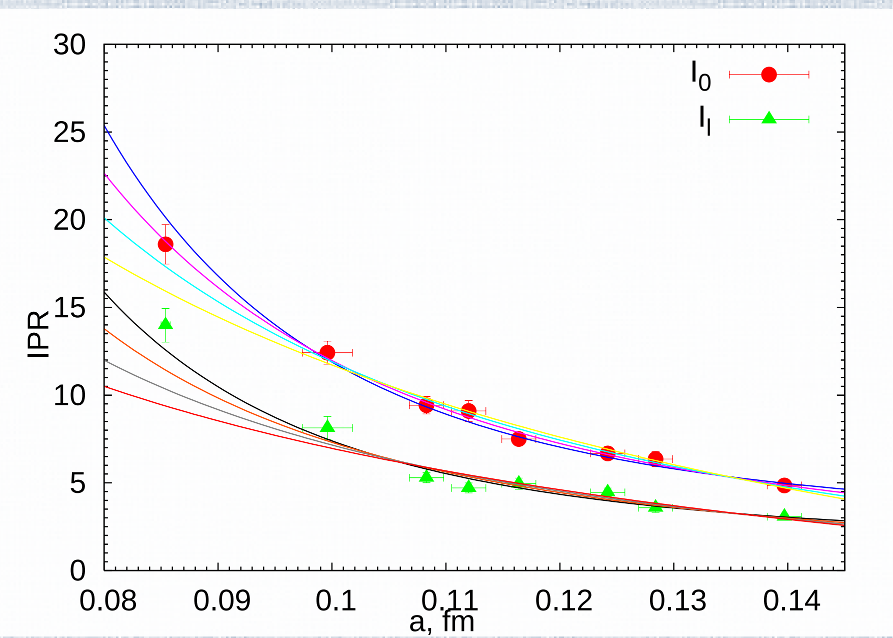
something happens at 150Mev<●<200Mev for all lattice spacings

$$I_\lambda \propto (\lambda_n - \lambda_{mobility\ edge})^{-\alpha}$$

Localization volume



IPR vs volume



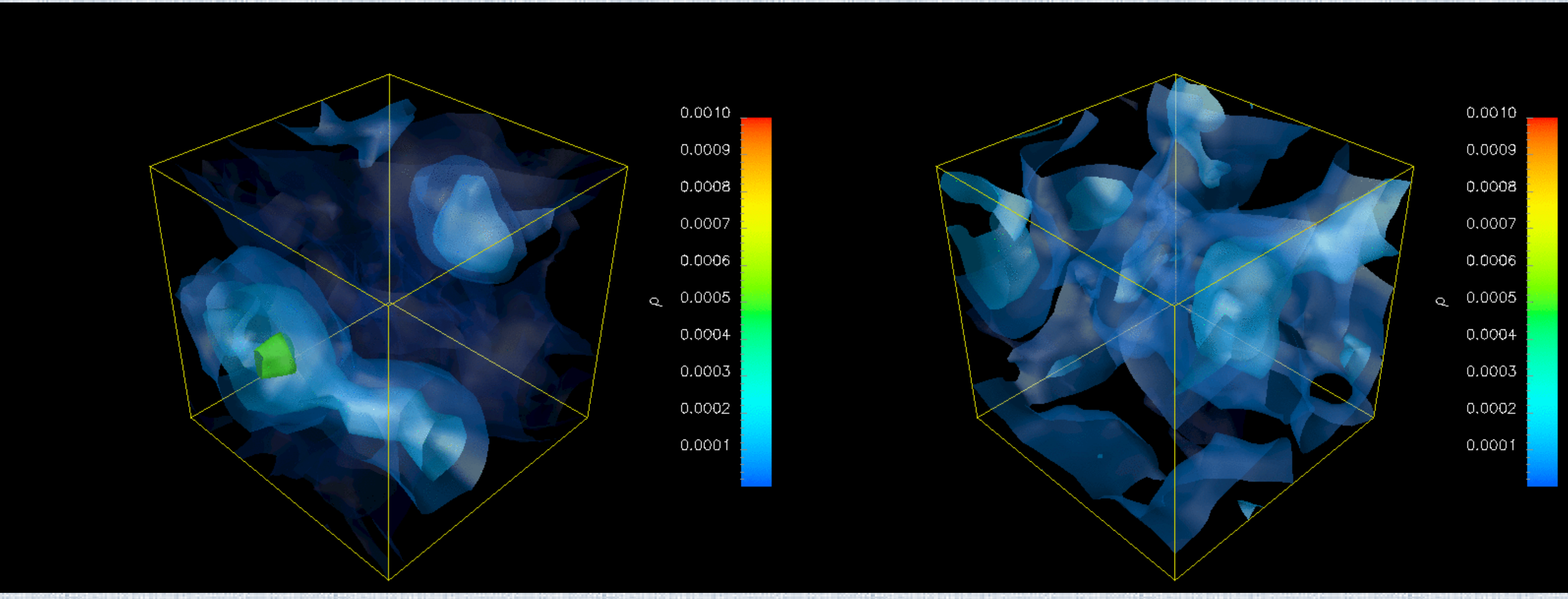
IPR vs lattice spacing, V=const

$$I_l = \frac{1}{N} \sum_{0<\lambda\leq 50\,Mev} < I_\lambda > \text{ IPR for low lying modes}$$

$< I_\lambda >= C_0 + C_1 a^{d-4}$, d is the dimensionalyty of localization volume

The best fit for I_l is $d = 0$, for I_0 $d = 1$

Time slices for $\rho^2,\ \rho_\lambda(x) = \psi_\lambda^+(x)\psi_\lambda(x)$



IPR=5.13
chirality=-1

IPR=1.45
chirality=0

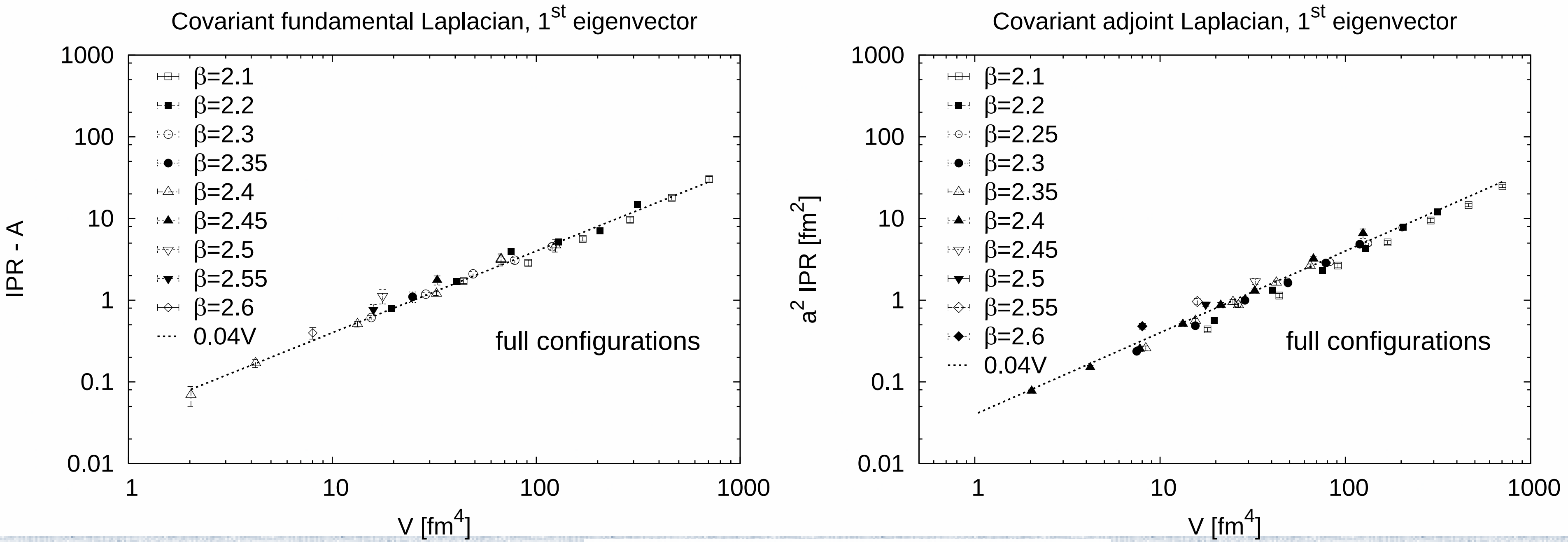
Localization properties of overlap fermions and scalars (fundamental representation, j=1/2) are qualitatively (not quantitatively) similar

Scalars

adjoint (j=1) and j=3/2 representation SU(2) lattice gluodynamics

$$\Delta(A)\varphi_{\lambda}(x) = \lambda\varphi_{\lambda}(x)$$

gauge field A_{μ} is generated by MC on the lattice

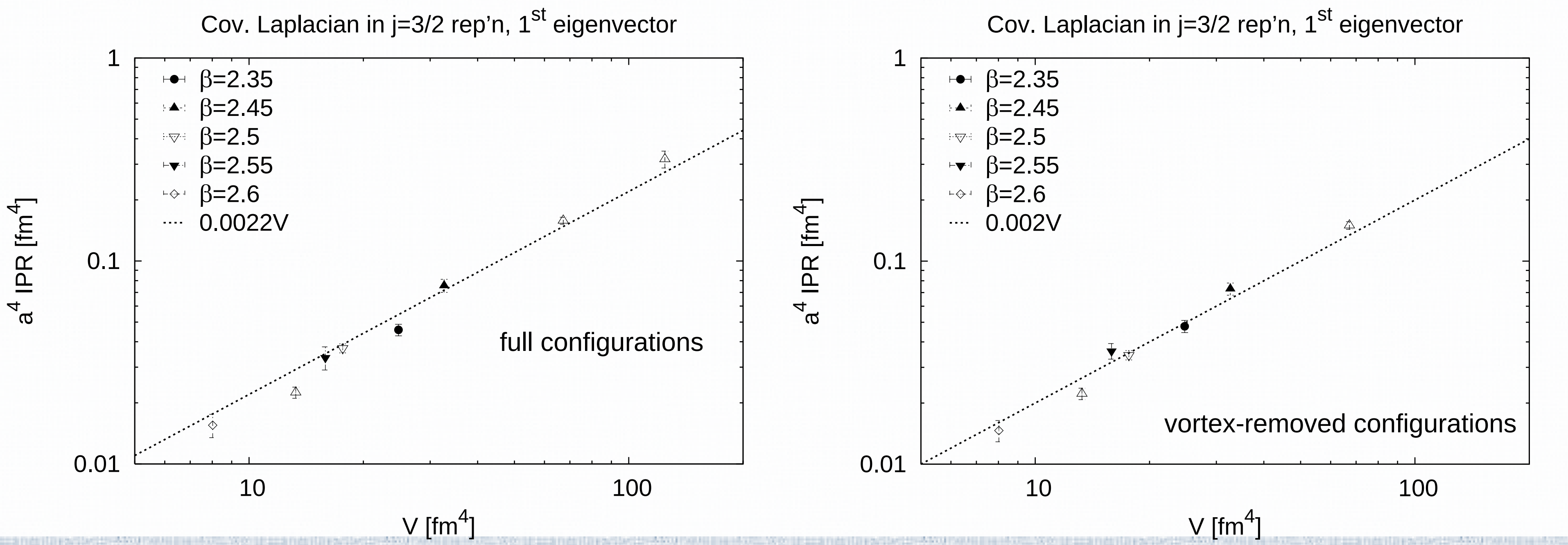


fundamental $$< I_\lambda >= C_0 + C_1 a^{d-4}$$ **adjoint**

Adjoint localization volume d=2? No!

Additional analysis (hep-lat/0504008) show that adjoint localization volume is 4d, but shrinks to zero in the continuum limit

j=3/2 representation



$$< I_\lambda > = C_0 + C_1 a^{d-4}$$

$$d = 0\,?$$

Or very strong localization (d=0) or localization volume very fast shrinks to zero when a ♦ 0

Low dimensional structures in lattice gluodynamics by other groups

1. I. Horvath et al. hep-lat/0410046, hep-lat/0308029, Phys.Rev. D68:114505,2003
2. MILC collaboration hep-lat/0410024
3. Y. Koma, E.-M. Ilgenfritz, K. Koller, G. Schierholz, T. Streuer, V. Weinberg hep-lat/0509164

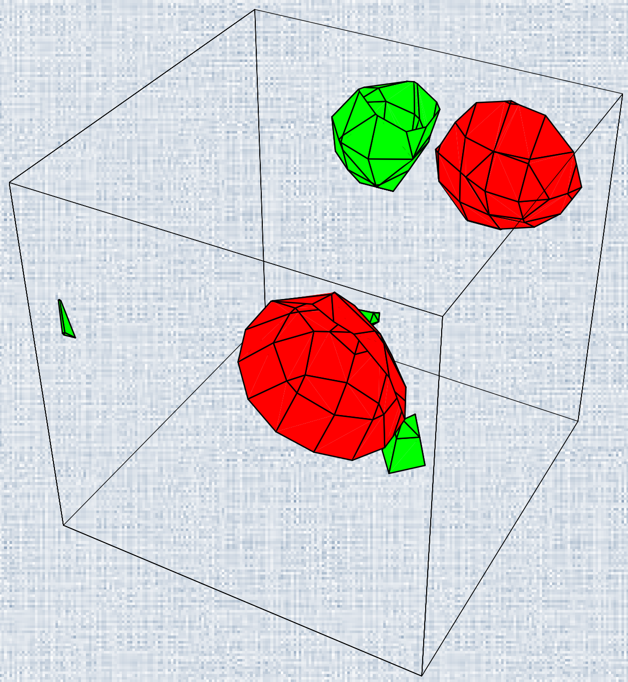
Summary

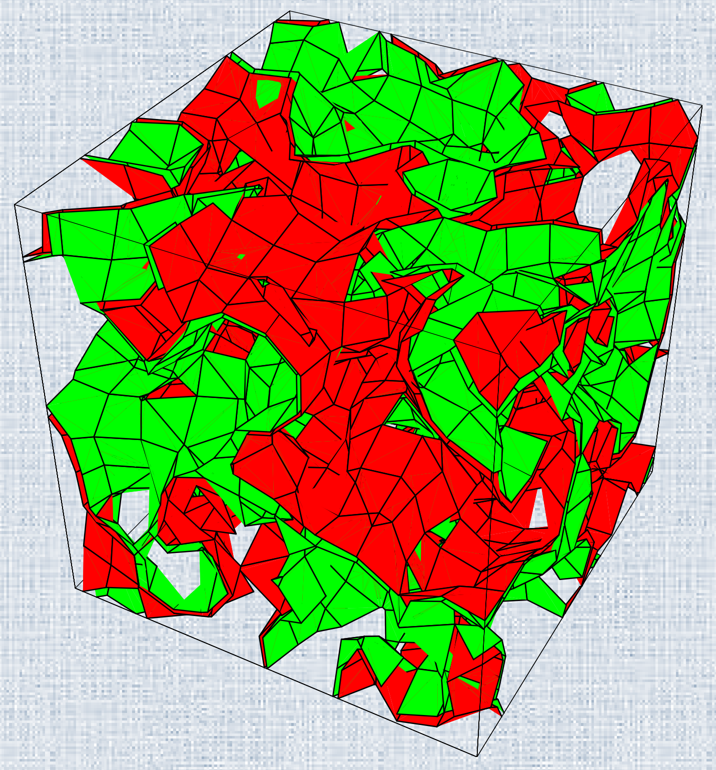
0. Analogue of the Anderson localization in quantum field theory is observed.

1. Localization of eigenfunctions of the Laplacian and Dirac operator is a manifestation of the possible existence of low dimensional objects in the QCD vacuum. Dependence of results on lattice spacing is clearly seen.

2. The density of the states is in physical units, while the localization volume of the modes tends to zero in physical units hep-lat/0505016 "Fine tuning phenomenon" and "Holography".

Lowdimensional objects Instanton vacuum -> ->3d submanyfolds





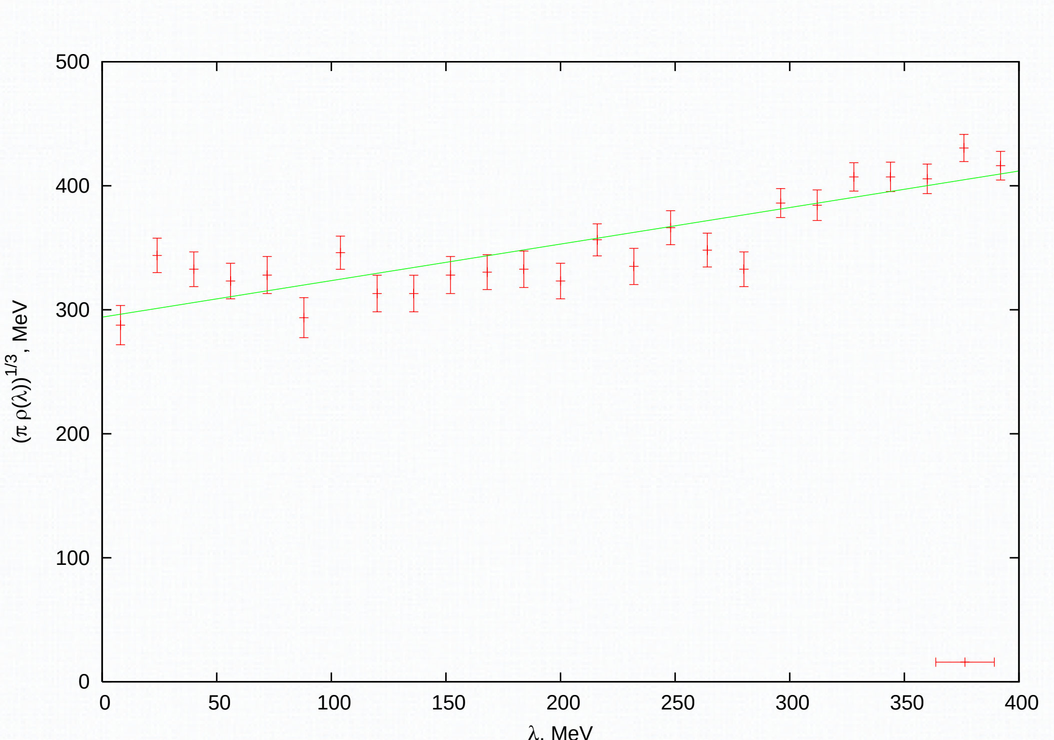
Dyakonov, Petrov, Shuriak

Y. Koma, E.-M. Ilgenfritz, K. Koller, G. Schierholz, T. Streuer, V. Weinberg hep-lat/0509164

Fine tuning

$$\text{Size of } |\psi|^2 \text{ is} \propto a = \frac{1}{\Lambda_{UV}}$$

$$\text{Energy of } \psi \text{ is} \propto \Lambda_{\mathrm{QCD}}$$



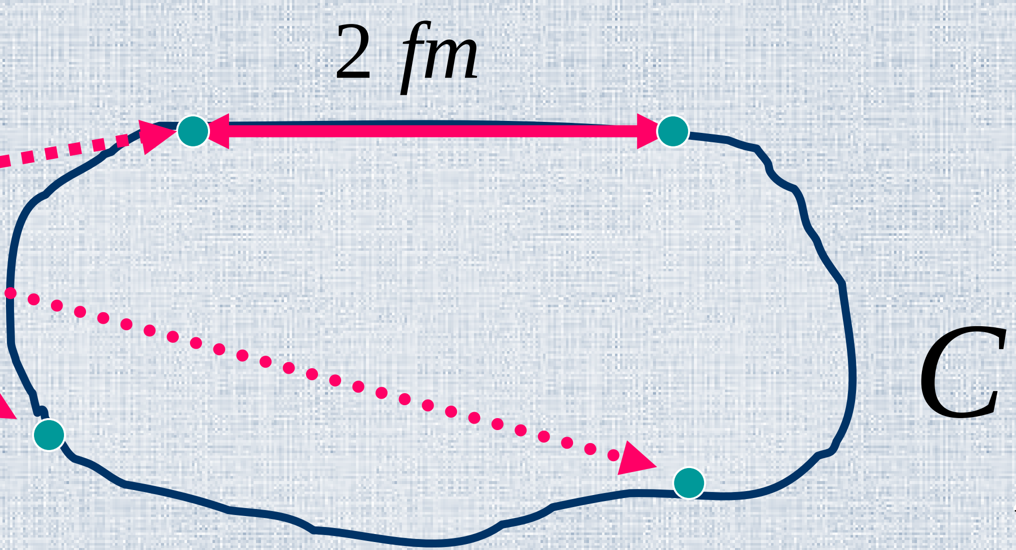
$$<\bar{\psi}\psi> = -\pi\rho(\lambda_n \to 0)$$

Banks-Casher (1980)

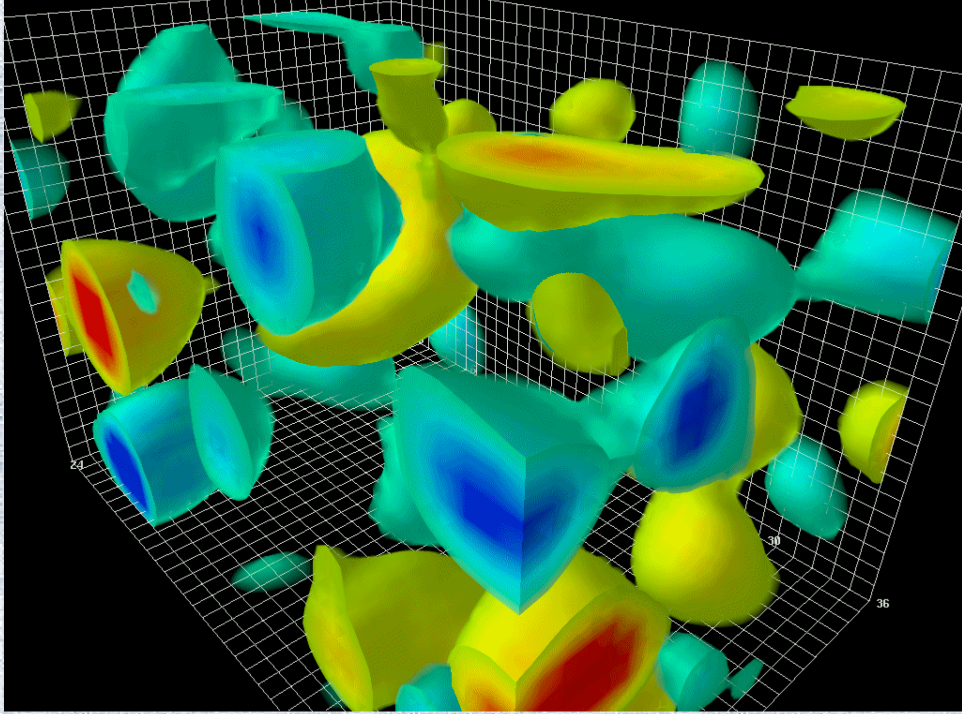
All information about confinement, quark condensate and any Wilson loop is encoded in 3d branes

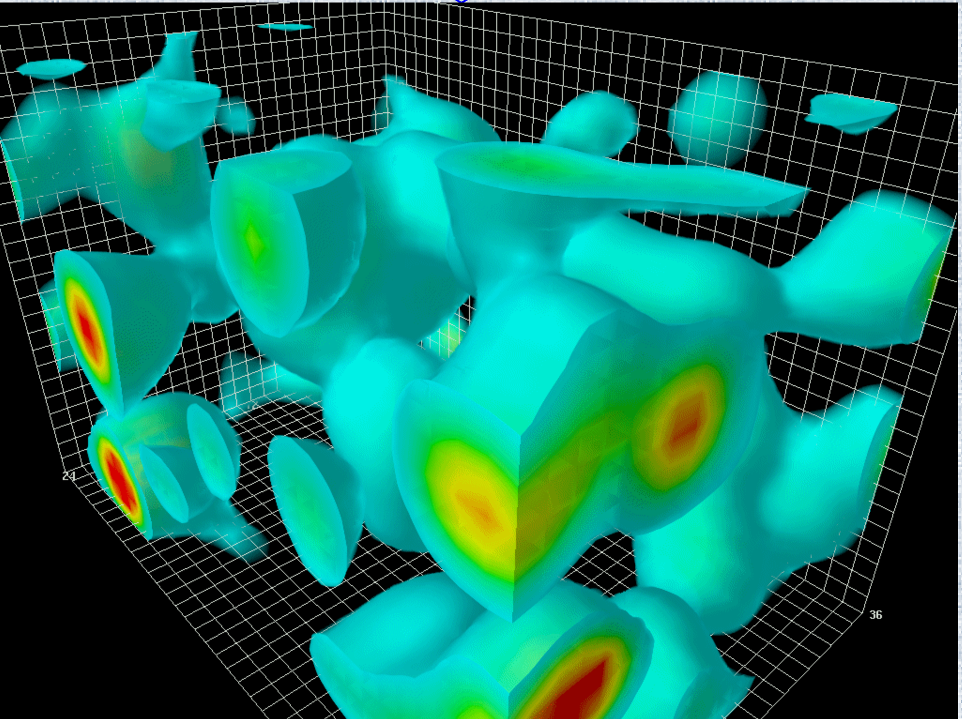
$$\boxed{\textbf{Holography}}$$

$$A_\mu \cong \frac{c}{a}$$



Action



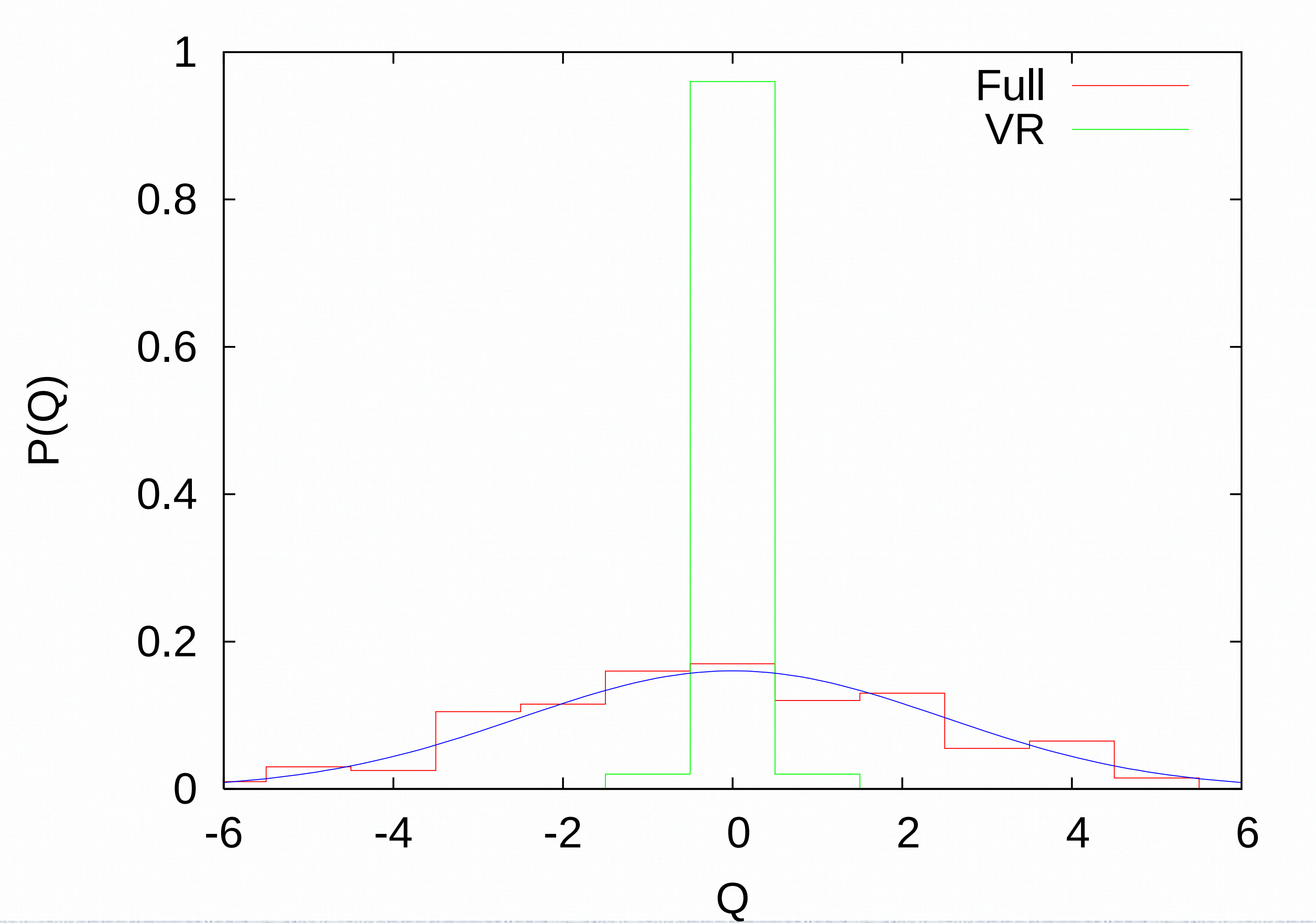


Topological charge density

Appendix

Removing center vortices

Confinement and chiral condensate disappears after removing center vortices (P. de Forcrand and M. d'Ellia (1999); J. Gattnar et al. (2005),
what happens with localization?

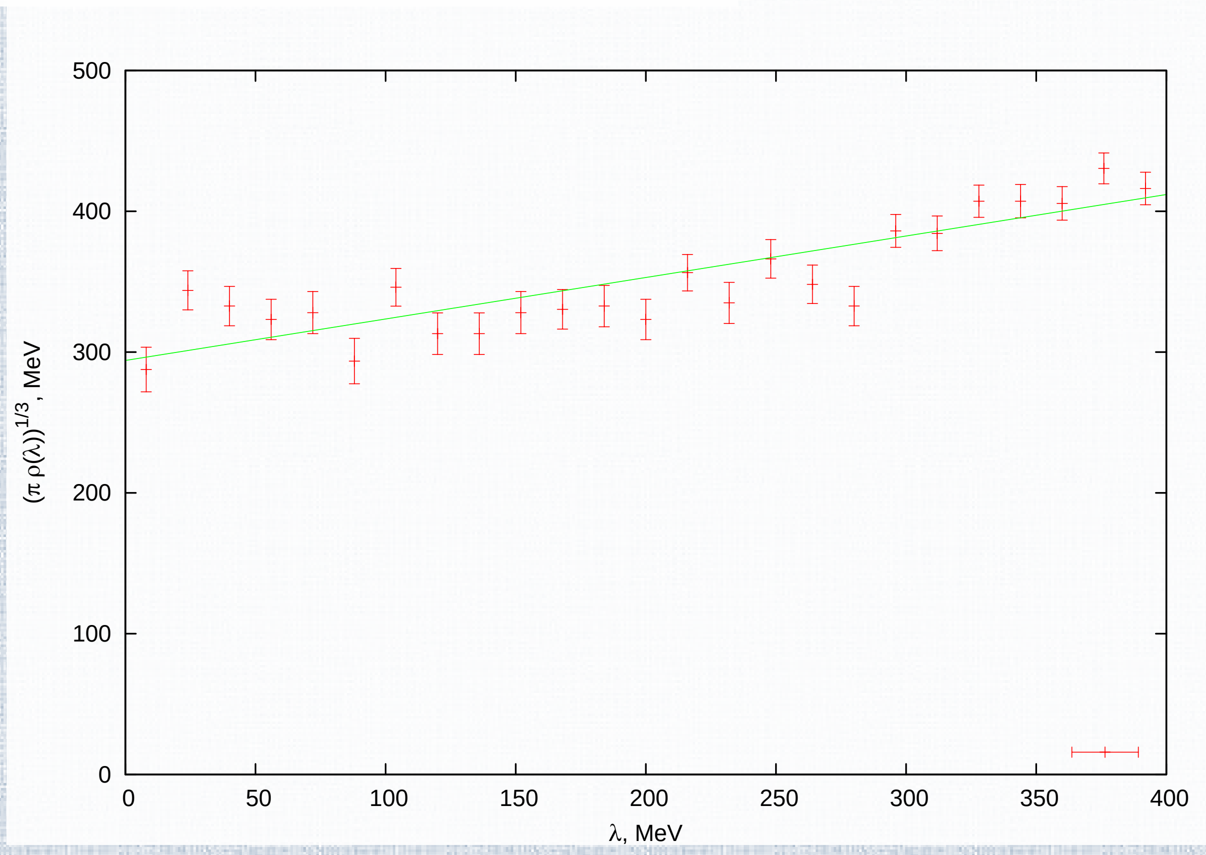


Topological charge disappears

Quark condensate

$$<\bar{\psi}\psi> = -\pi\rho(\lambda_n \to 0)$$

Banks-Casher (1980)



Result is in agreement with S.J.Hands and M.Teper (1990), (Wilson fermions)

The action of monopoles and center vortices **is singular**, they exist due to energy-entropy balance (the entropy of lines and surfaces is also singular)

$$\boxed{\frac{1}{a} = \Lambda_{UV}}$$

Monopoles

$$L_{mon} \approx 31 \frac{V_4}{fm^3}, \qquad S_{mon} \approx 1.9 \frac{L_{perc}}{a}$$

Center vortices

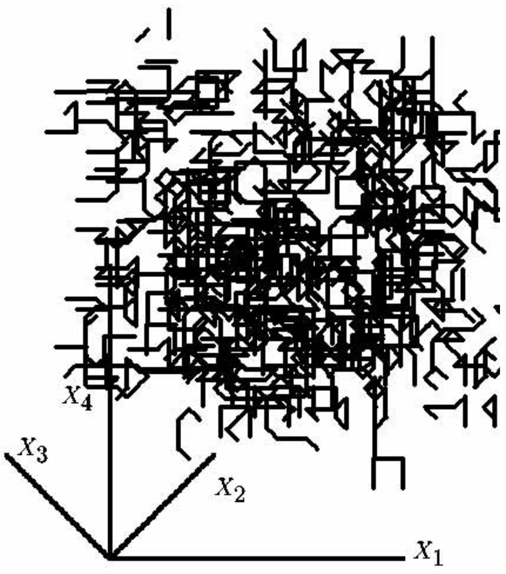
$$A_{vort} \approx 24 \frac{V_4}{fm^2}, \qquad S_{vort} \approx 0.53 \frac{A_{vort}}{a^2}$$

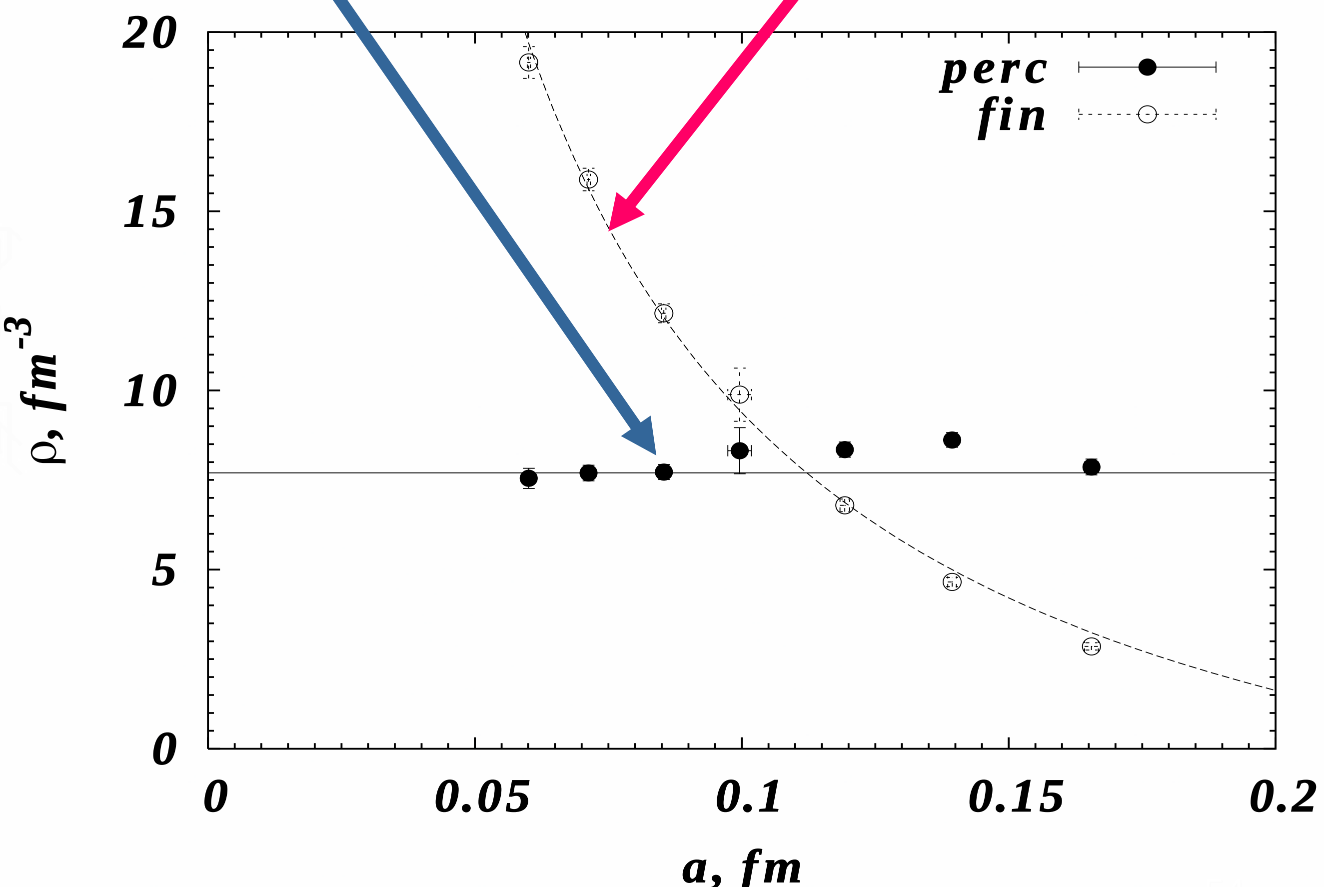
3D volumes

$$V_{3d} \approx 2 \frac{V_4}{fm}, \qquad S_{3d} \approx 0$$

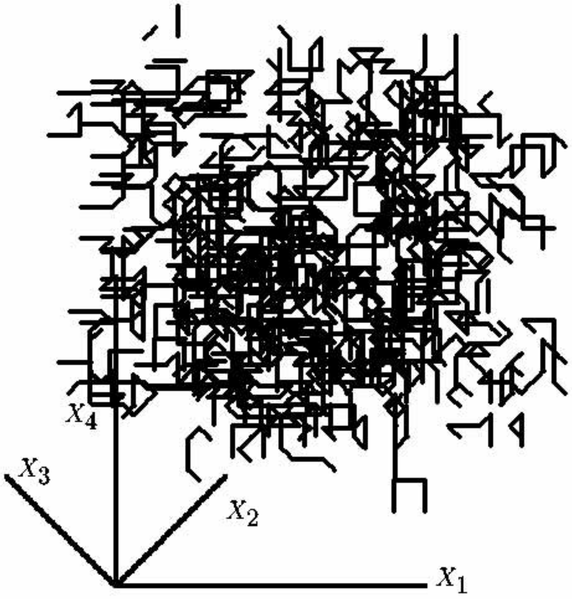
Length of IR monopole cluster

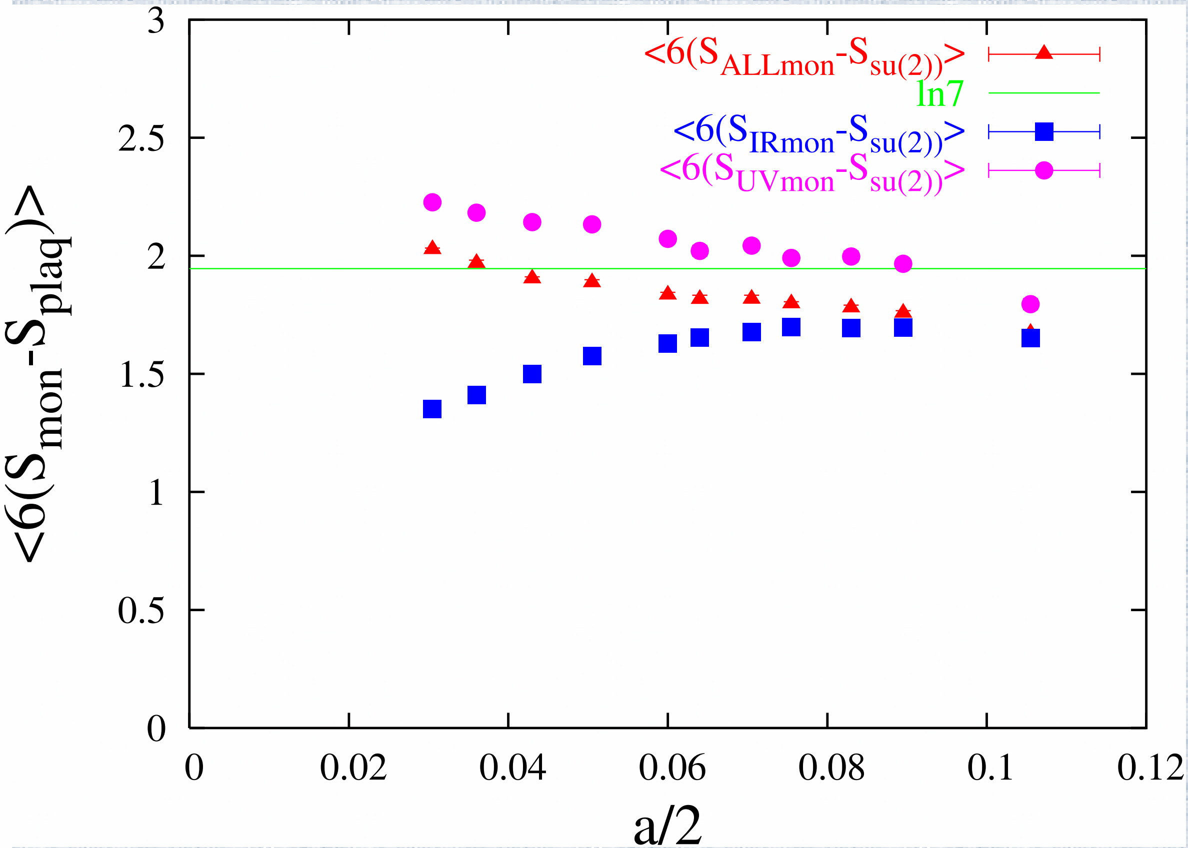
scales, $\rho_{IR} = \frac{L}{V_4}$ $\rho_{UV} = \frac{const}{a}$



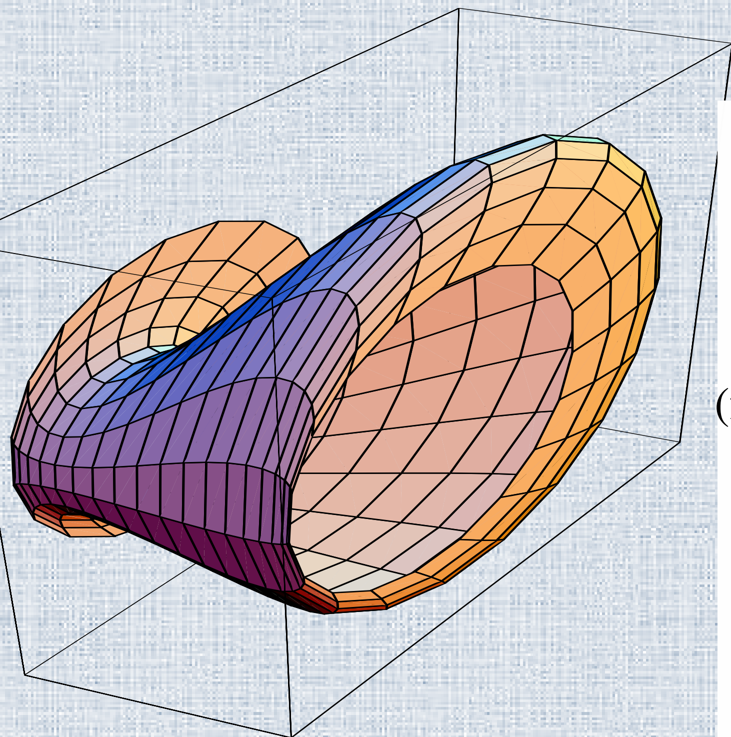


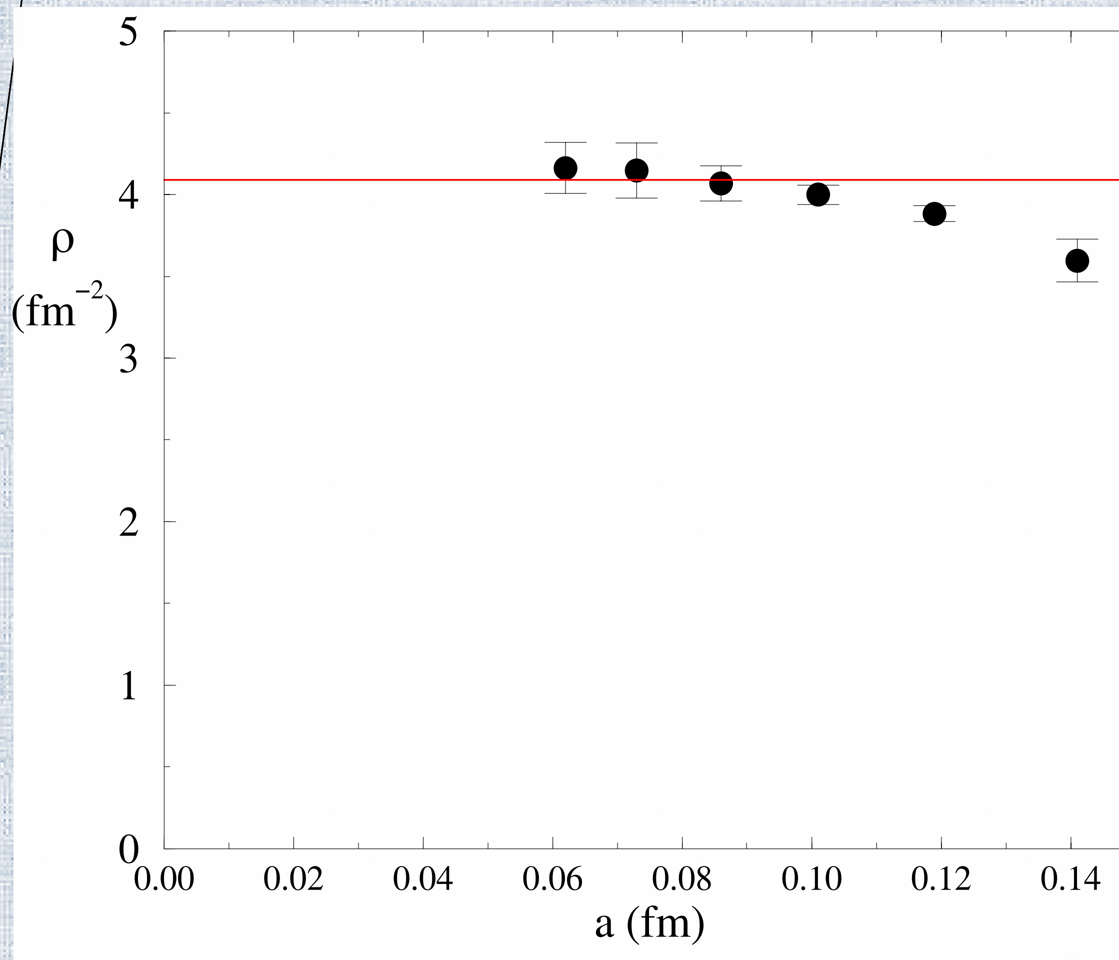
Monopoles have fine tuned action density:



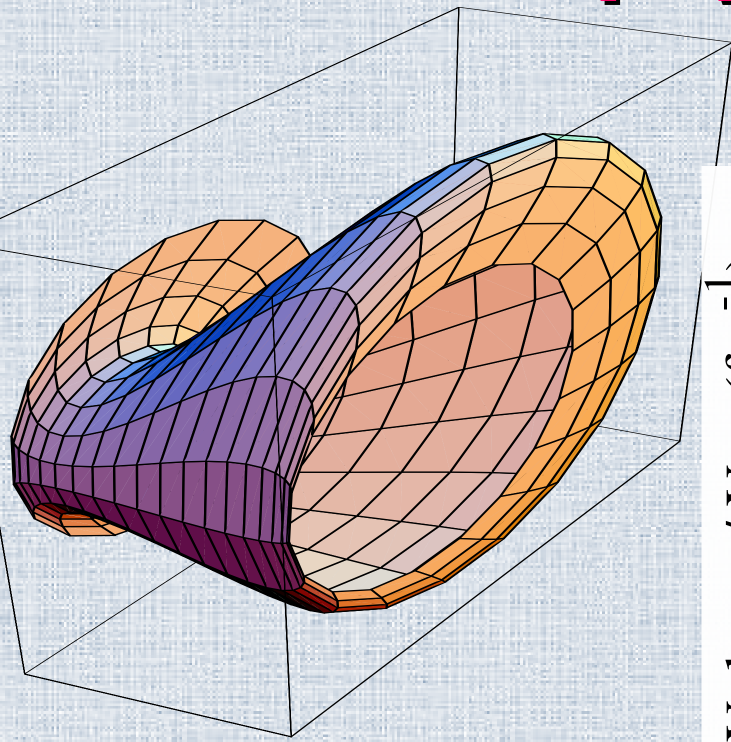


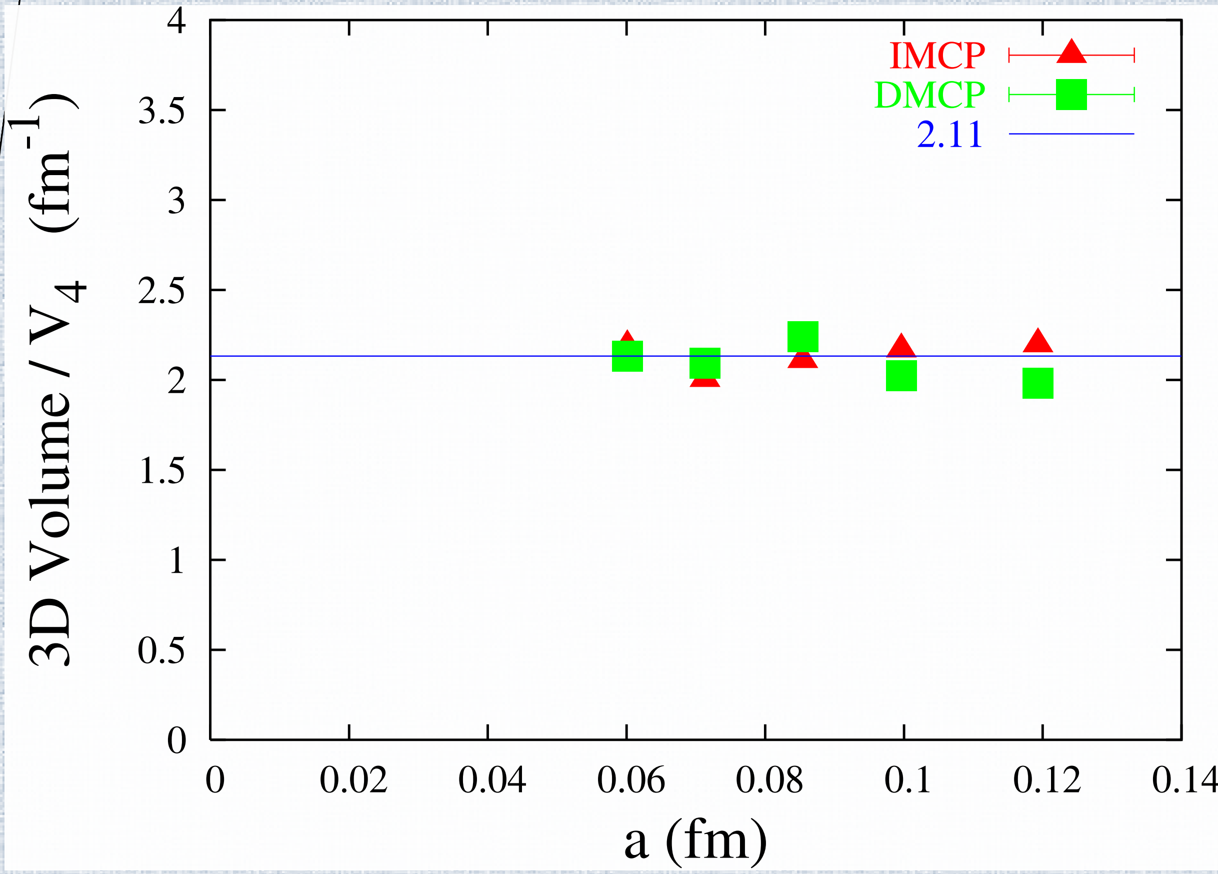
P-VORTEX density, Area / (6*V_4), scales:





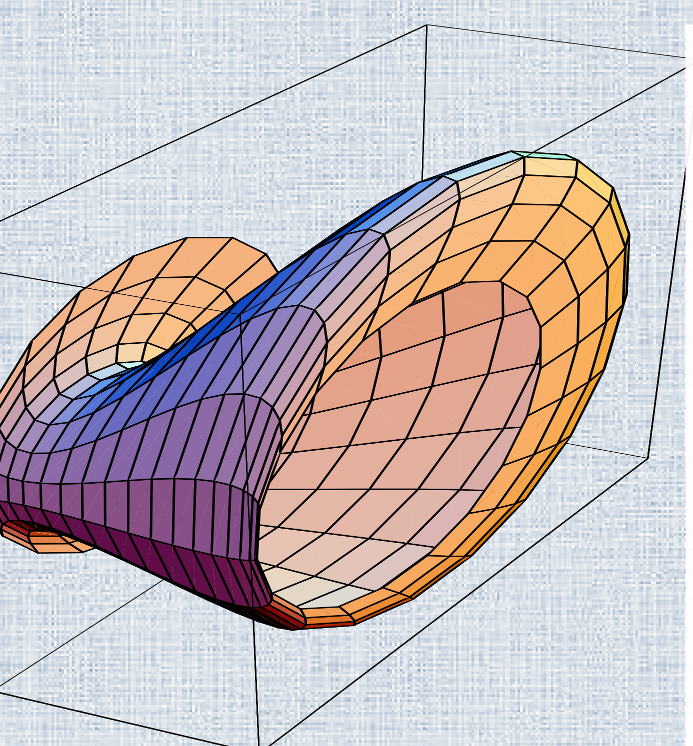
Minimal 3D Volumes bounded by P-vortices scale

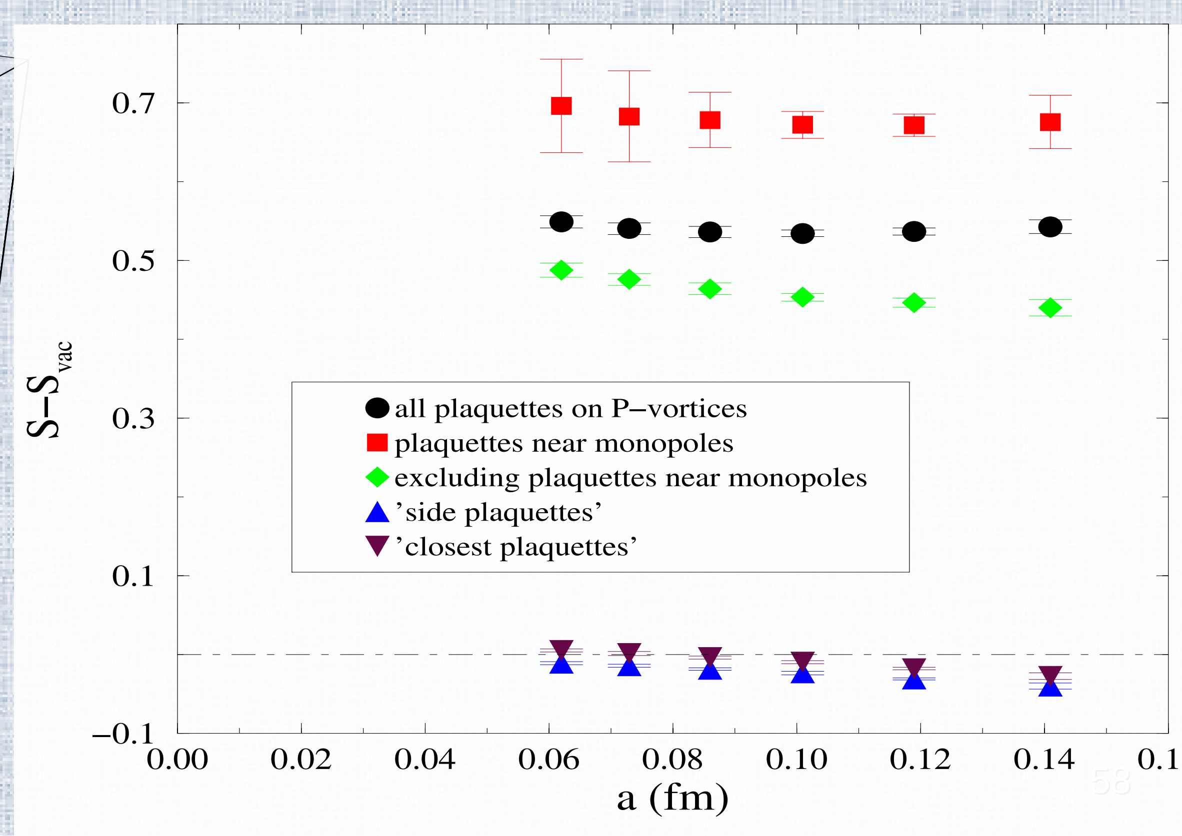




P-VORTEX has UV divergent action density: $(S\text{-}S_{vac})=Const./$♋^{2}

In lattice units





Monopoles belong to surfaces (center vortices).

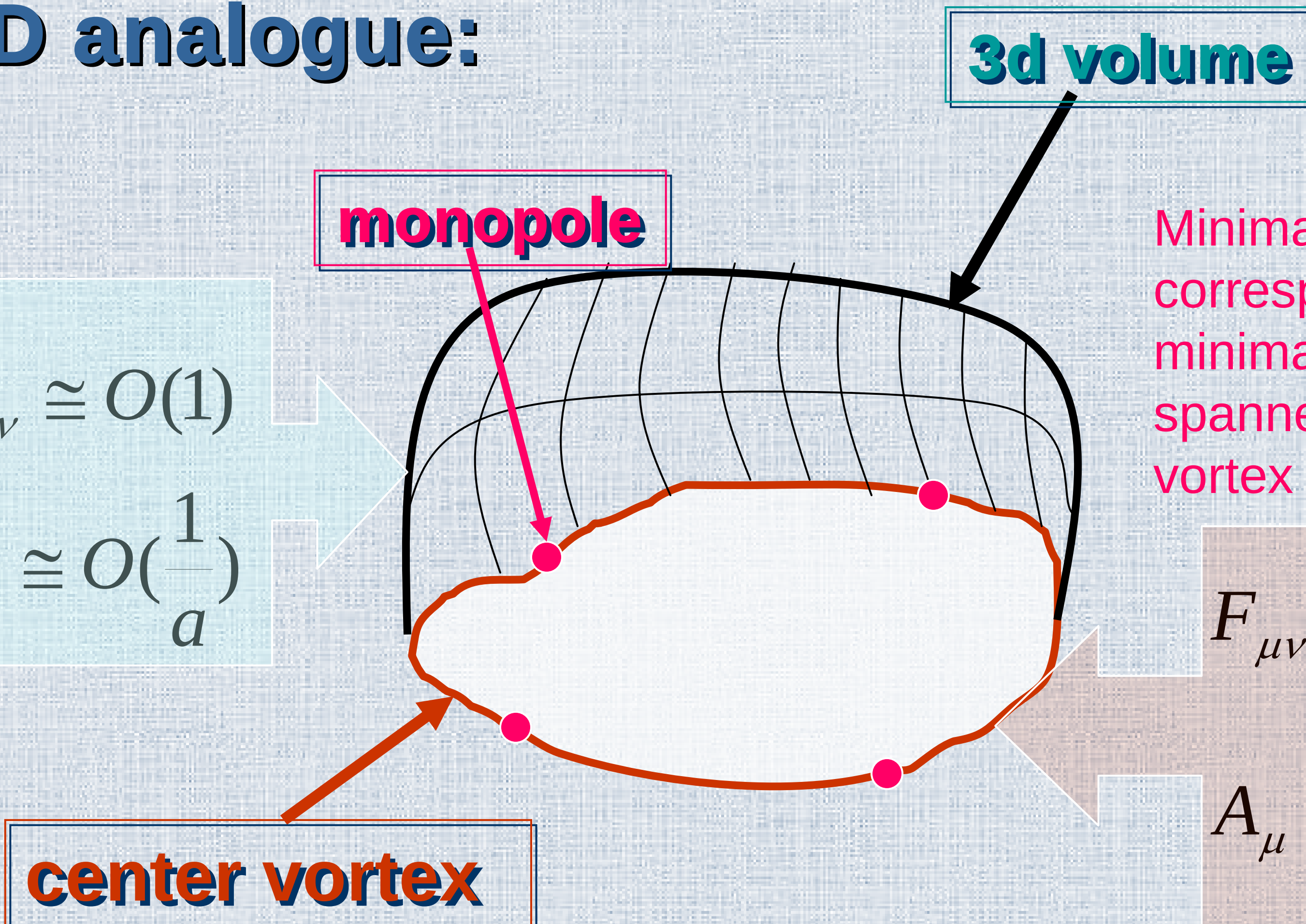
Surfaces are bounds of **minimal** 3d volumes

in Z(2) Landau gauge

3D analogue:

$$F_{\mu\nu} \cong O(1)$$

$$A_\mu \cong O(\frac{1}{a})$$



Minimal 3d volume corresponds to minimal surface spanned on center vortex

$$F_{\mu\nu} \cong O(\frac{1}{a^2})$$

$$A_\mu \cong O(\frac{1}{a})$$

Workshop on Computational Hadron Physics,
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Localization of the scalar and fermionic eigenmodes and confinement

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hep-lat/0505016, hep-lat/0504008